Dynamic Connectivity for Unit Disk Graphs

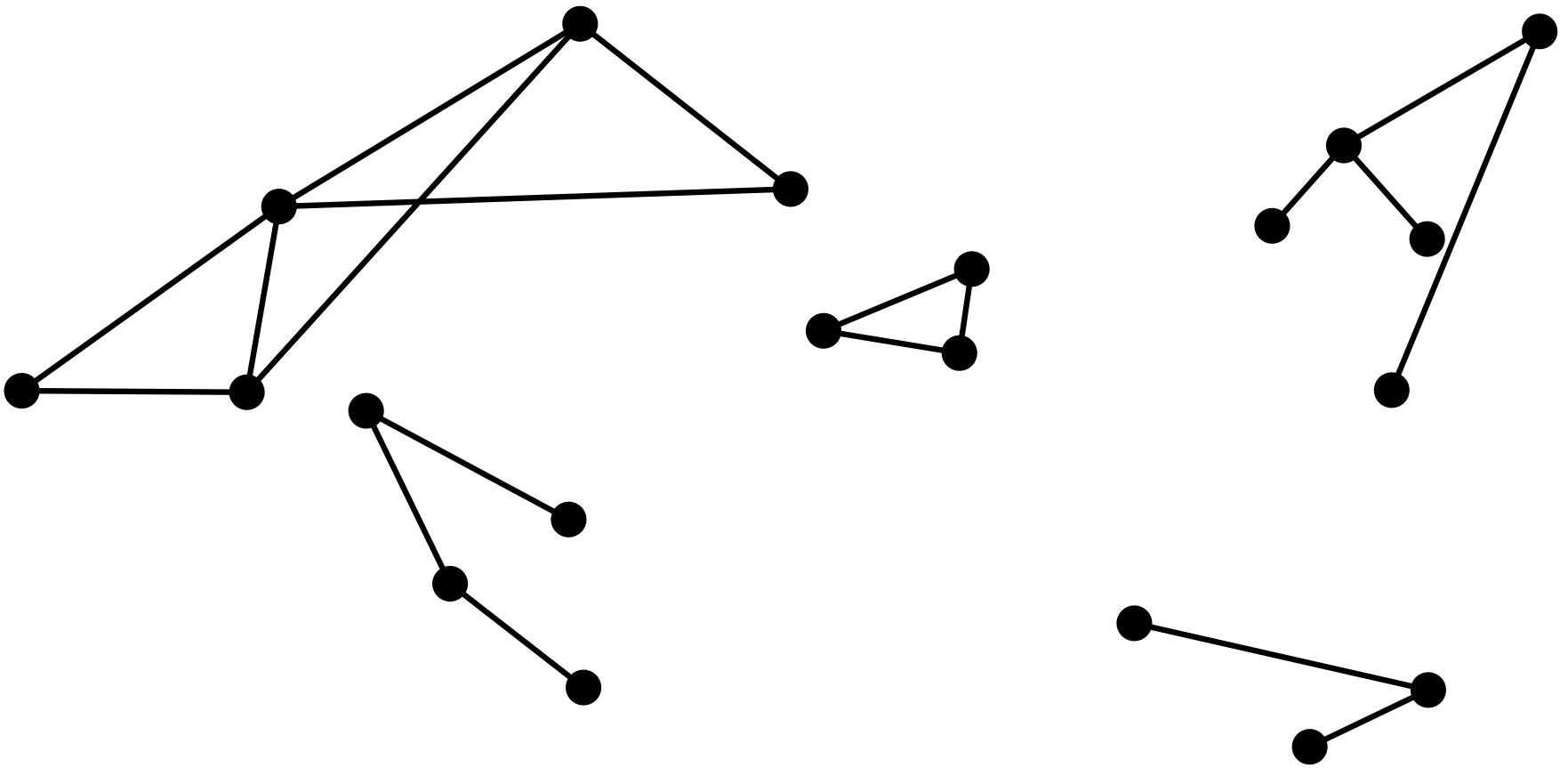
Haim Kaplan
Tel Aviv University
Tel Aviv, Israel

Wolfgang Mulzer
Freie Universität
Berlin, Germany

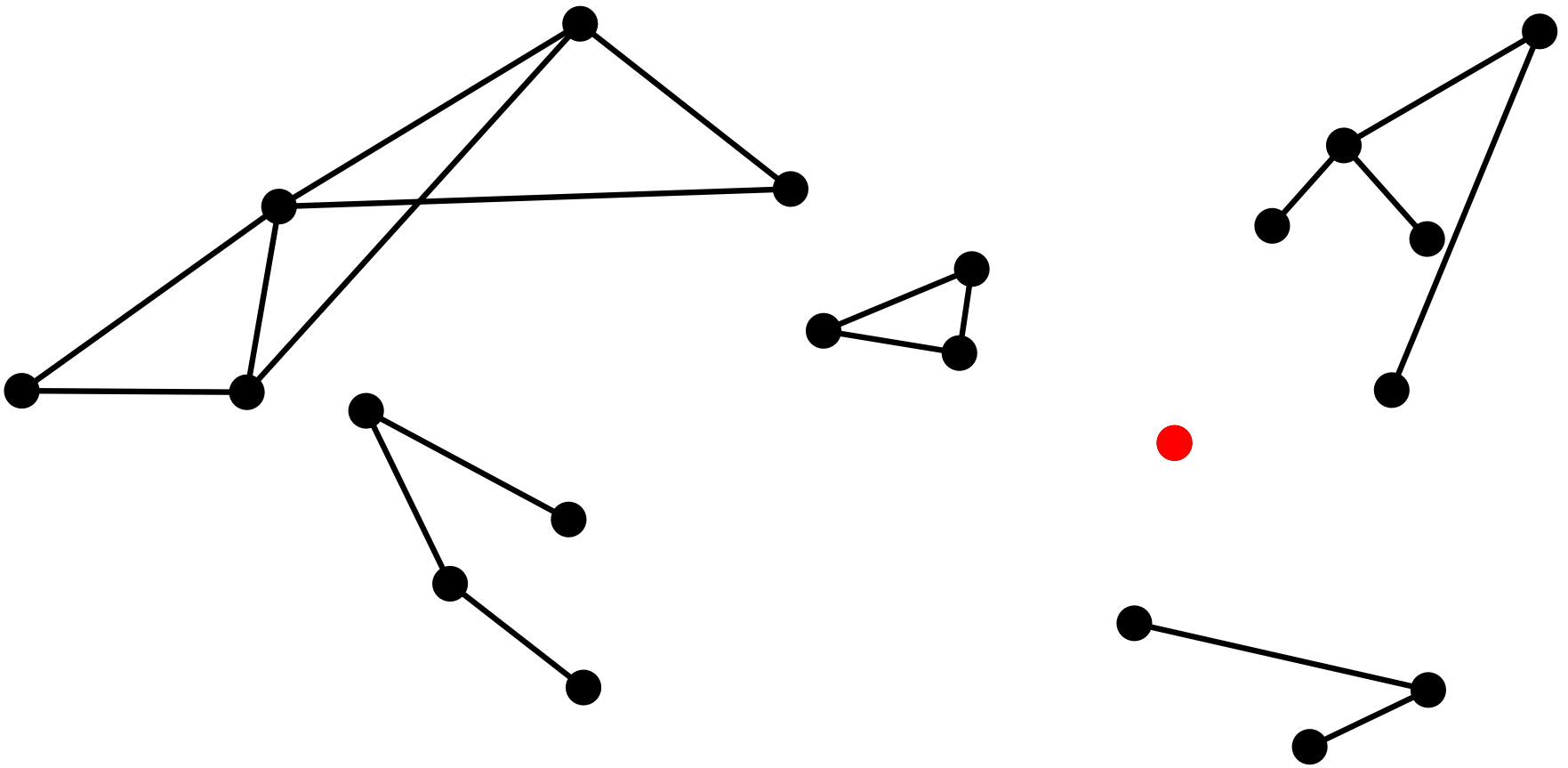
Liam Roditty
Bar-Ilan University
Tel Aviv, Israel

Paul Seiferth
Freie Universität
Berlin, Germany

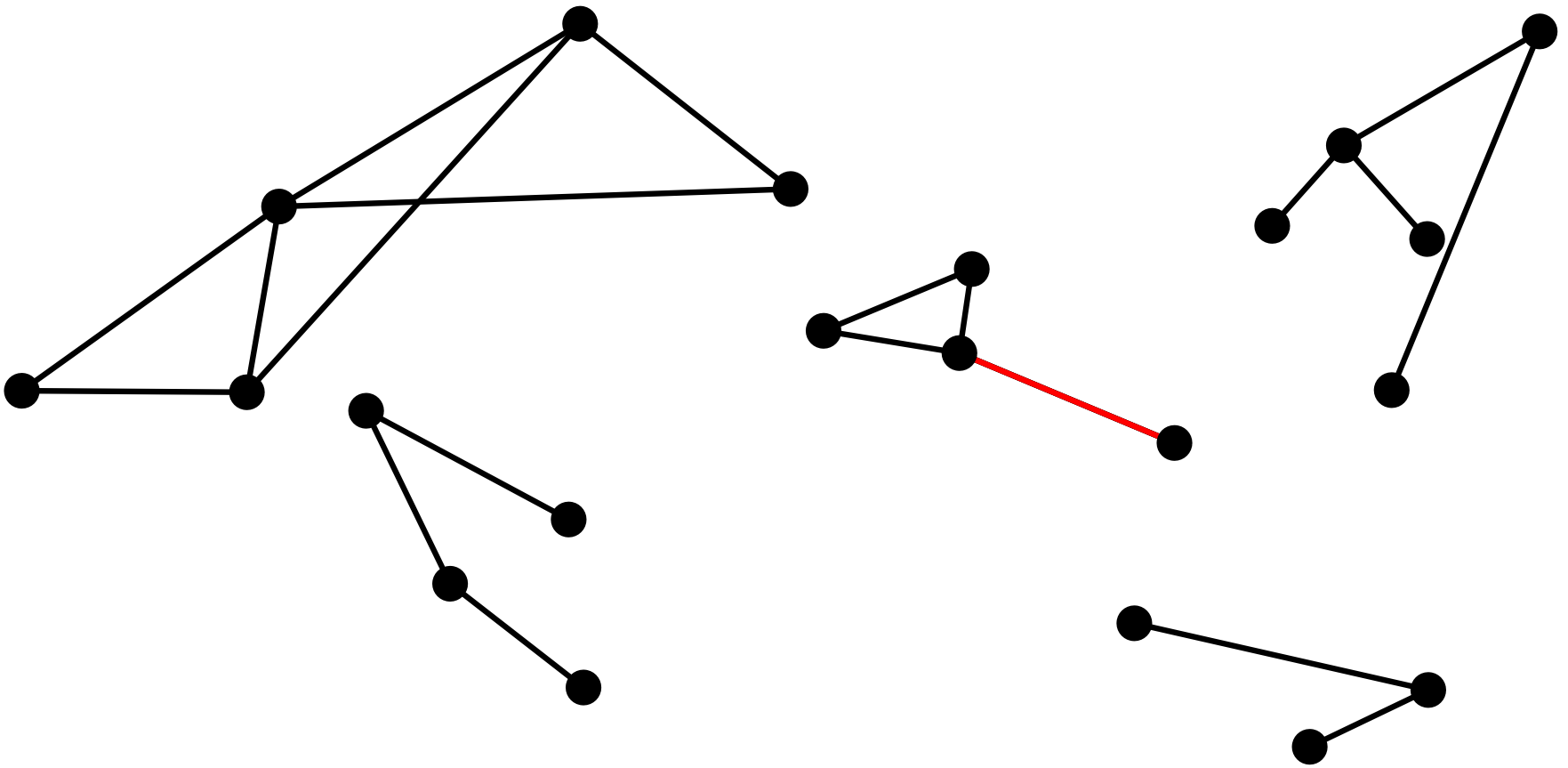
The Problem in General



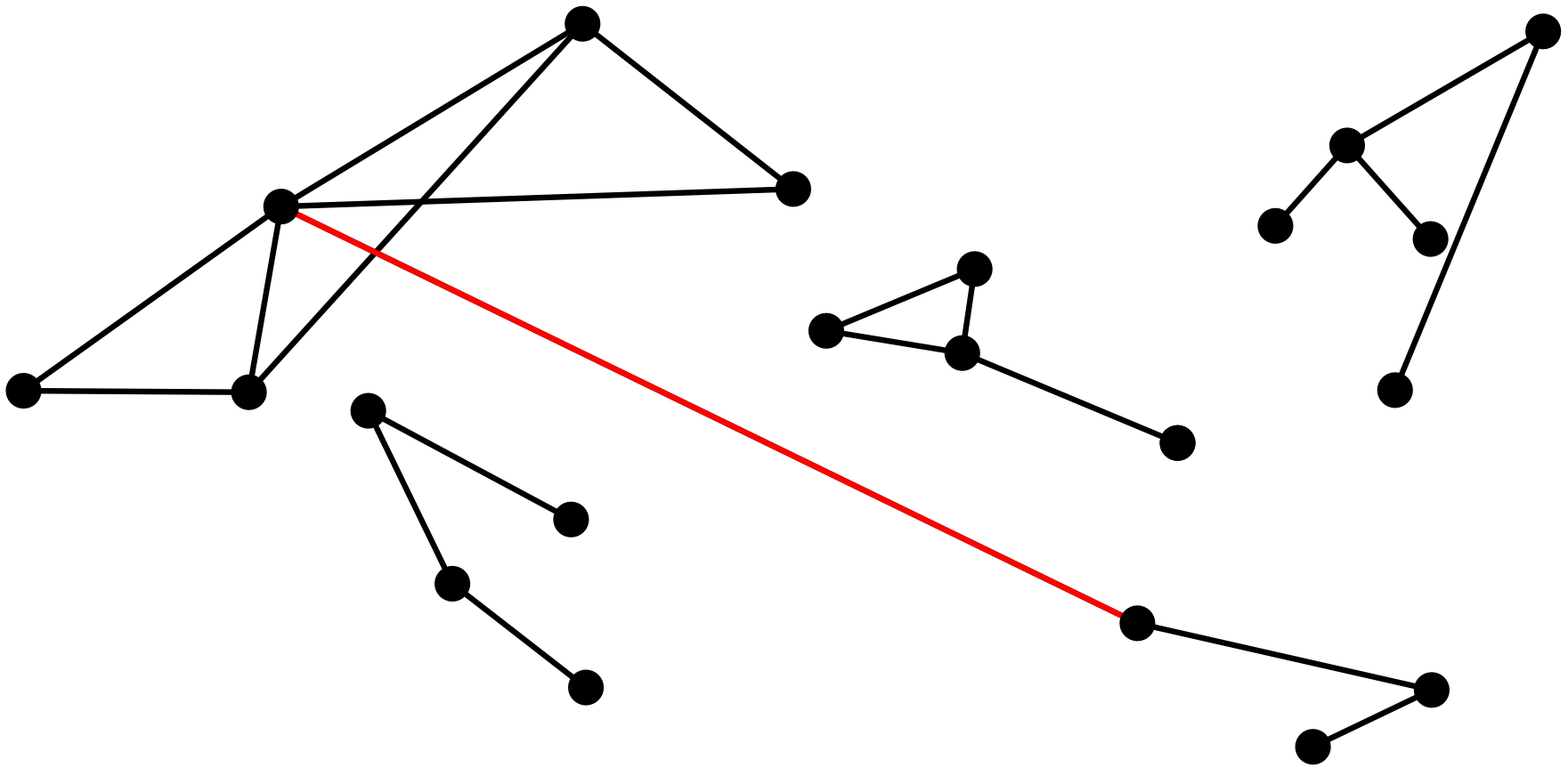
The Problem in General



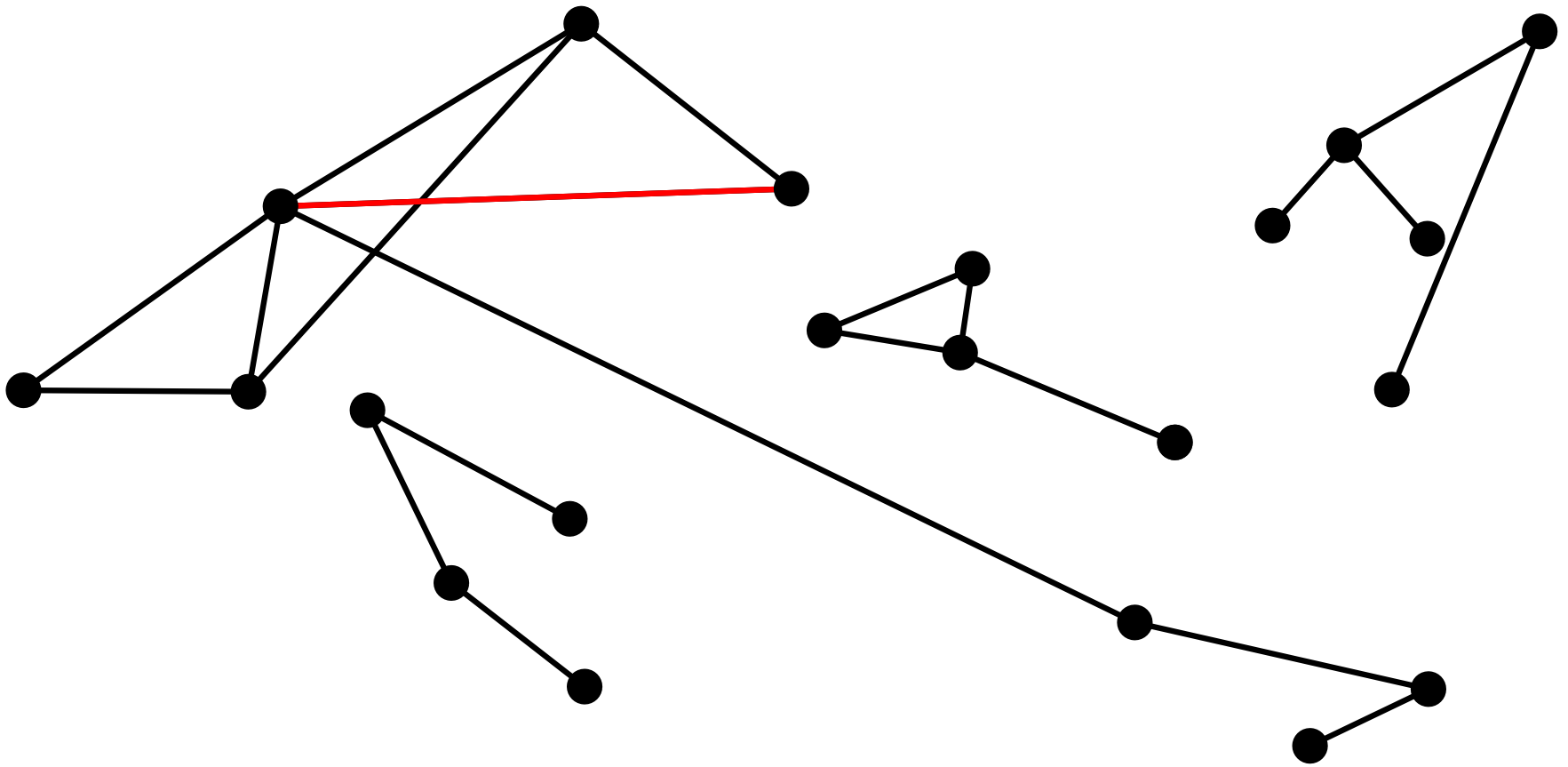
The Problem in General



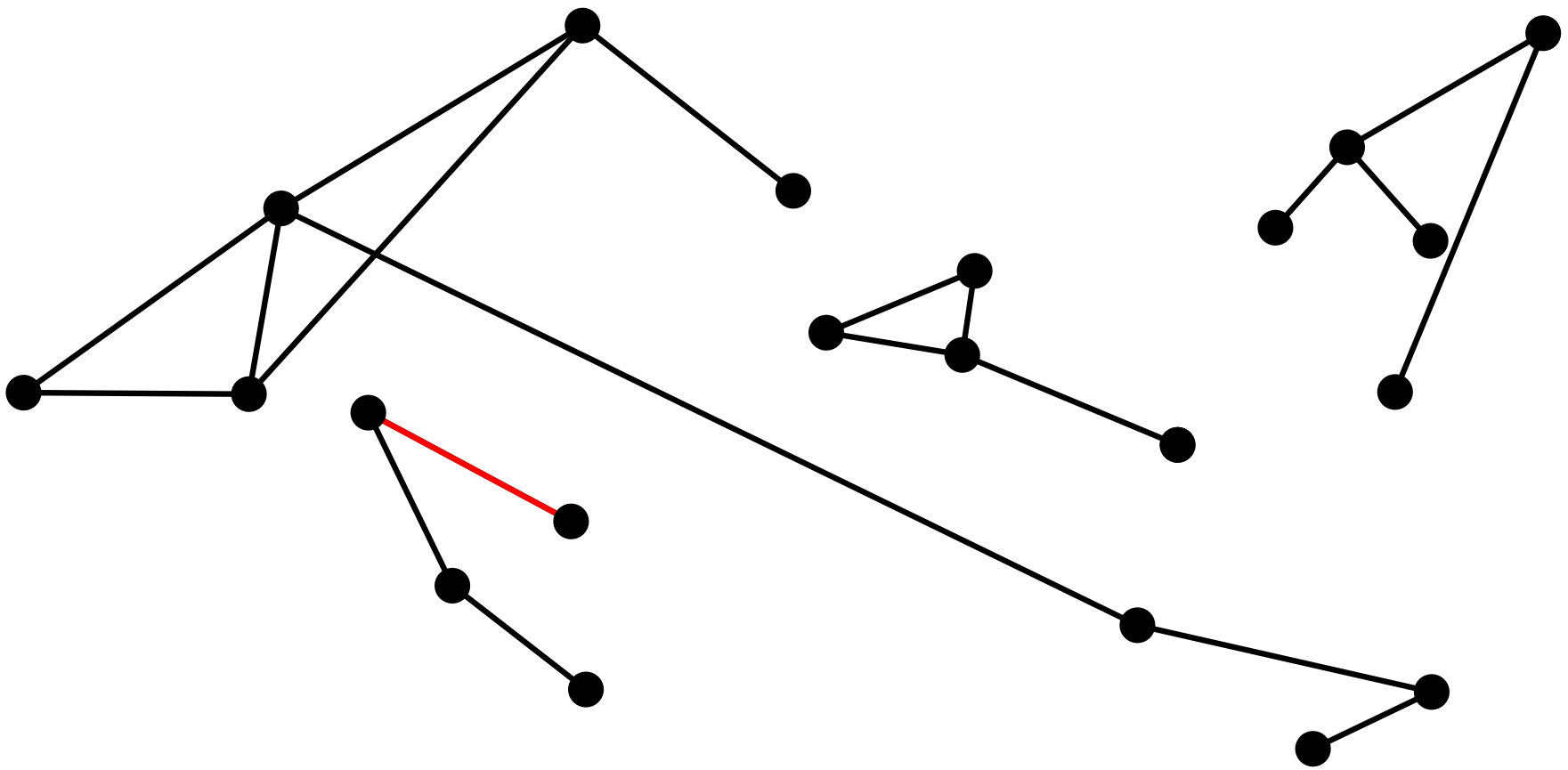
The Problem in General



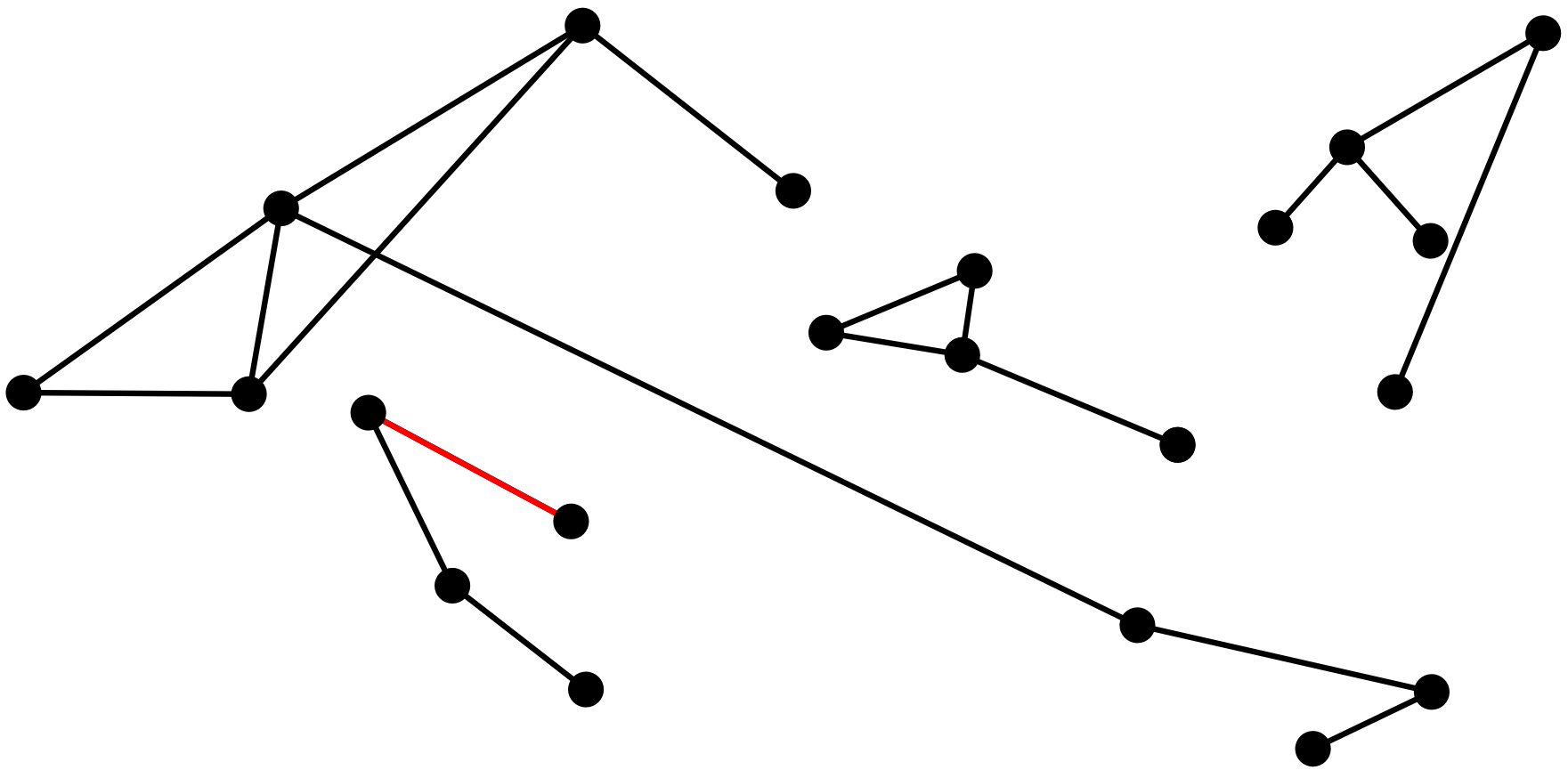
The Problem in General



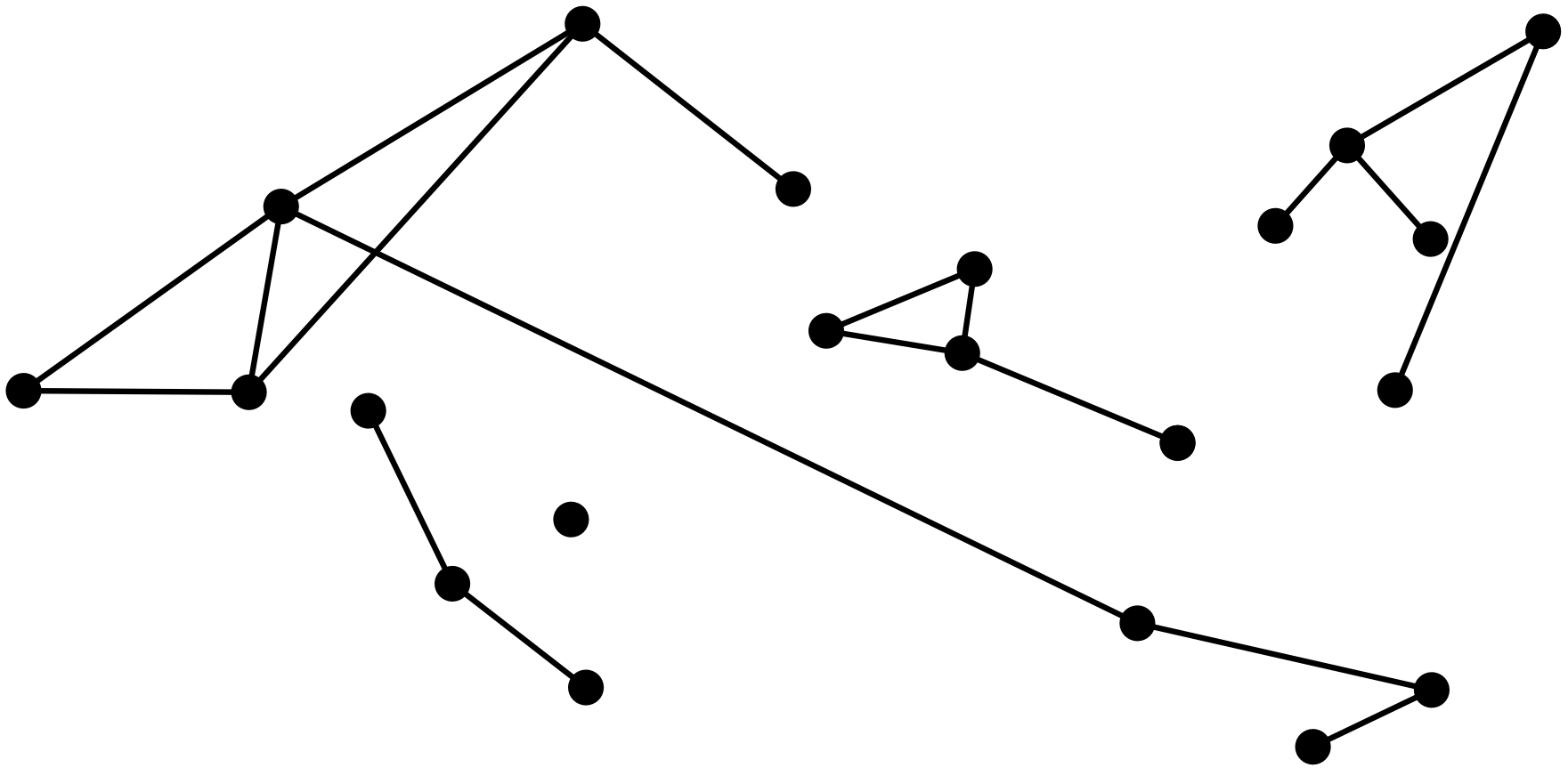
The Problem in General



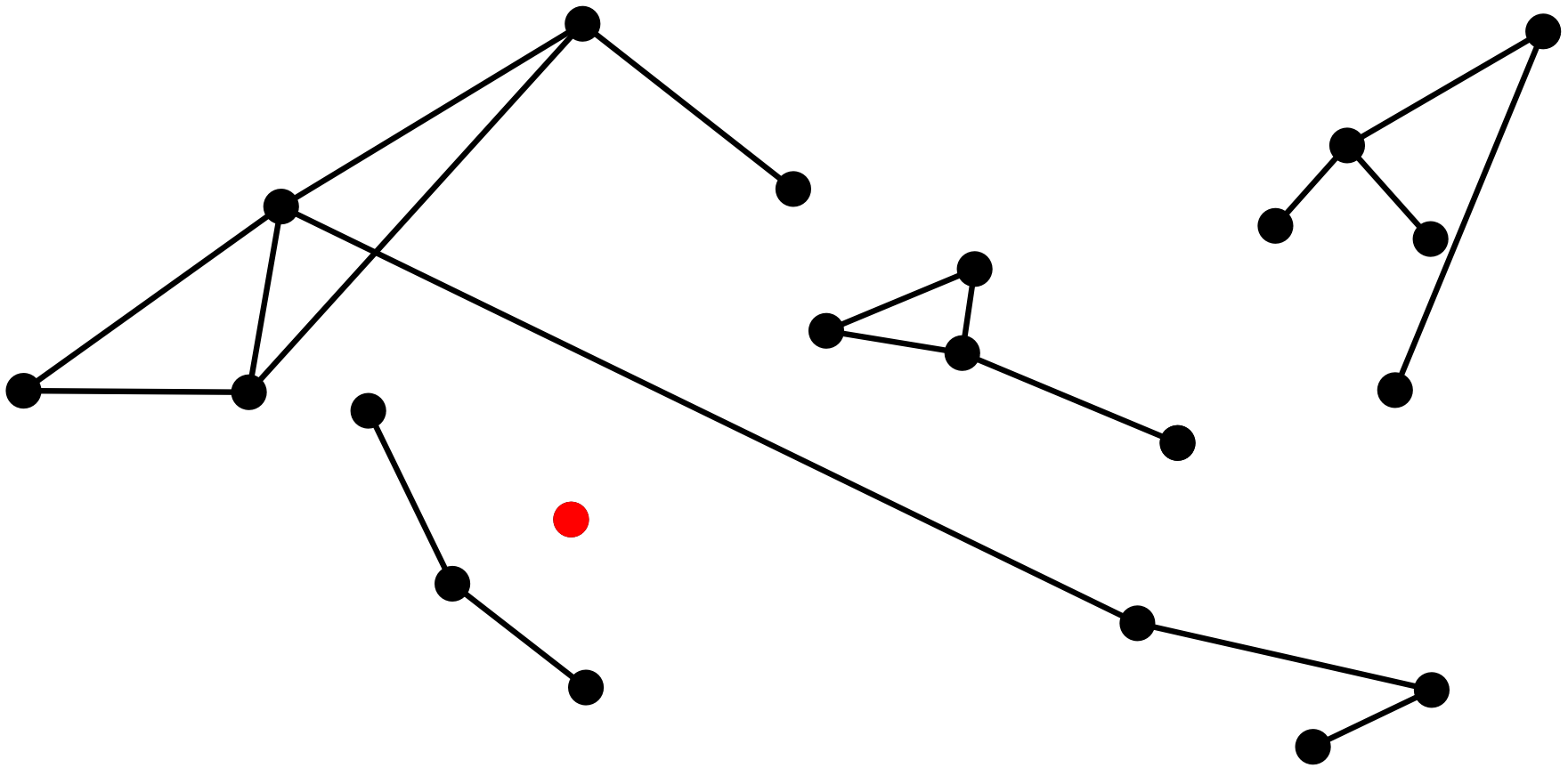
The Problem in General



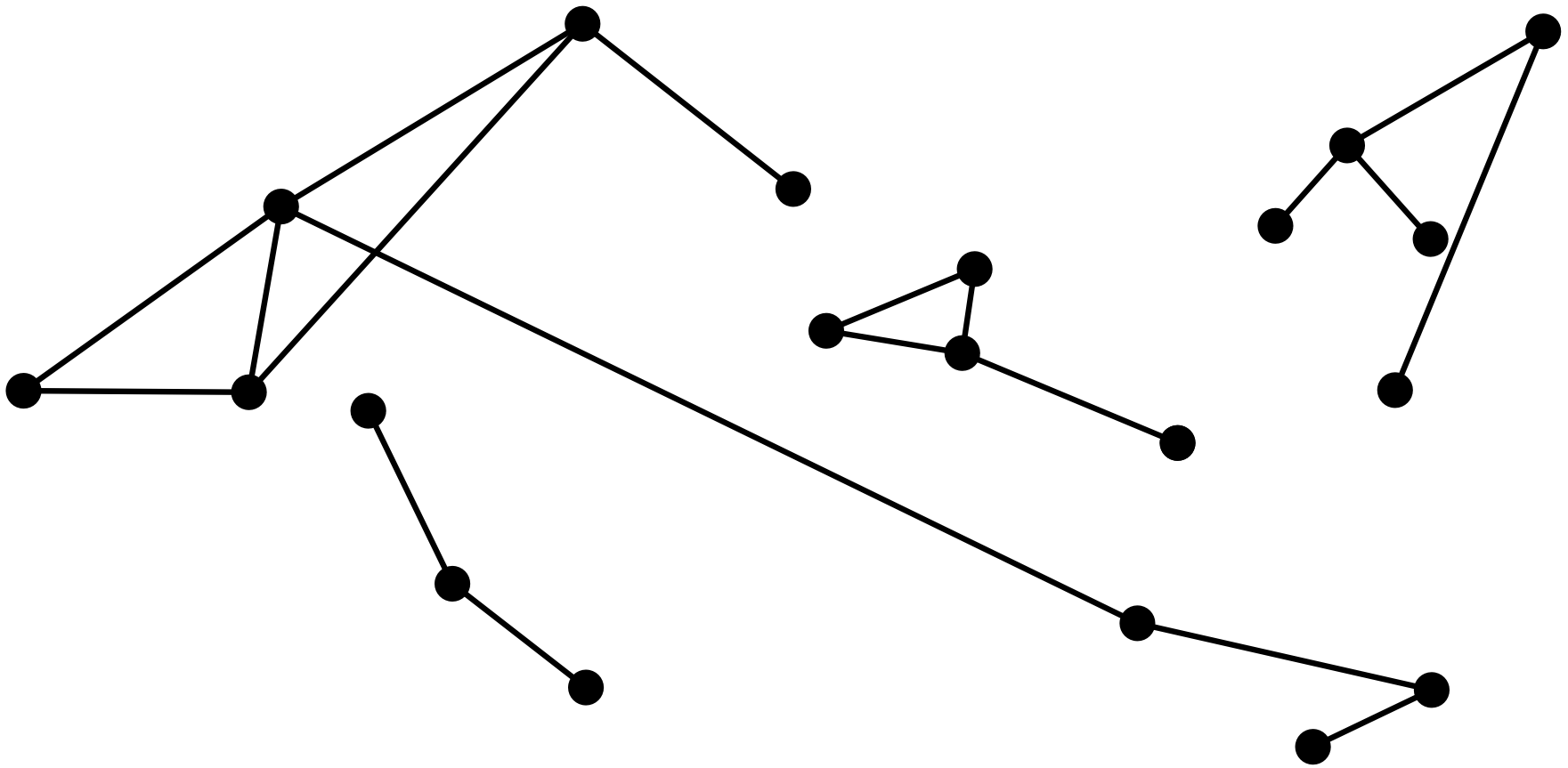
The Problem in General



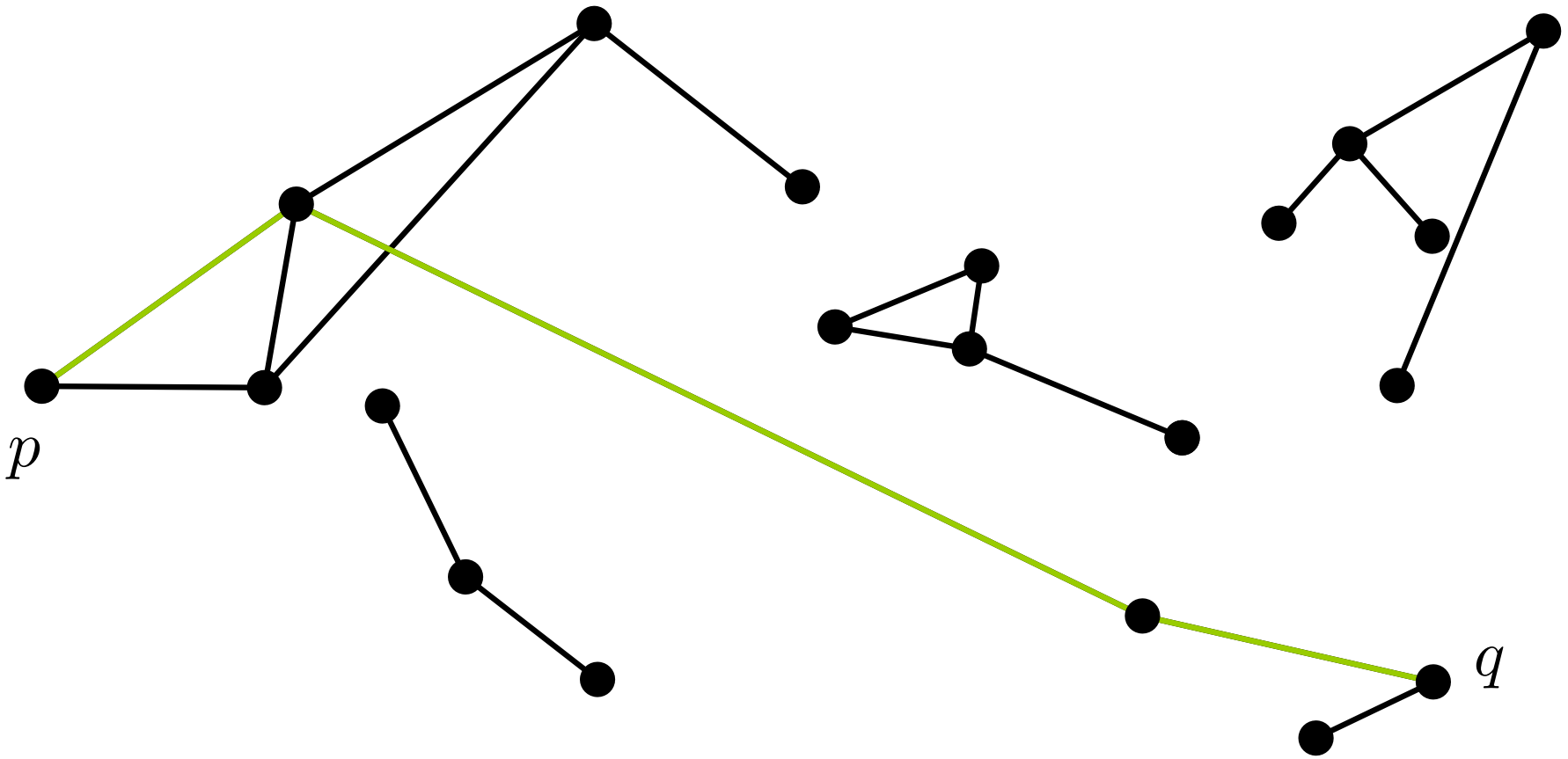
The Problem in General



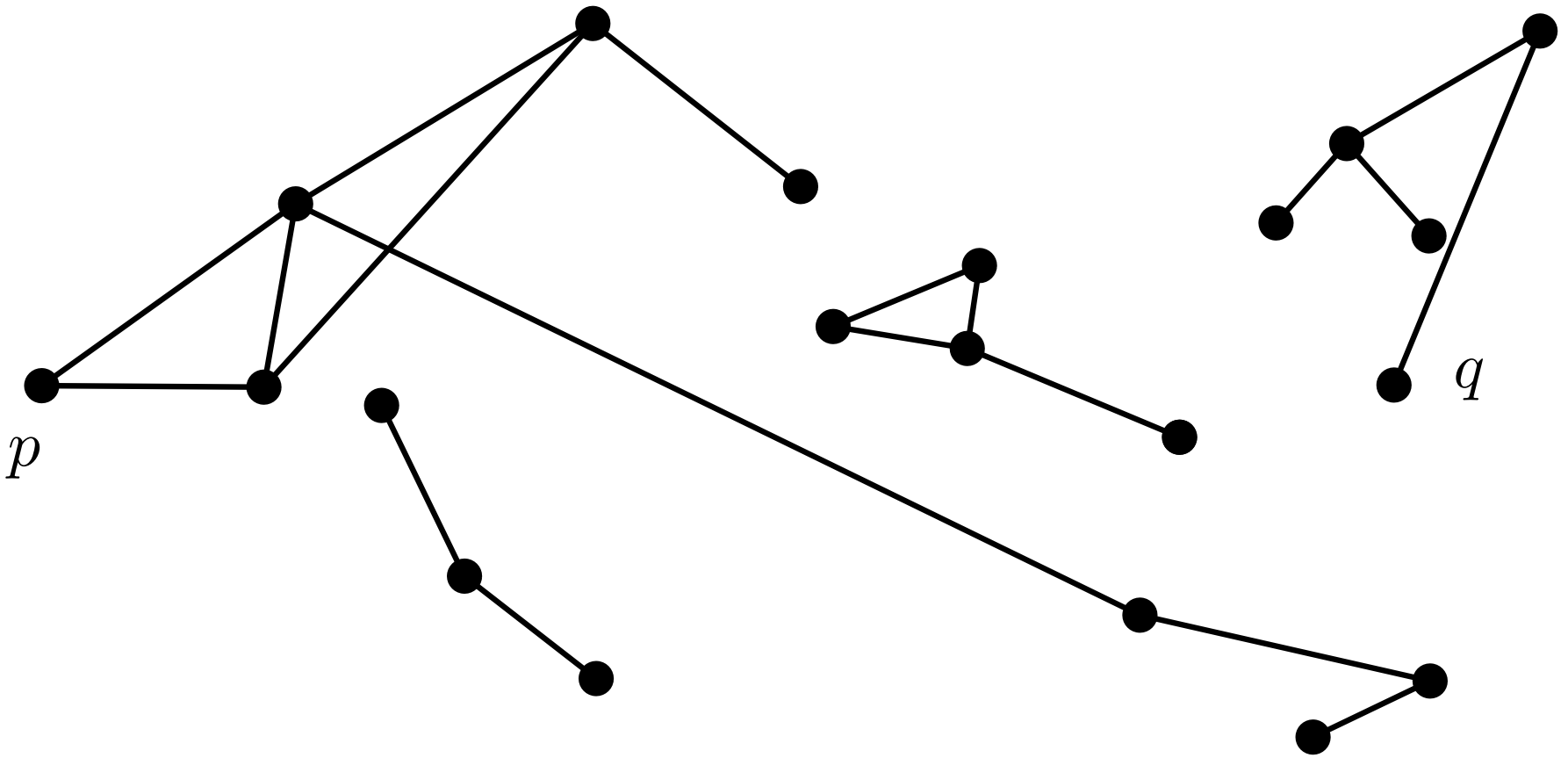
The Problem in General



The Problem in General

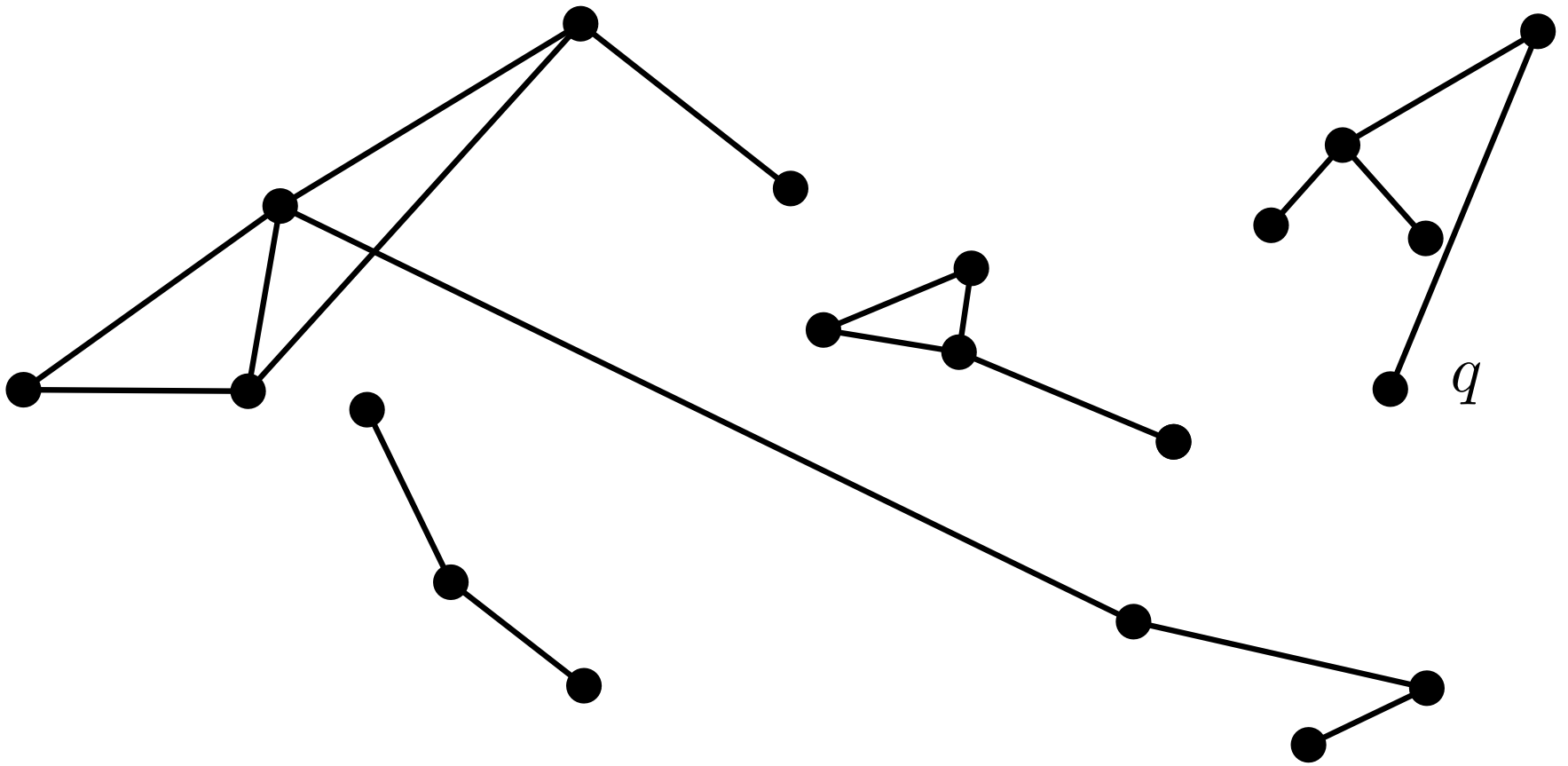


The Problem in General



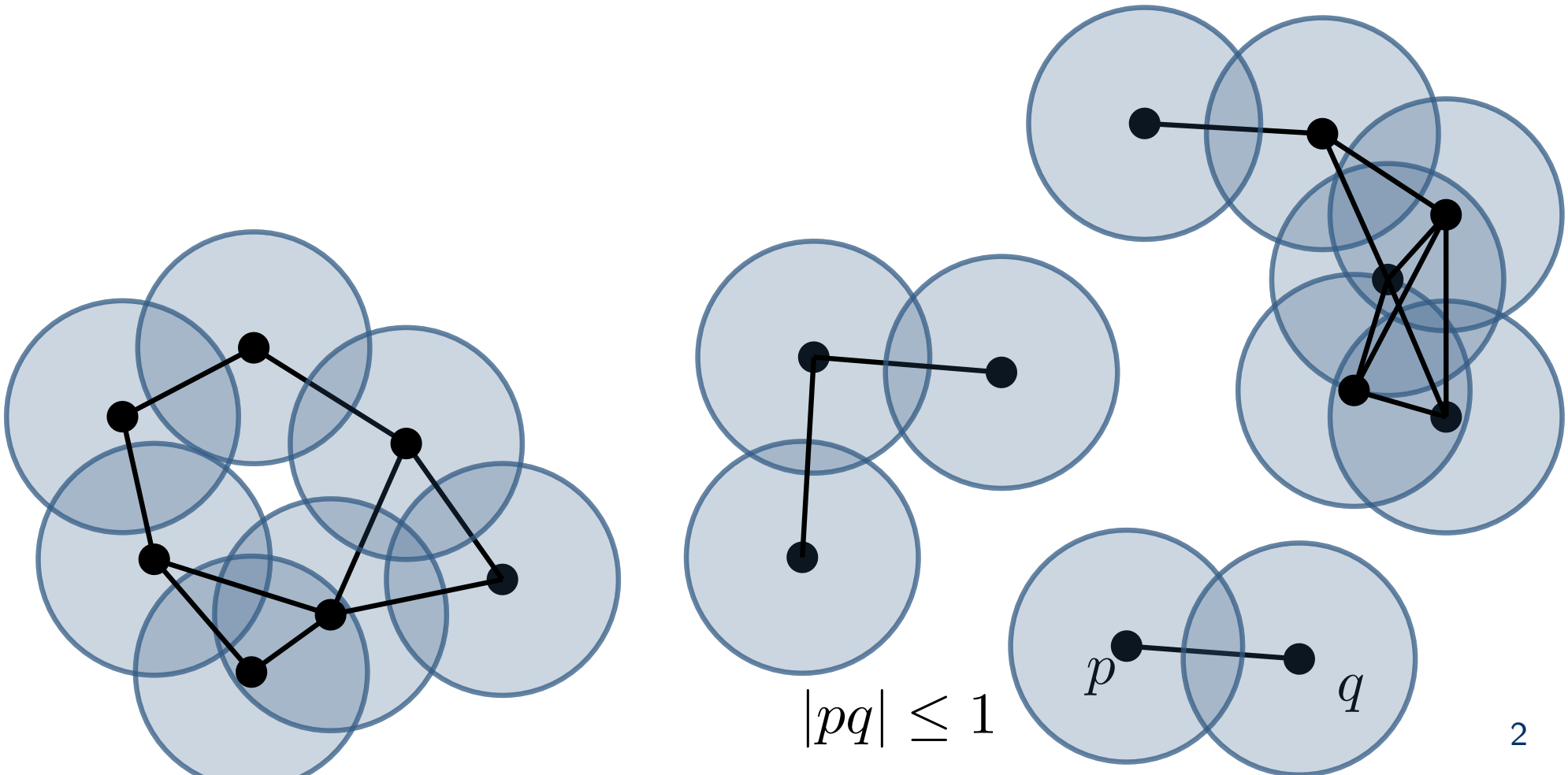
The Problem in General

	update	query	
gen.	$O(\log^2 n)$	$O(\log n / \log\log n)$	Holm et al.
planar	$O(\log n)$	$O(\log n)$	Eppstein et al.



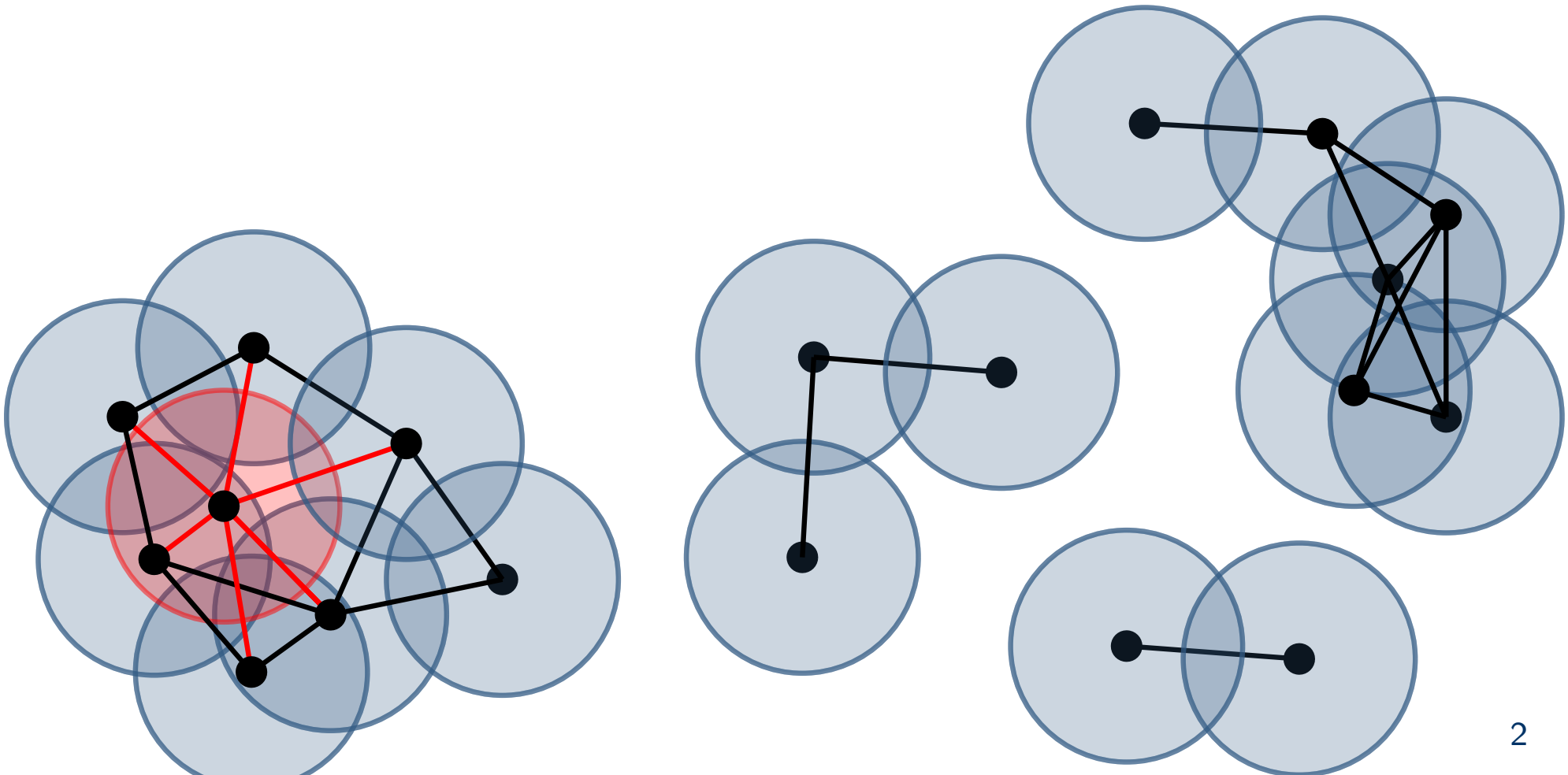
Connectivity in Unit Disk Graphs

- dynamic set $P \subset \mathbb{R}^2$, $|P| = n$



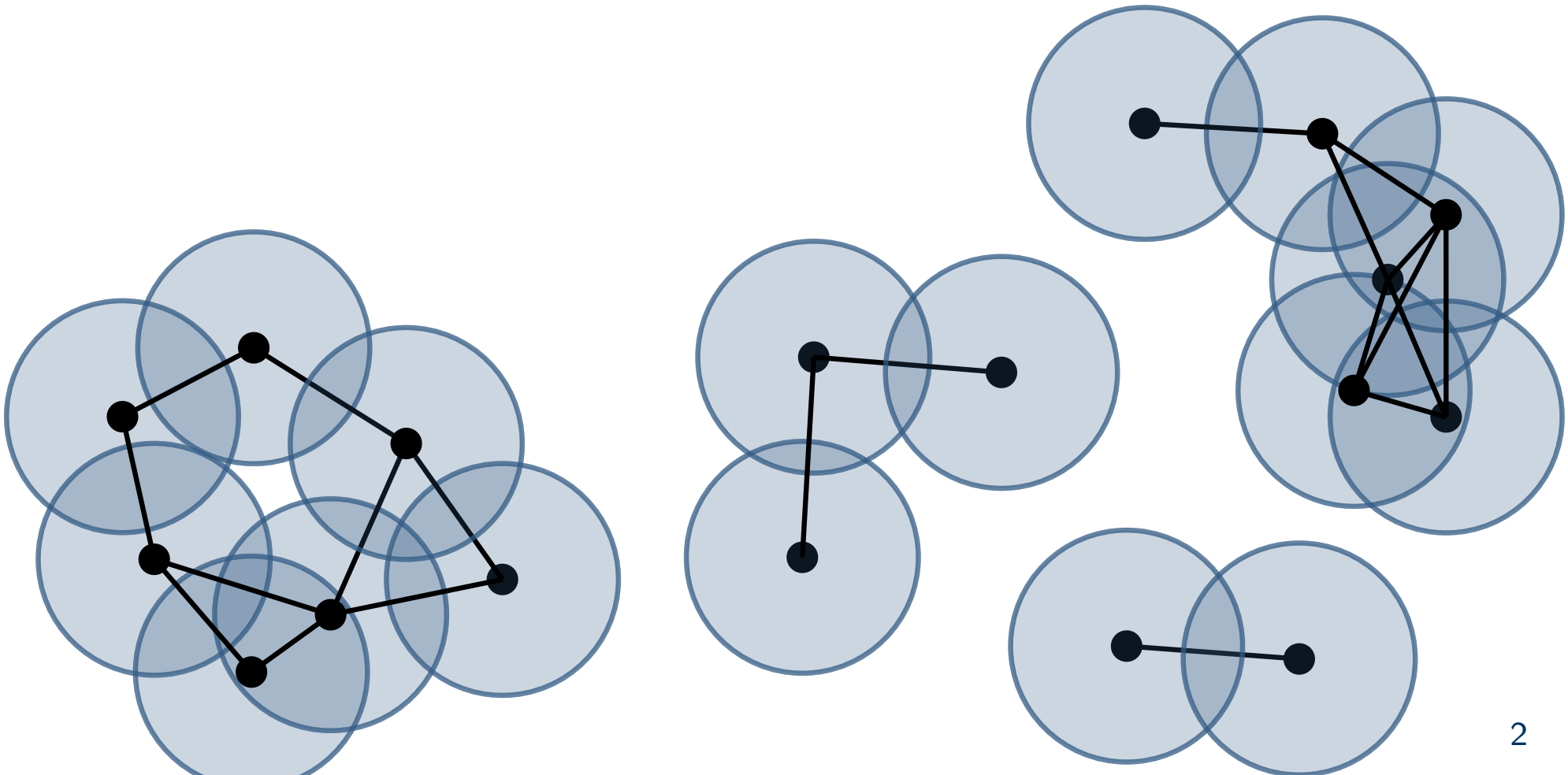
Connectivity in Unit Disk Graphs

- dynamic set $P \subset \mathbb{R}^2$, $|P| = n$



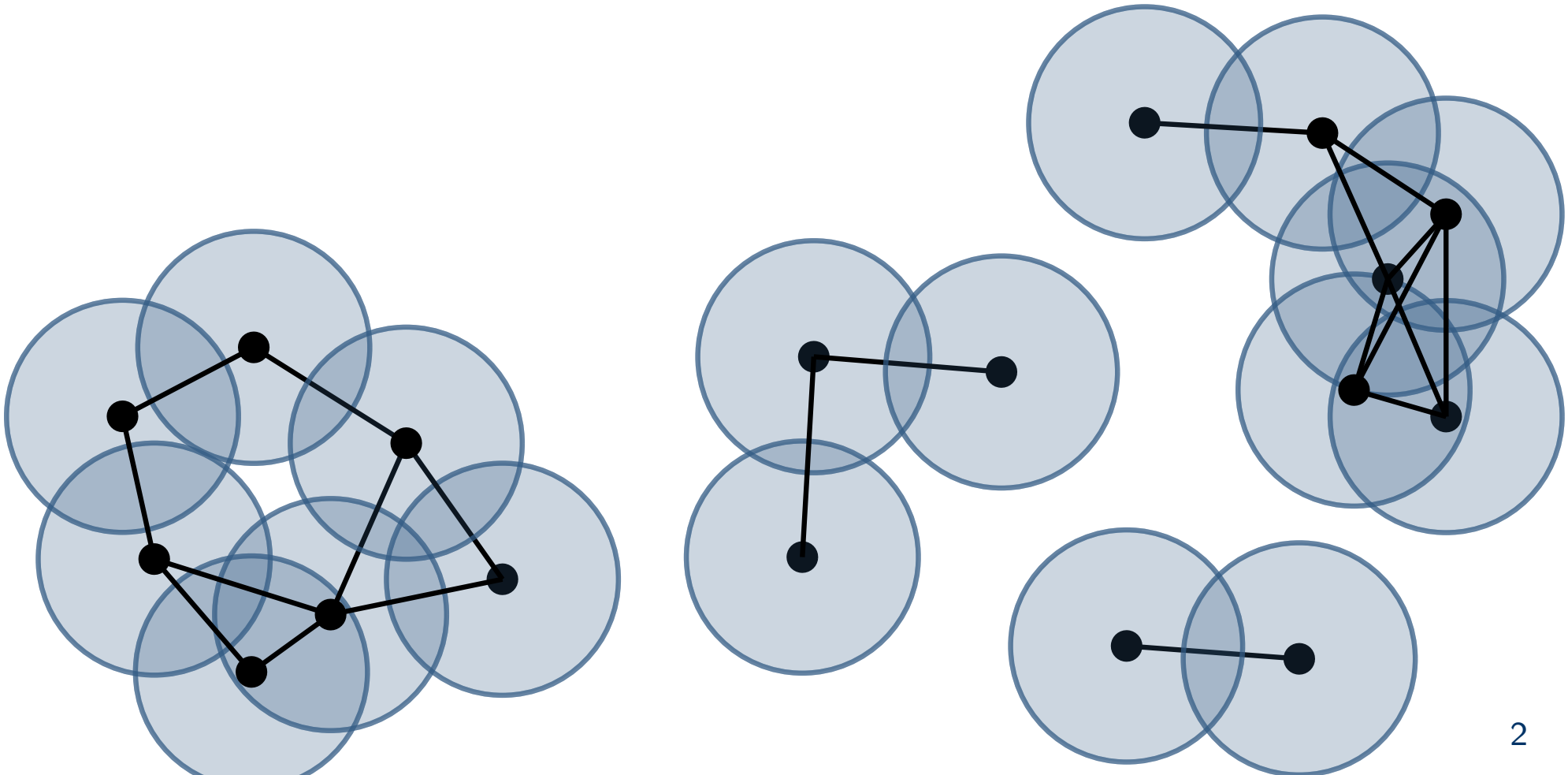
Connectivity in Unit Disk Graphs

- dynamic set $P \subset \mathbb{R}^2$, $|P| = n$



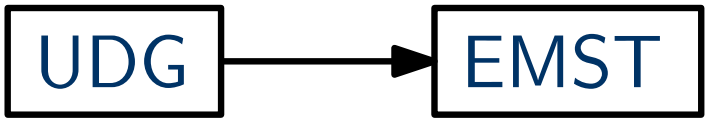
Connectivity in Unit Disk Graphs

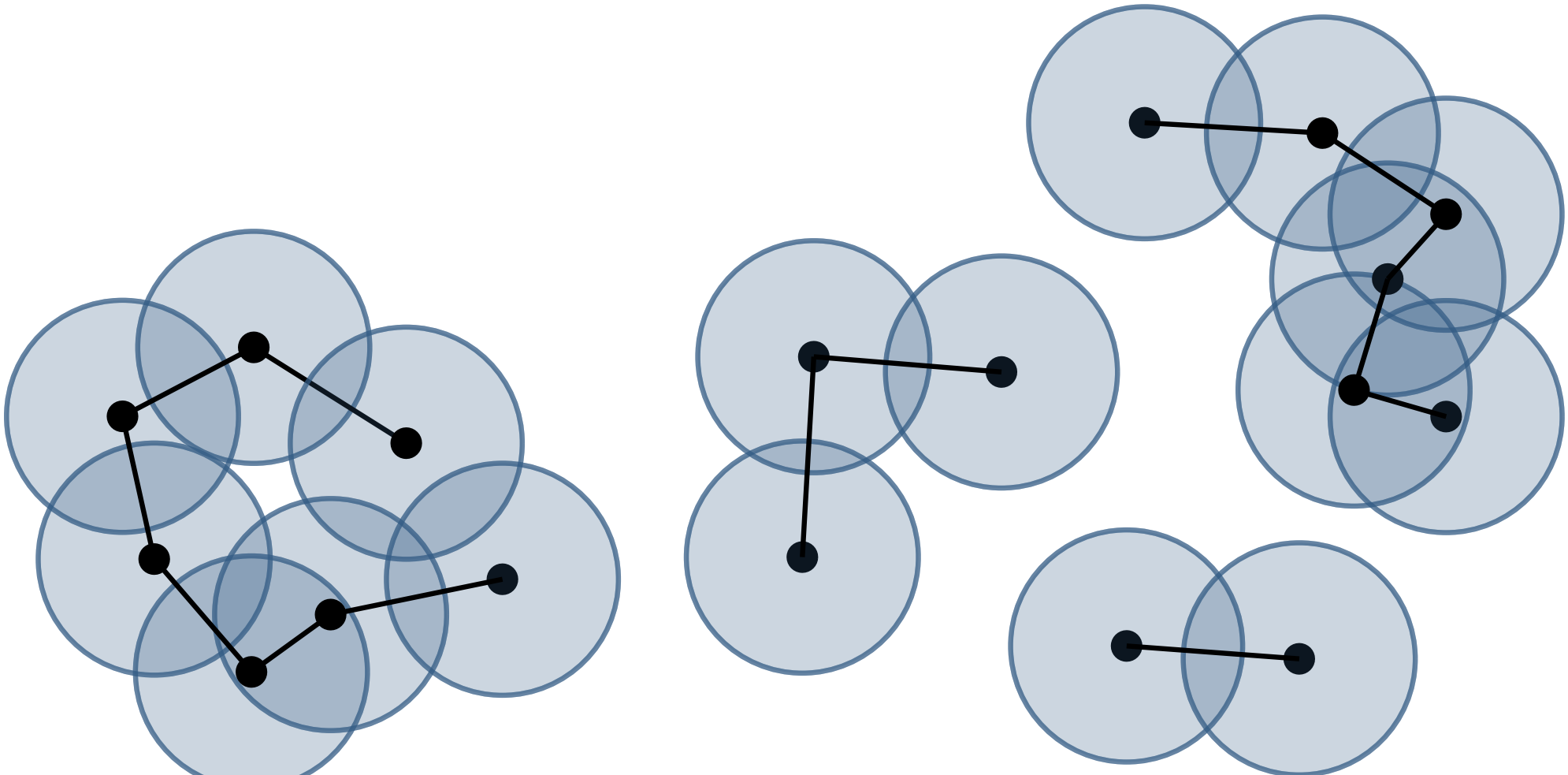
- dynamic set $P \subset \mathbb{R}^2$, $|P| = n$
- Chan et al.: update time $O(\log^{10} n)$



Connectivity in Unit Disk Graphs

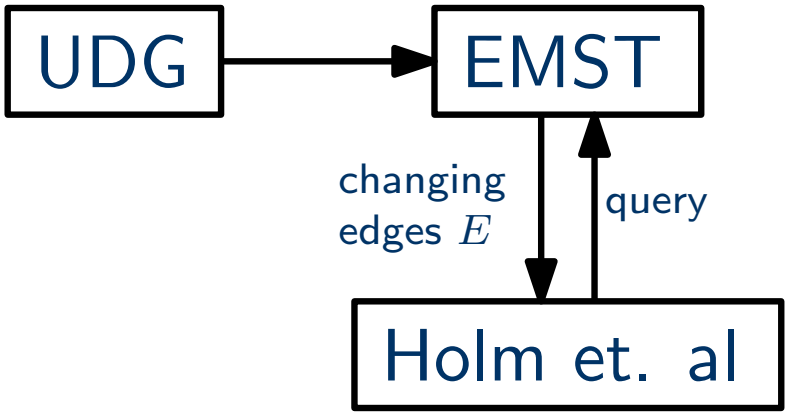
- dynamic set $P \subset \mathbb{R}^2$, $|P| = n$
- Chan et al.: update time $O(\log^{10} n)$





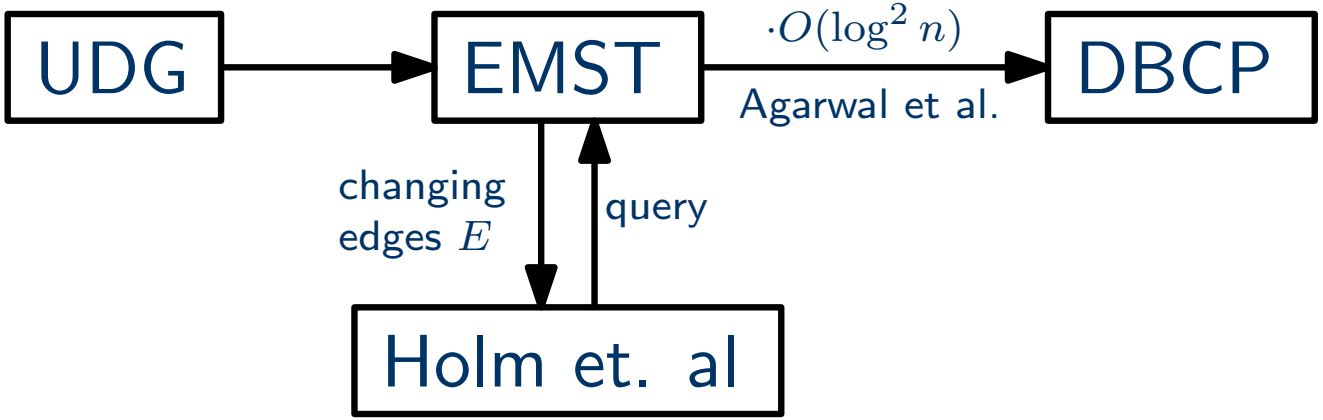
Connectivity in Unit Disk Graphs

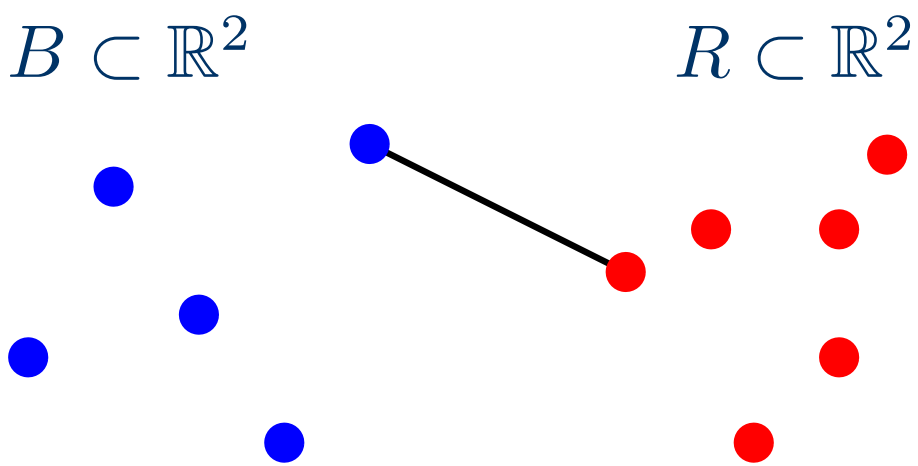
- dynamic set $P \subset \mathbb{R}^2$, $|P| = n$
- Chan et al.: update time $O(\log^{10} n)$



Connectivity in Unit Disk Graphs

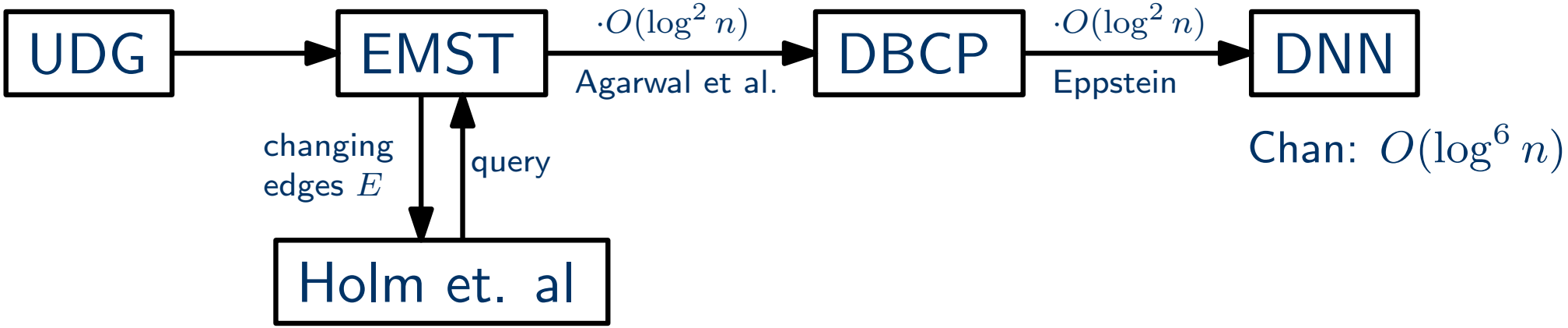
- dynamic set $P \subset \mathbb{R}^2$, $|P| = n$
- Chan et al.: update time $O(\log^{10} n)$

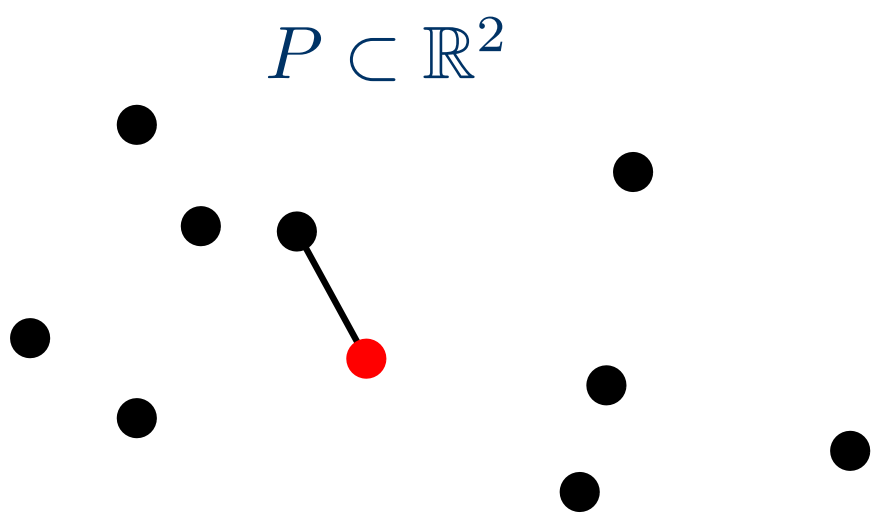




Connectivity in Unit Disk Graphs

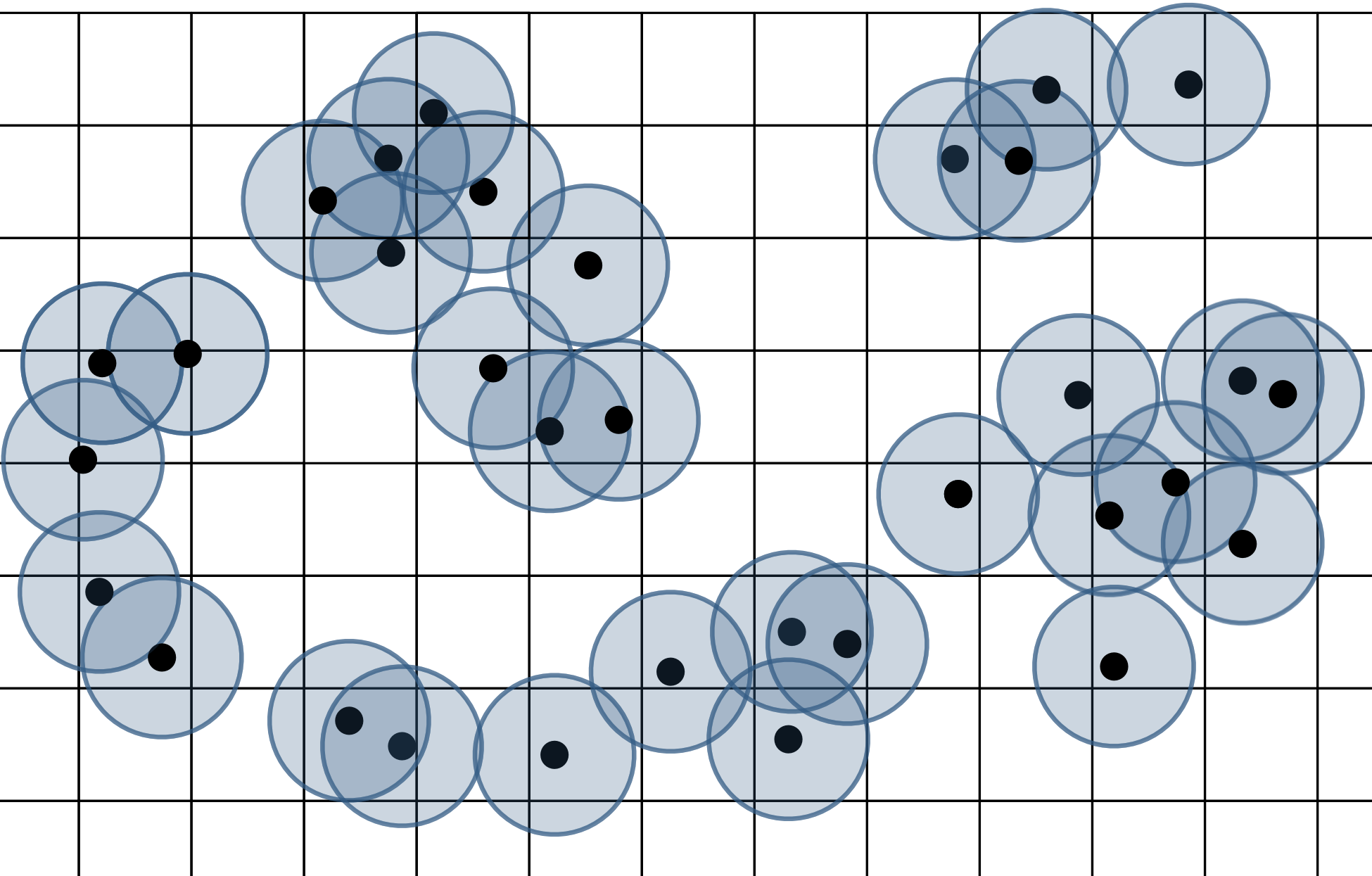
- dynamic set $P \subset \mathbb{R}^2$, $|P| = n$
- Chan et al.: update time $O(\log^{10} n)$





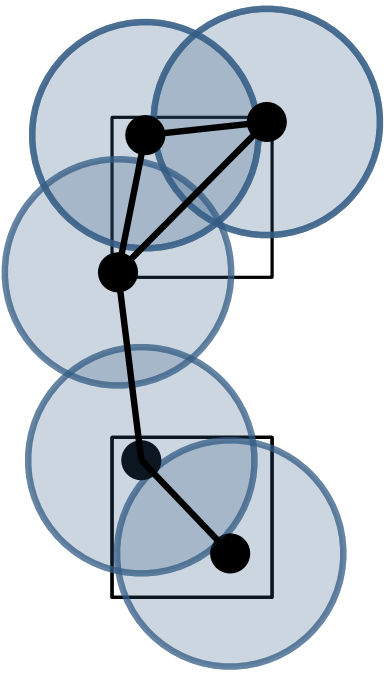
The Grid Graph

- use grid with diameter 1 to define grid graph



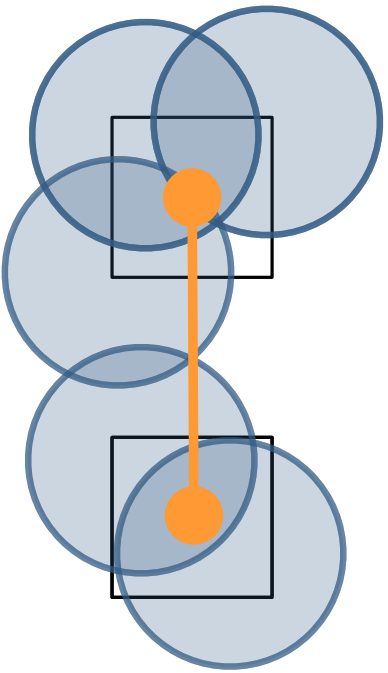
The Grid Graph

- use grid with diameter 1 to define grid graph



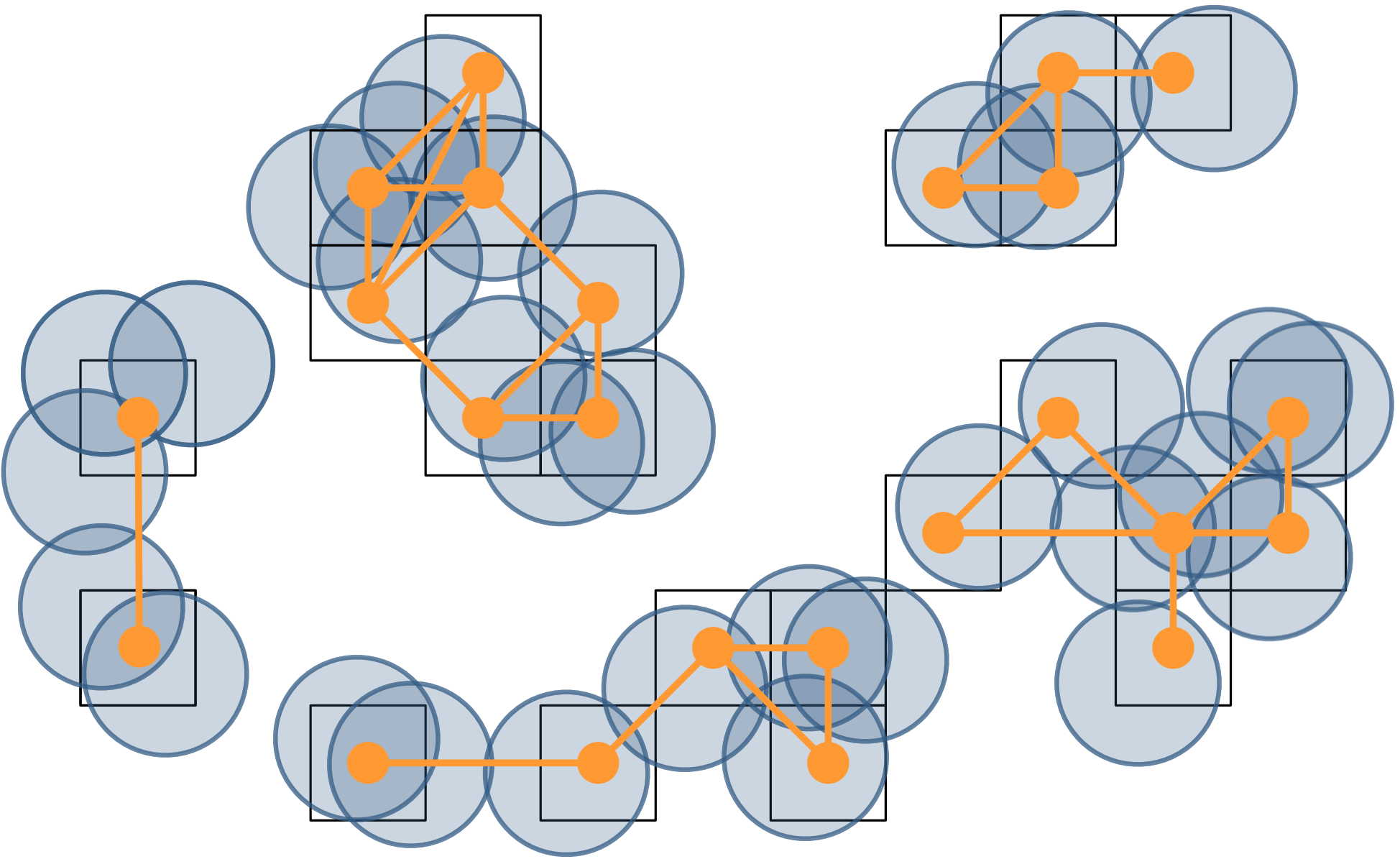
The Grid Graph

- use grid with diameter 1 to define grid graph



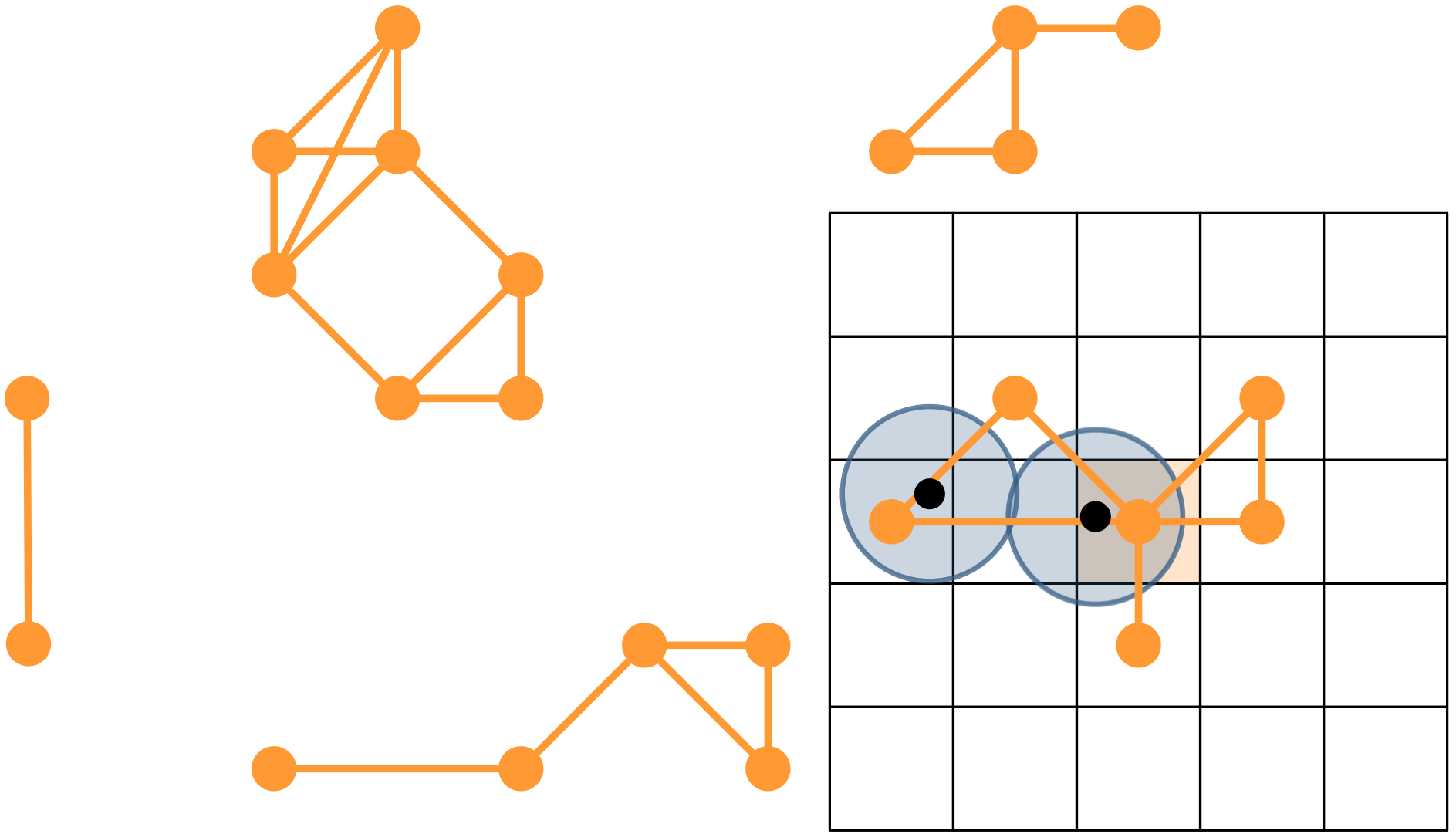
The Grid Graph

- use grid with diameter 1 to define grid graph

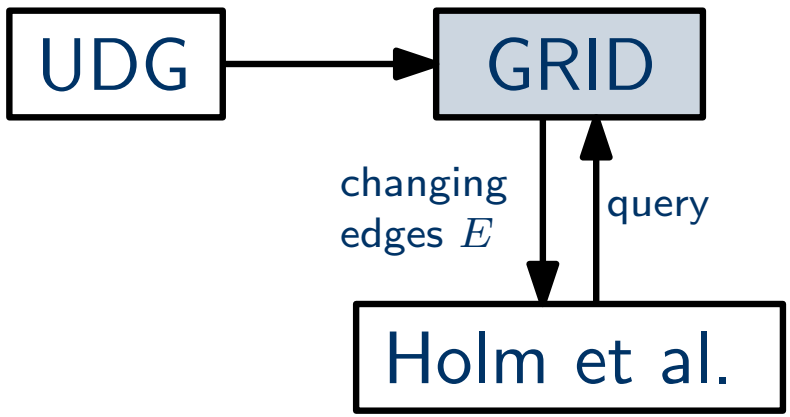


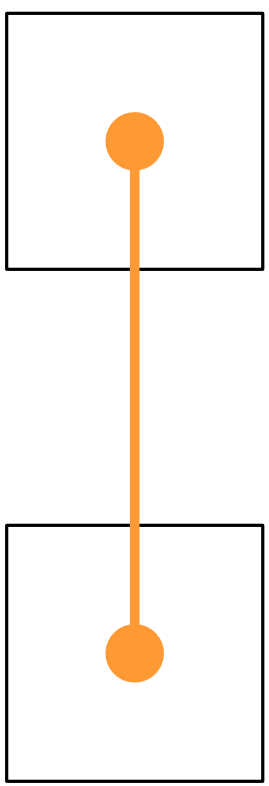
The Grid Graph

- use grid with diameter 1 to define grid graph



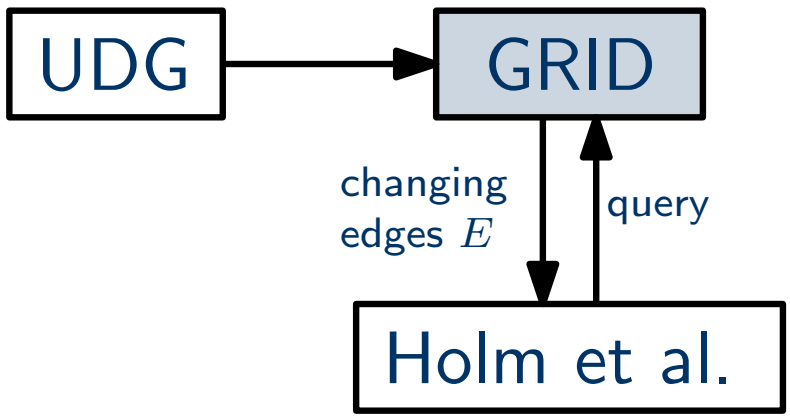
Connectivity in Unit Disk Graphs

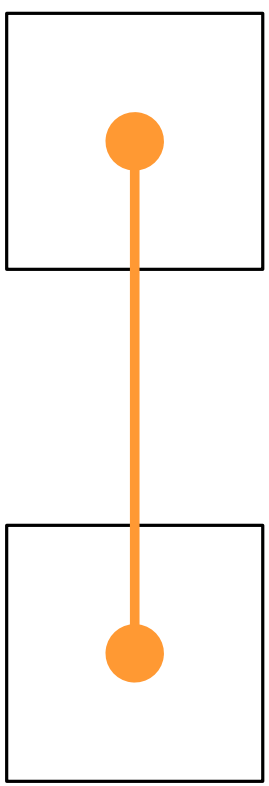




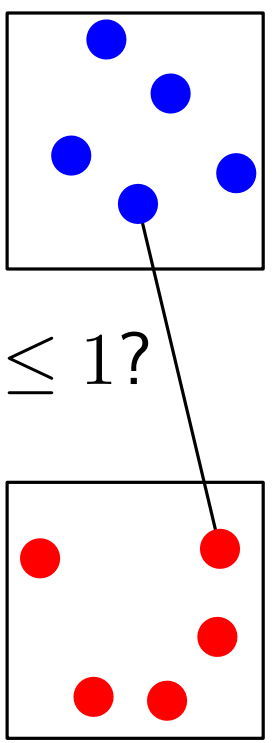
Connectivity in Unit Disk Graphs

- update time $O(\log^{10} n)$





$\Leftrightarrow$



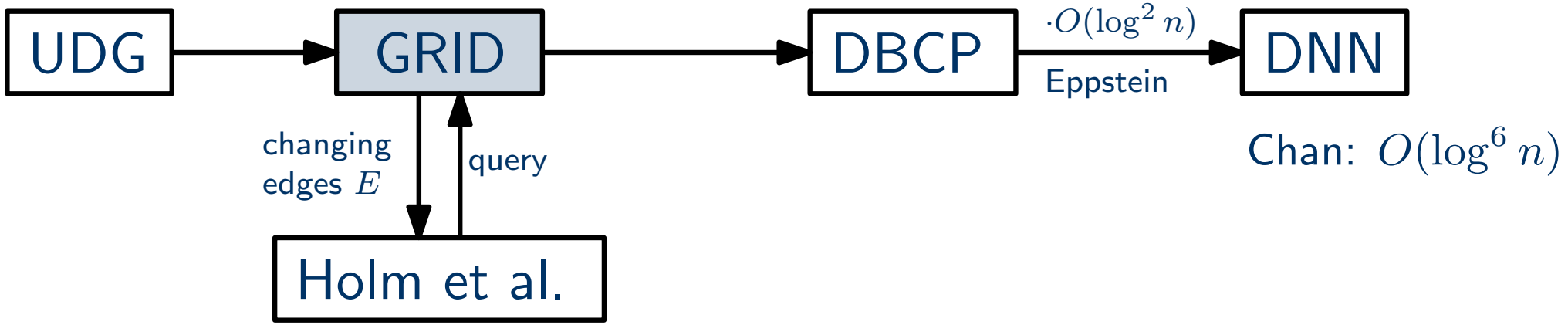
$B \subset \mathbb{R}^2$

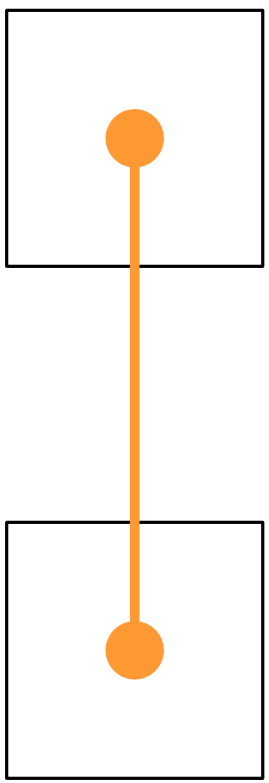
≤ 1?

$R \subset \mathbb{R}^2$

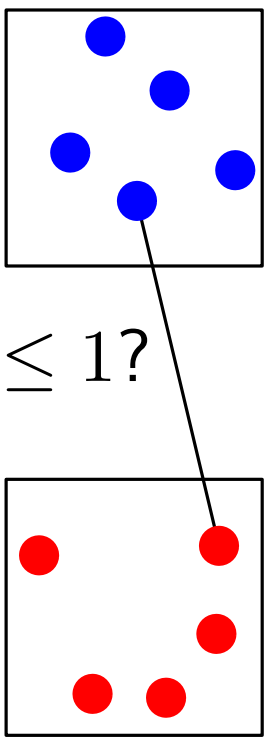
Connectivity in Unit Disk Graphs

- update time ~~$O(\log^{10} n)$~~ $O(\log^8 n)$



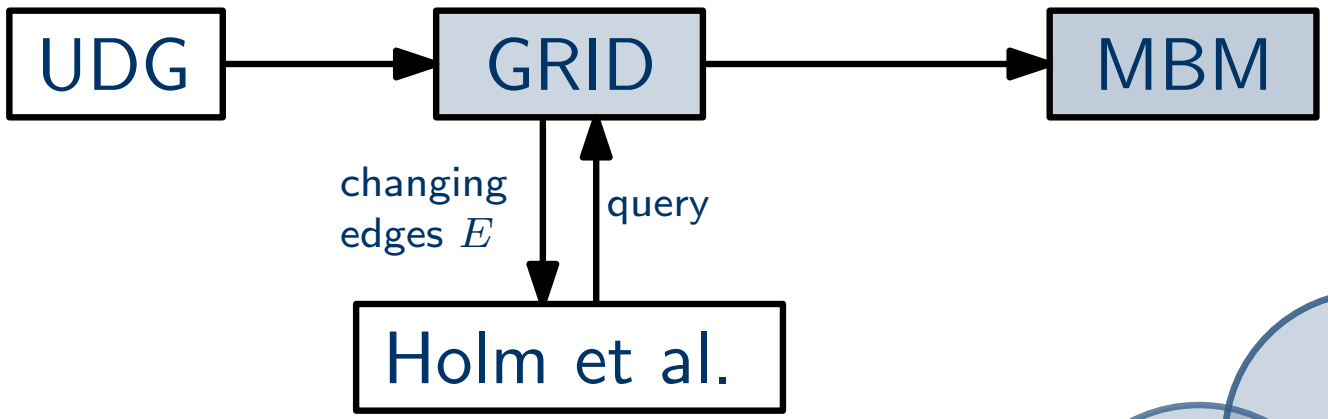


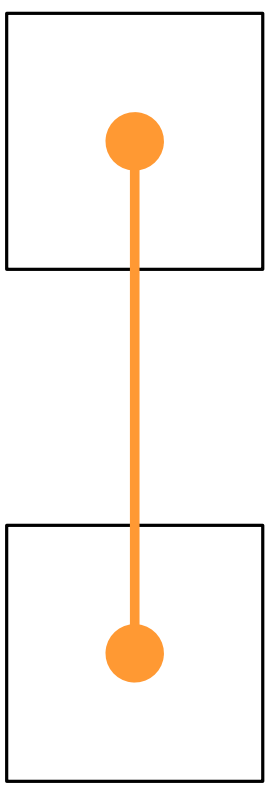
$\Leftrightarrow$

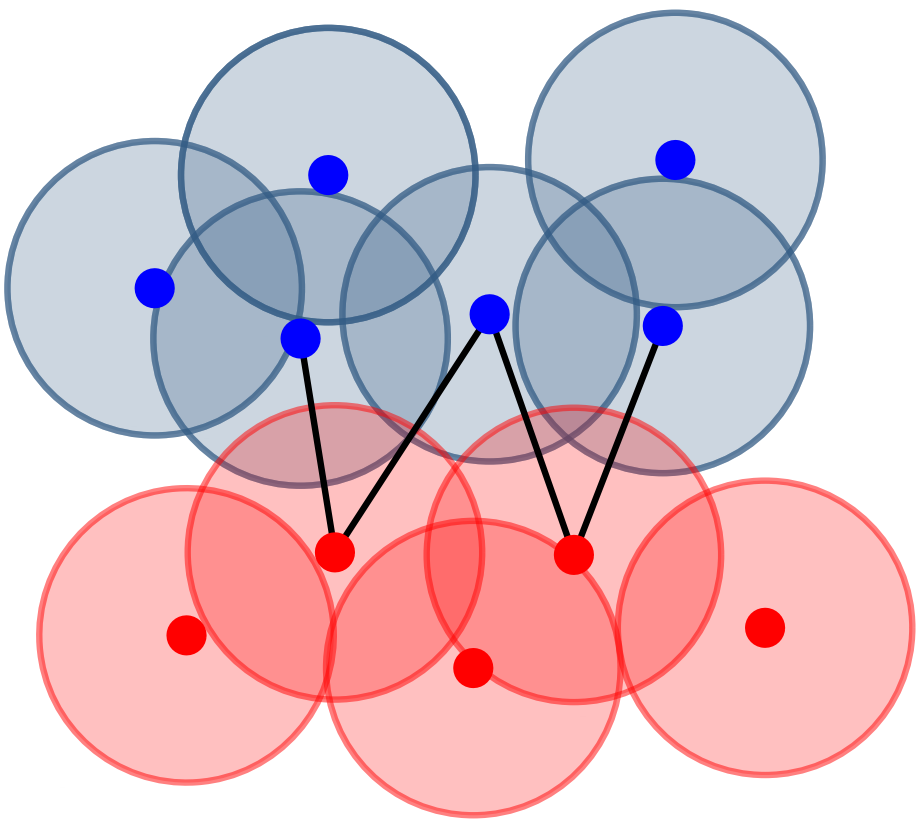


Connectivity in Unit Disk Graphs

- update time ~~$O(\log^{10} n)$~~ $O(\log^8 n)$

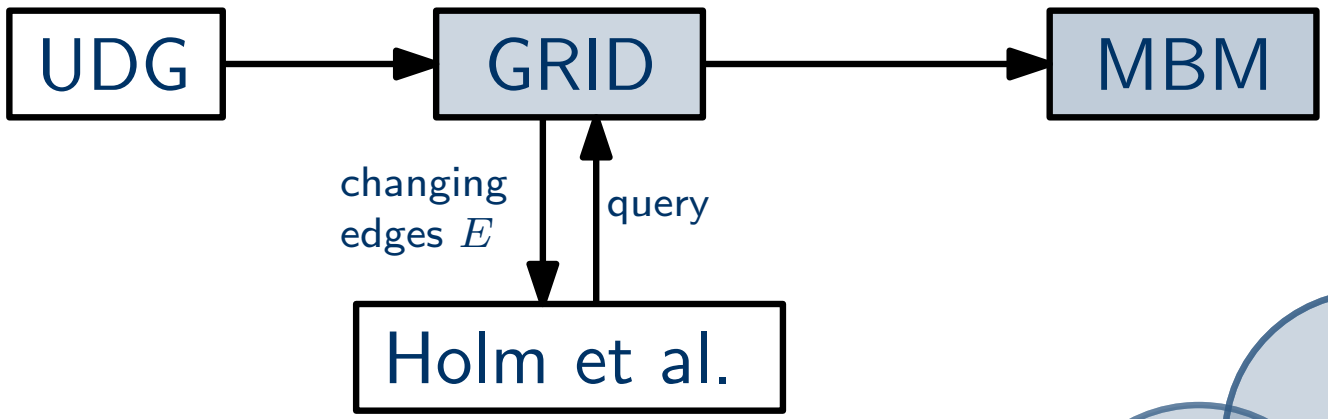


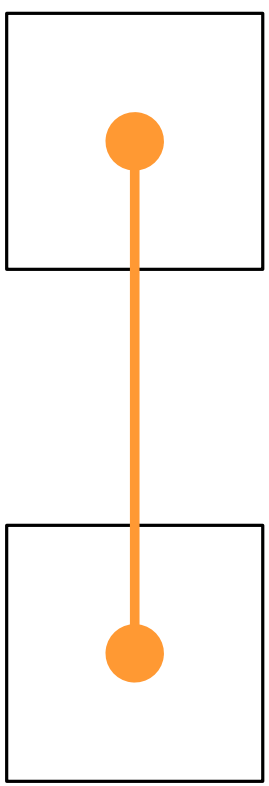




Connectivity in Unit Disk Graphs

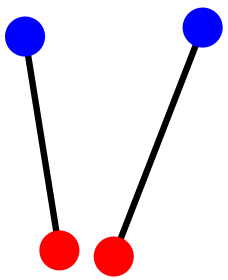
- update time ~~$O(\log^{10} n)$~~ $O(\log^8 n)$

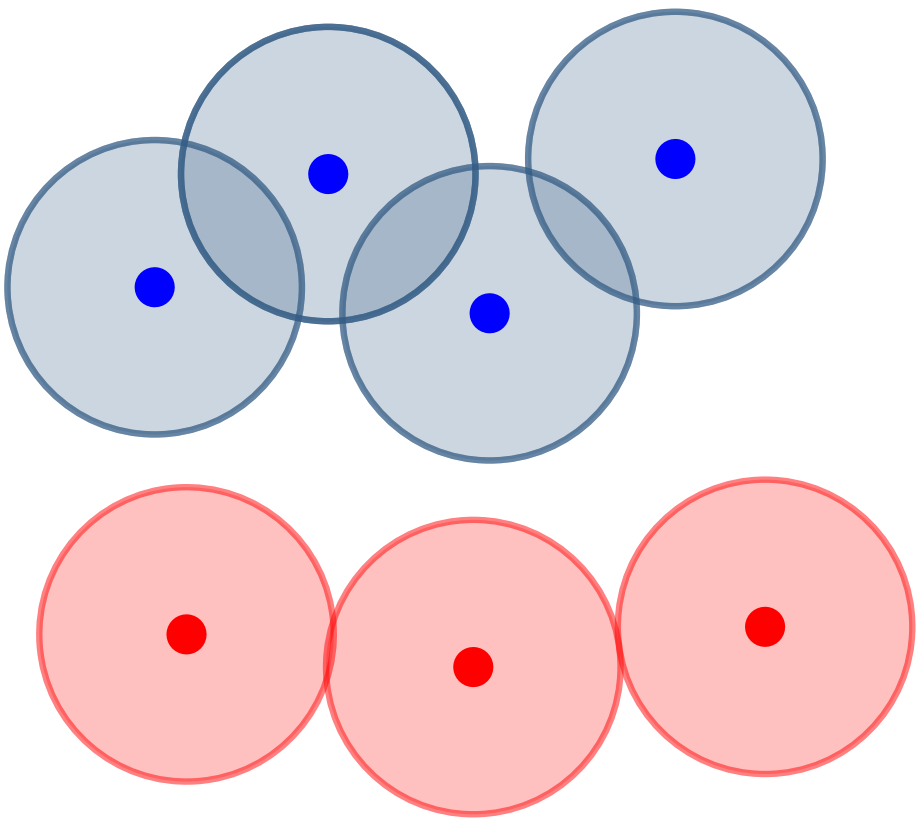




Matching M

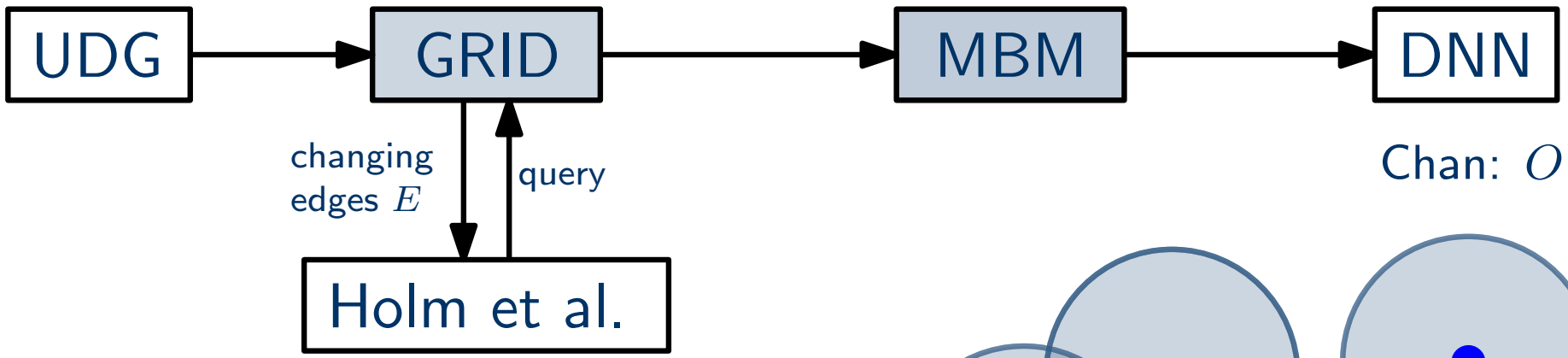
$\Leftrightarrow M \neq \emptyset$



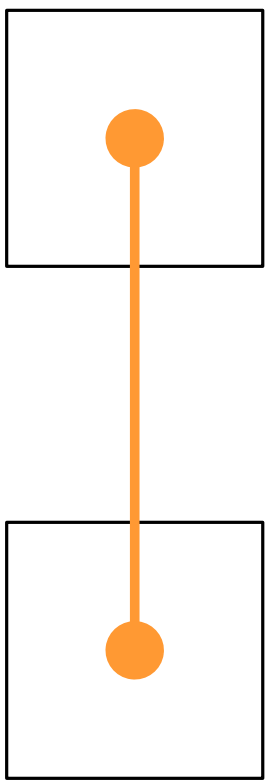


Connectivity in Unit Disk Graphs

- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ $O(\log^{6} n)$

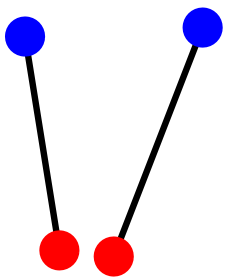


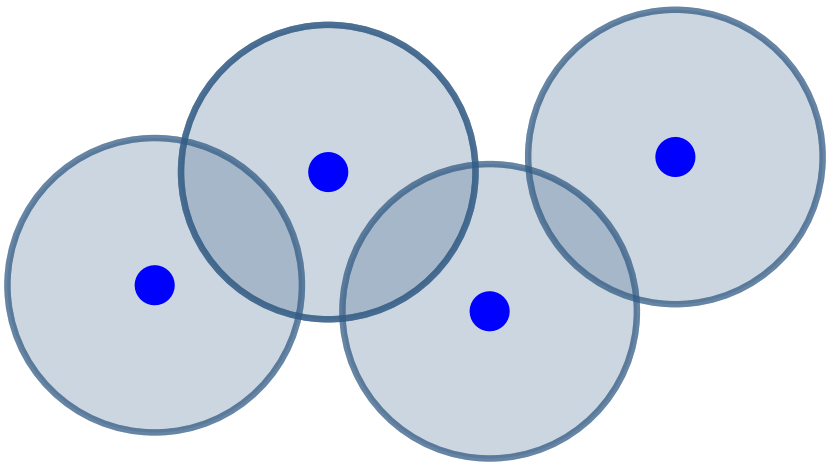
Chan: $O(\log^{6} n)$

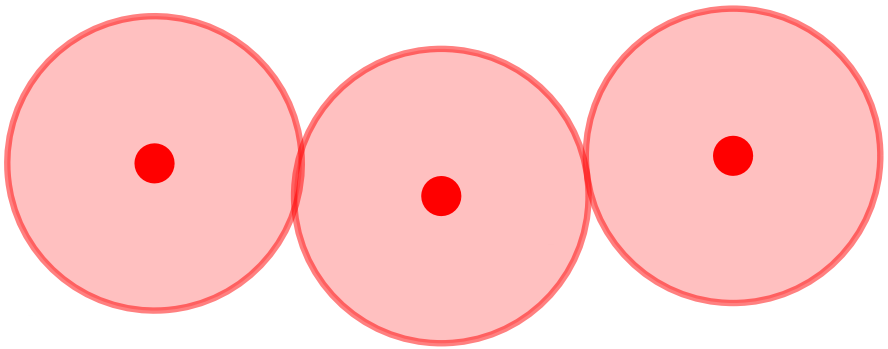


Matching M

$\Leftrightarrow M \neq \emptyset$

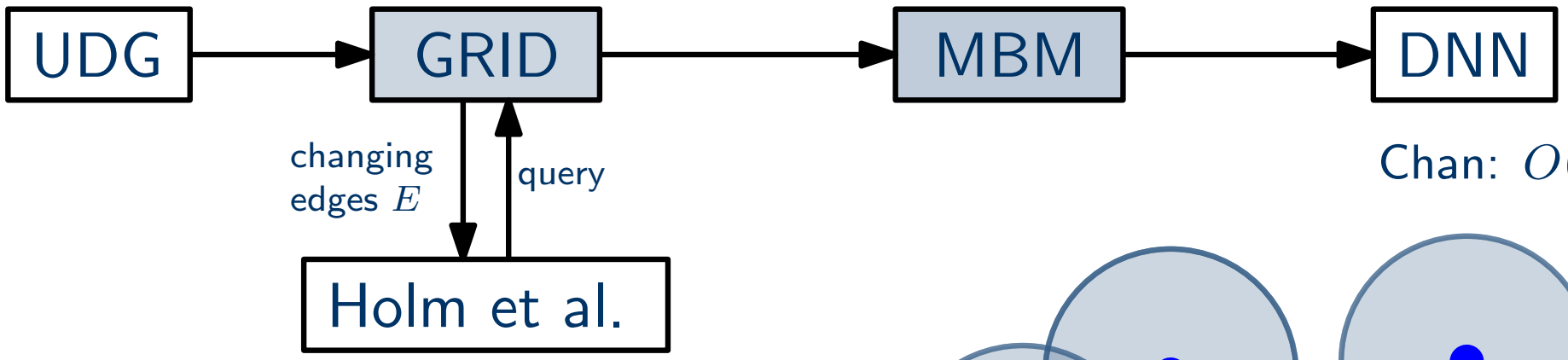




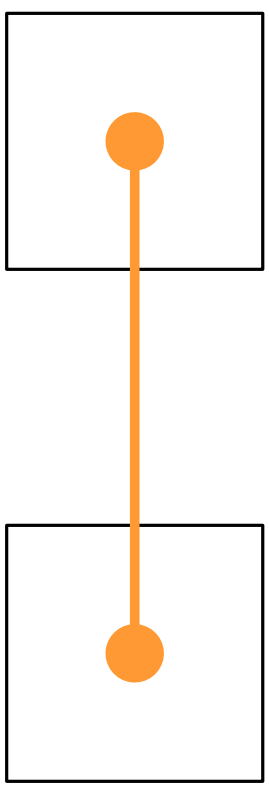


Connectivity in Unit Disk Graphs

- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ $O(\log^{6} n)$

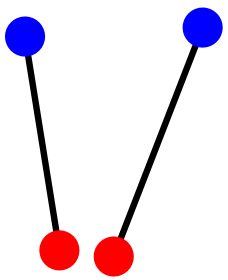


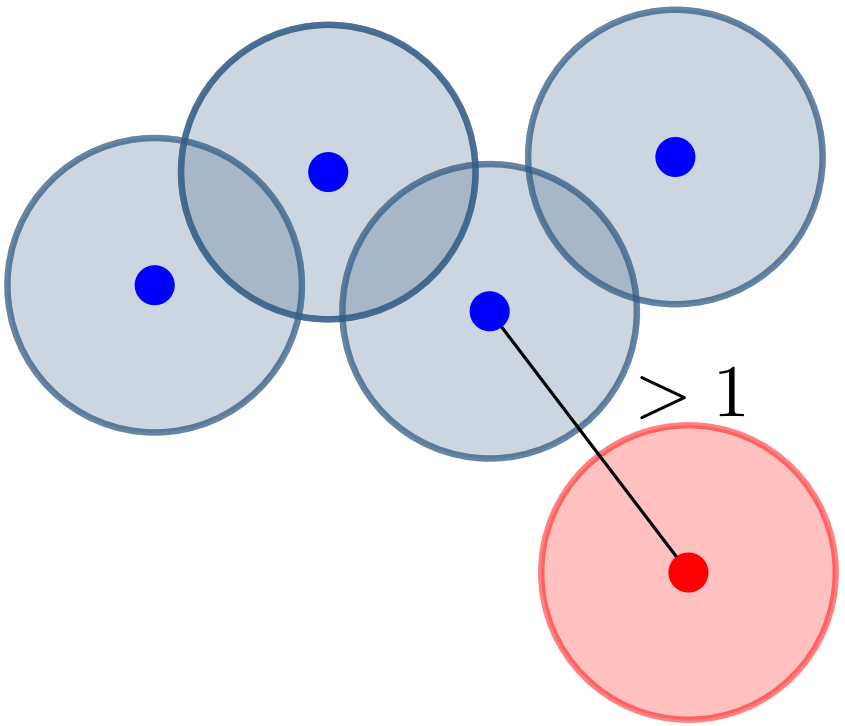
Chan: $O(\log^{6} n)$



Matching M

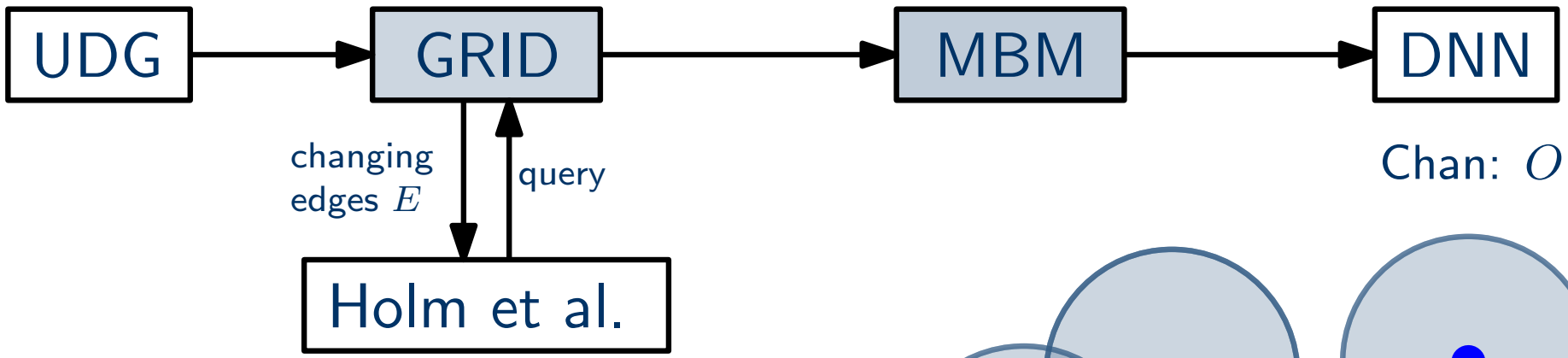
$\Leftrightarrow M \neq \emptyset$



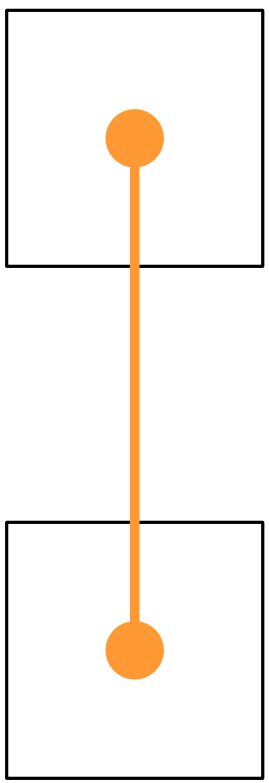


Connectivity in Unit Disk Graphs

- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ $O(\log^{6} n)$

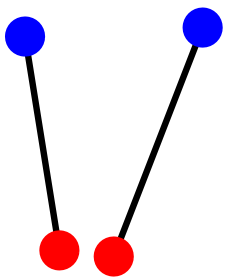


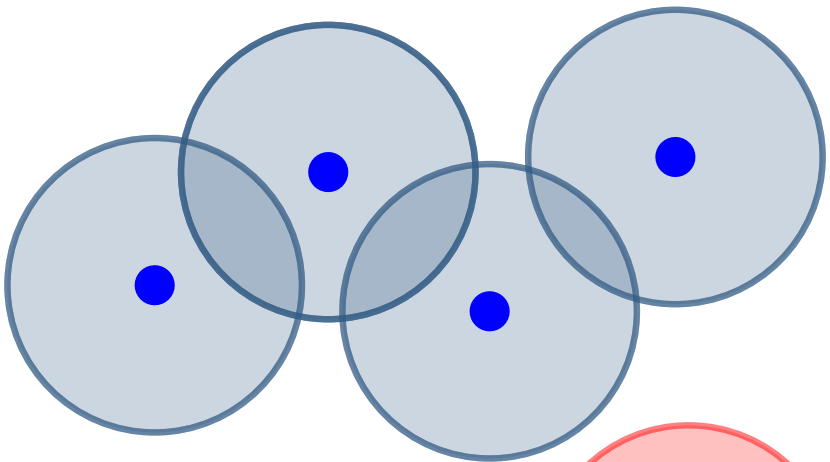
Chan: $O(\log^{6} n)$

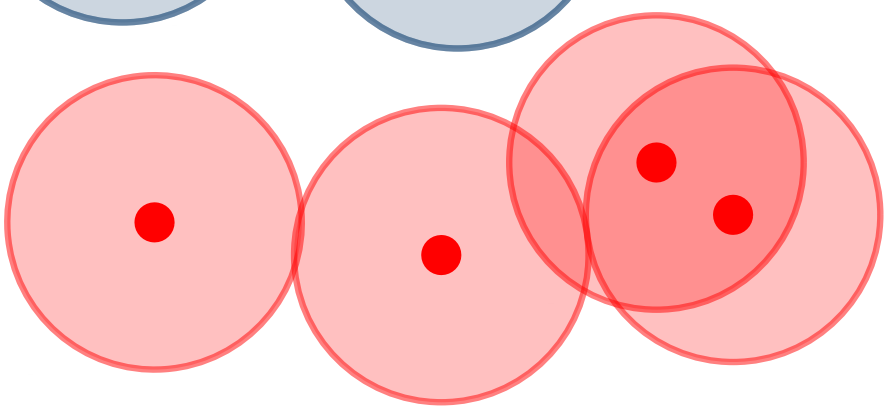


Matching M

$\Leftrightarrow M \neq \emptyset$

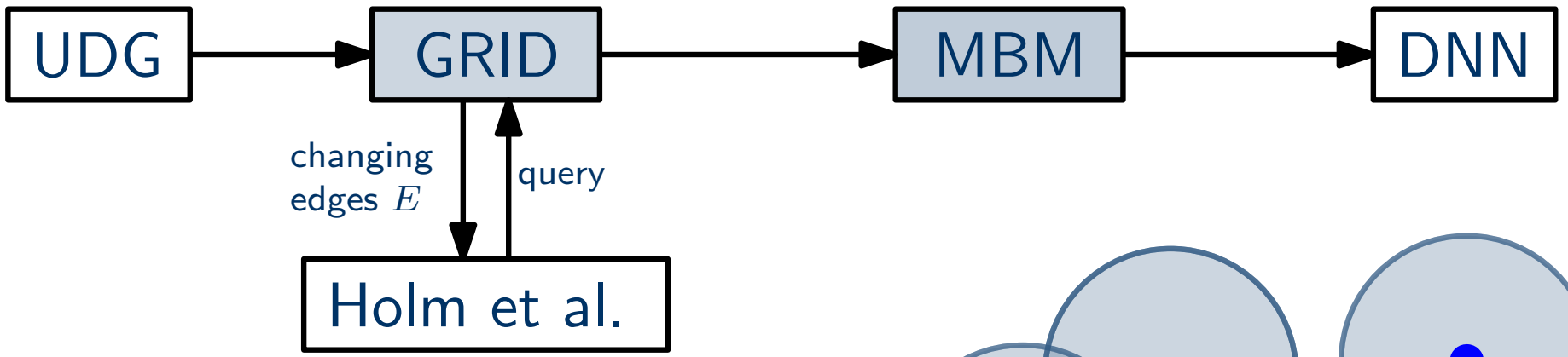


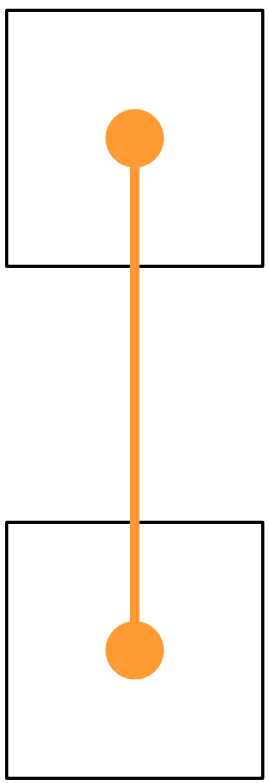




Connectivity in Unit Disk Graphs

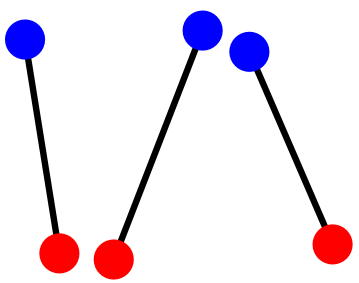
- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ $O(\log^{6} n)$

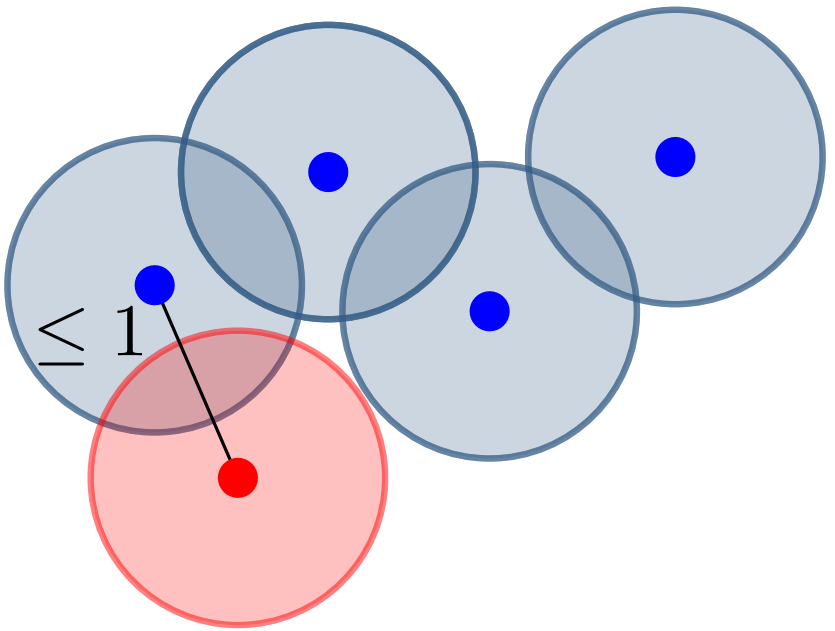




Matching M

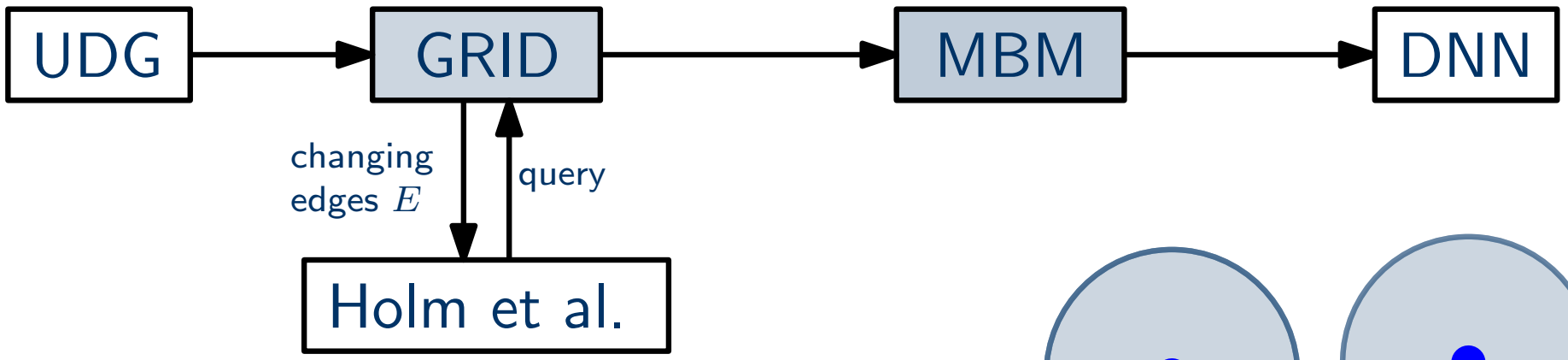
$\Leftrightarrow M \neq \emptyset$

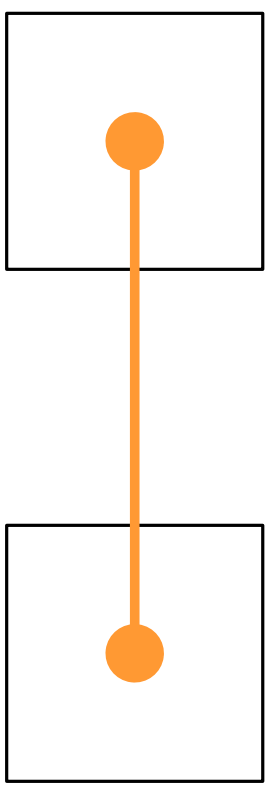




Connectivity in Unit Disk Graphs

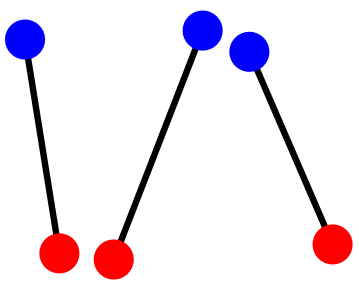
- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ $O(\log^{6} n)$

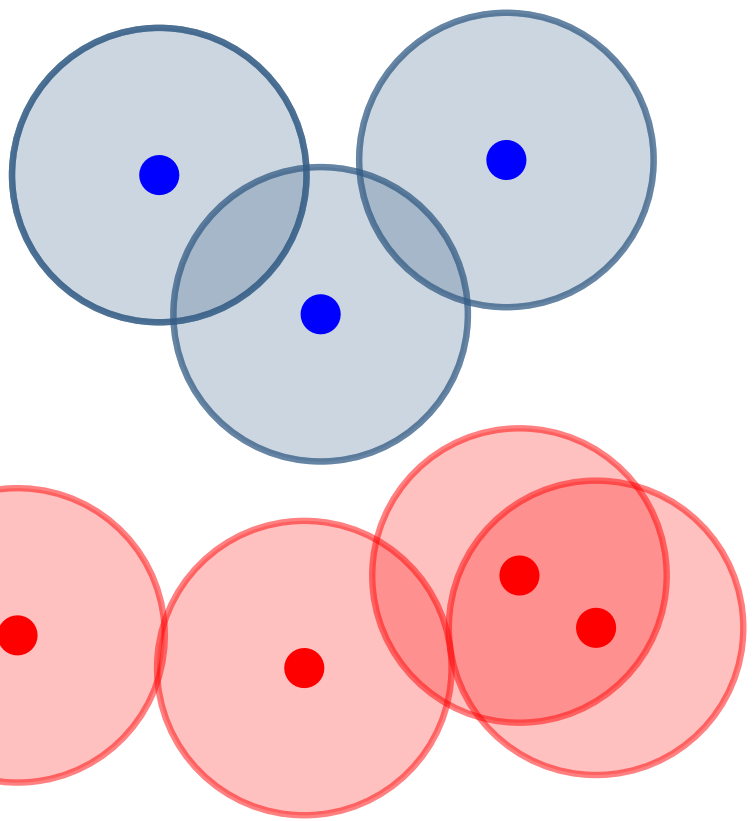




Matching M

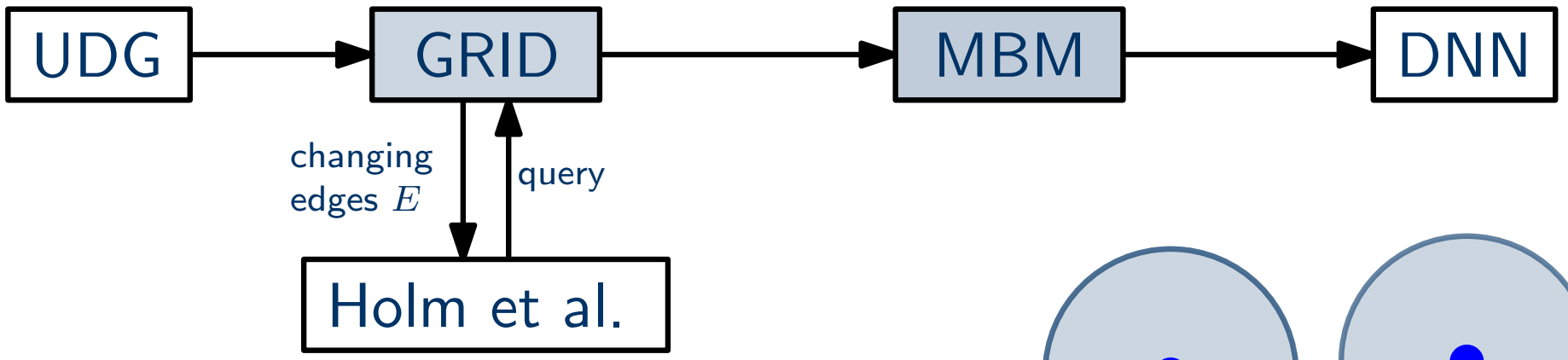
$\Leftrightarrow M \neq \emptyset$

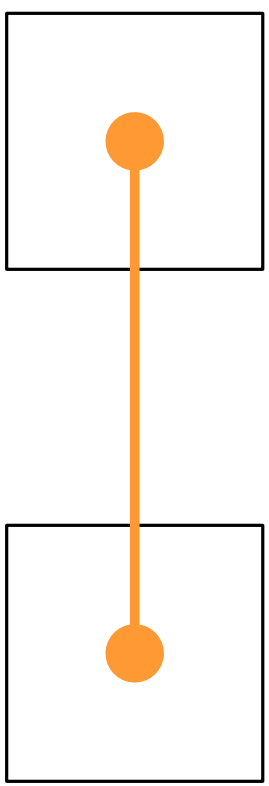




Connectivity in Unit Disk Graphs

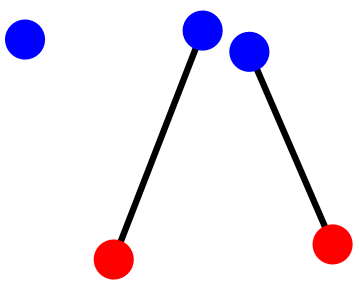
- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ $O(\log^{6} n)$

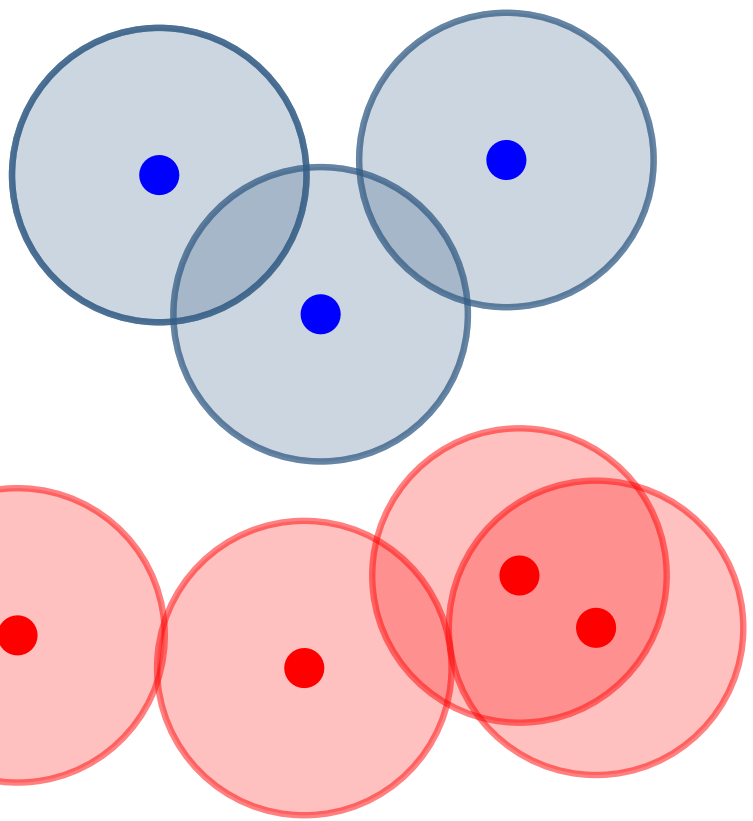




Matching M

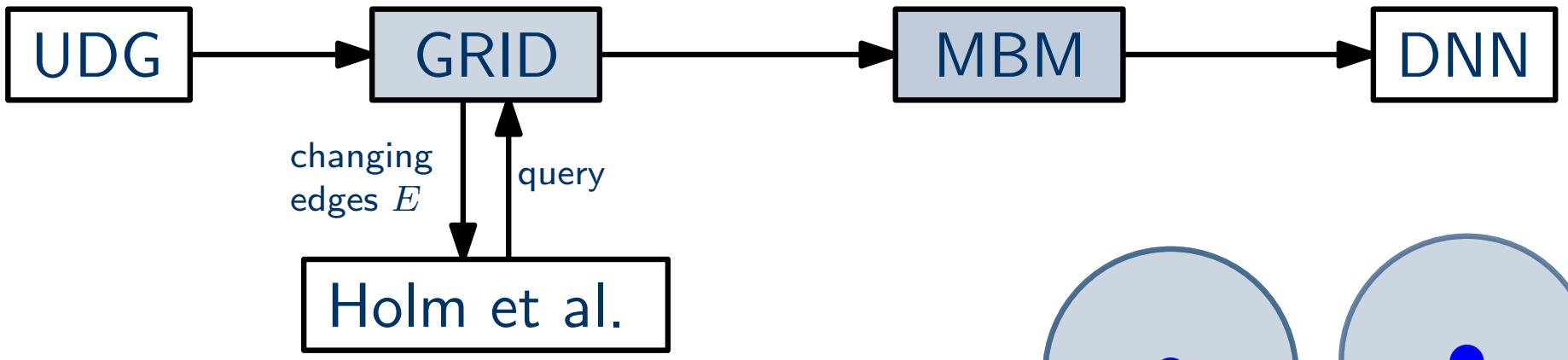
$\Leftrightarrow M \neq \emptyset$

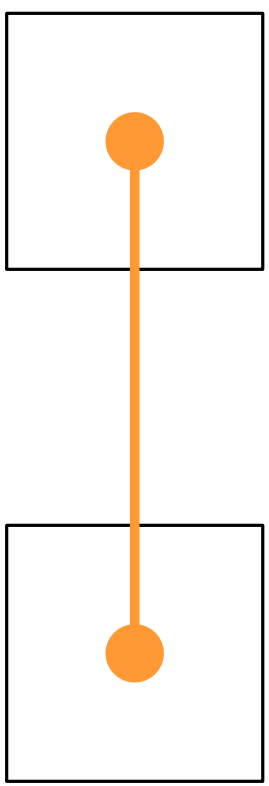




Connectivity in Unit Disk Graphs

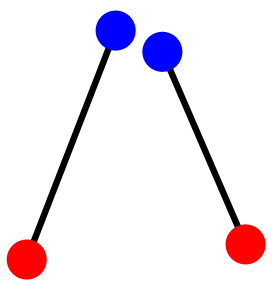
- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ $O(\log^{6} n)$

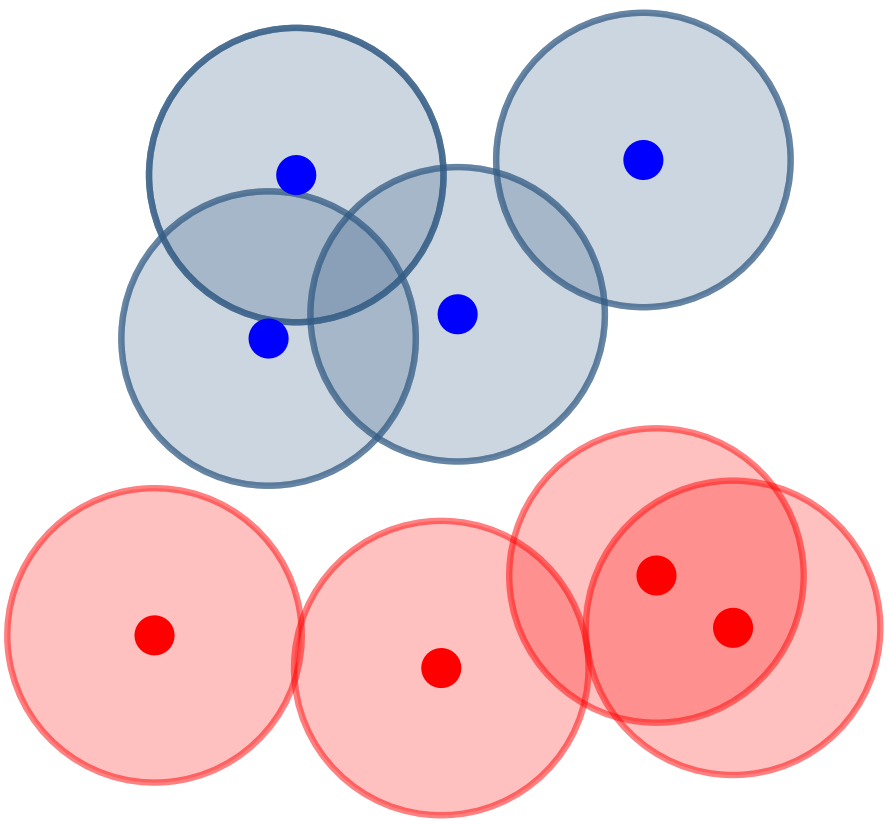




Matching M

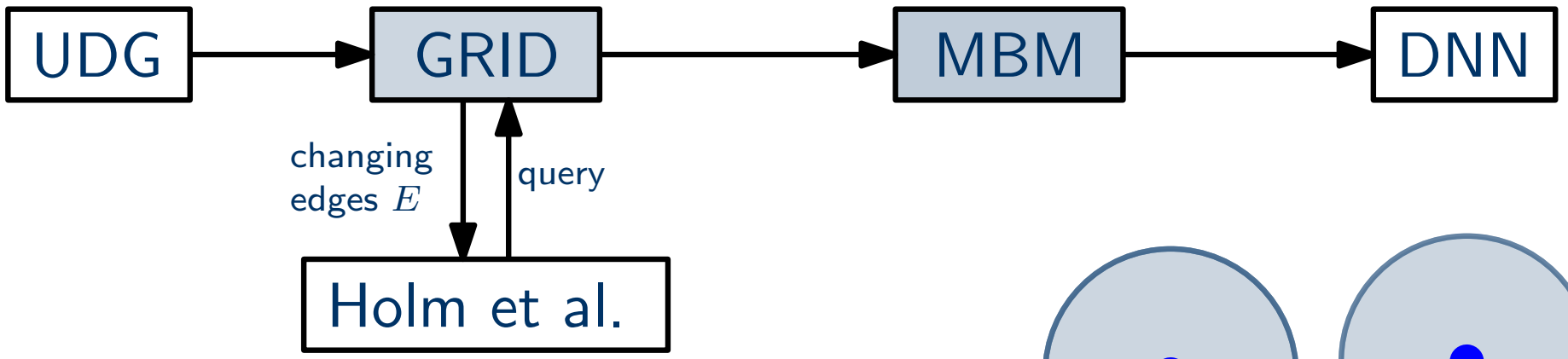
$\Leftrightarrow M \neq \emptyset$

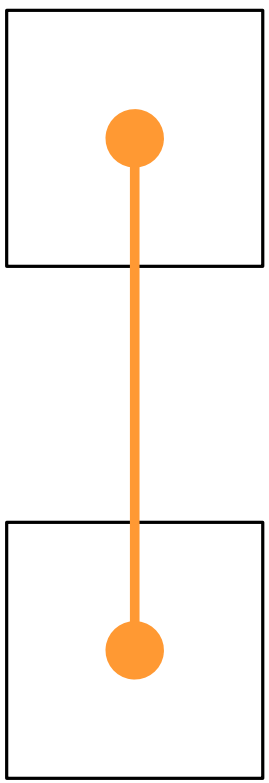




Connectivity in Unit Disk Graphs

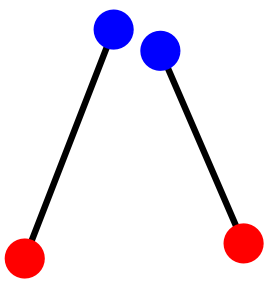
- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ $O(\log^{6} n)$

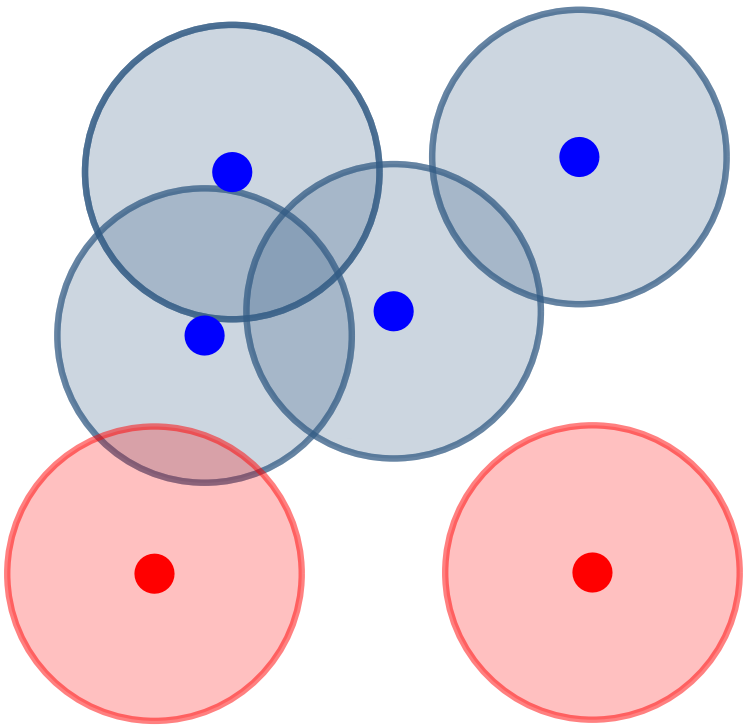




Matching M

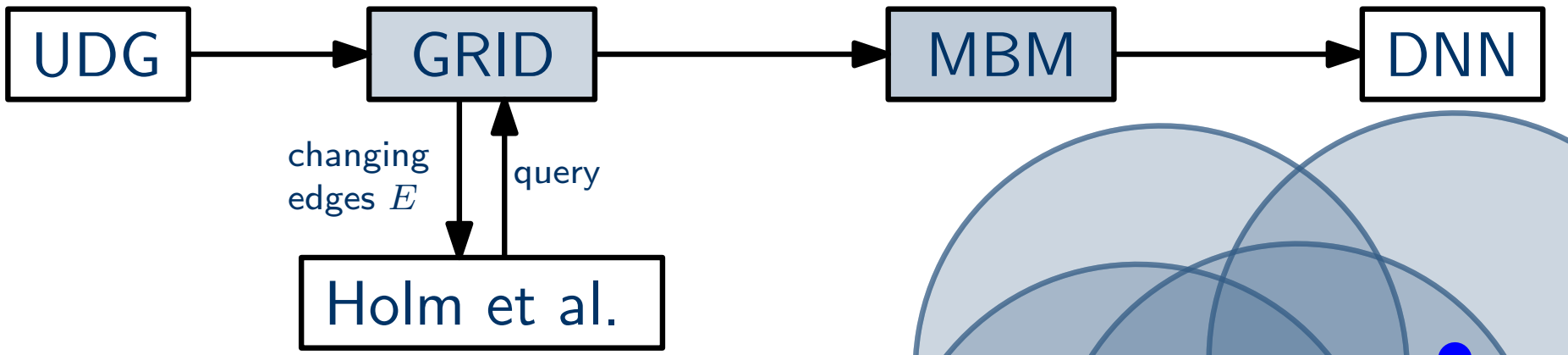
$\Leftrightarrow M \neq \emptyset$

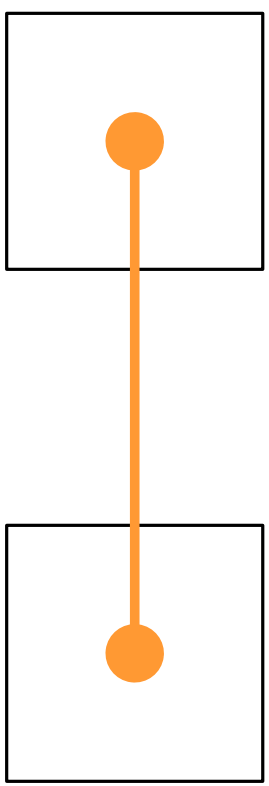




Connectivity in Unit Disk Graphs

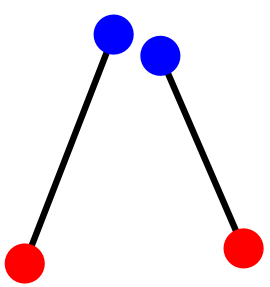
- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ $O(\log^{6} n)$

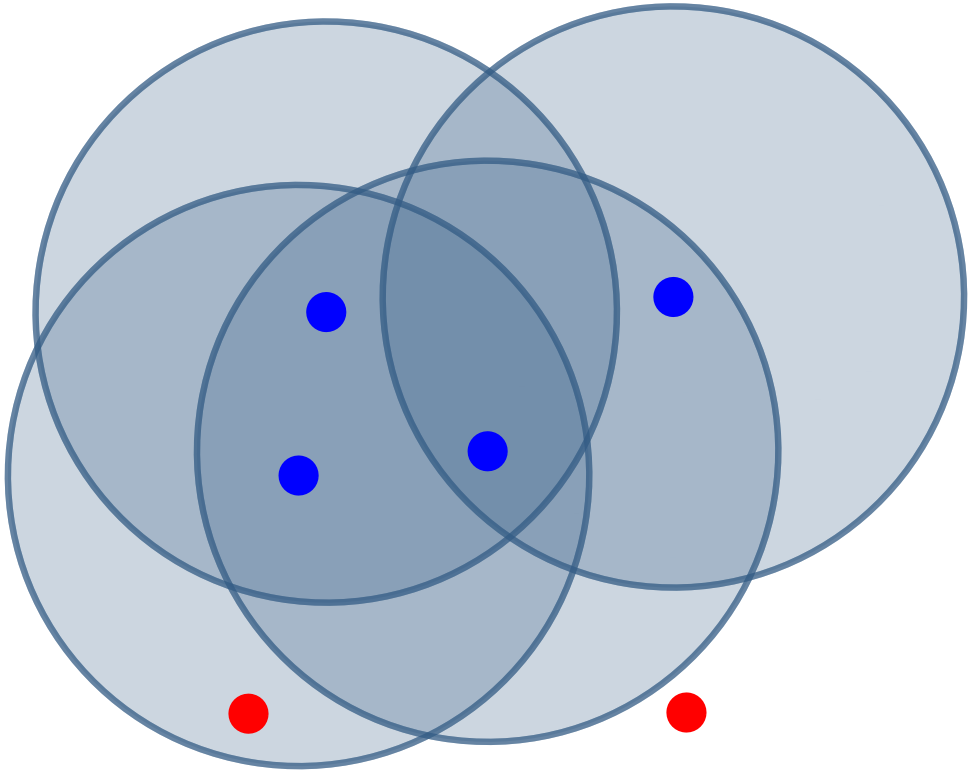




Matching M

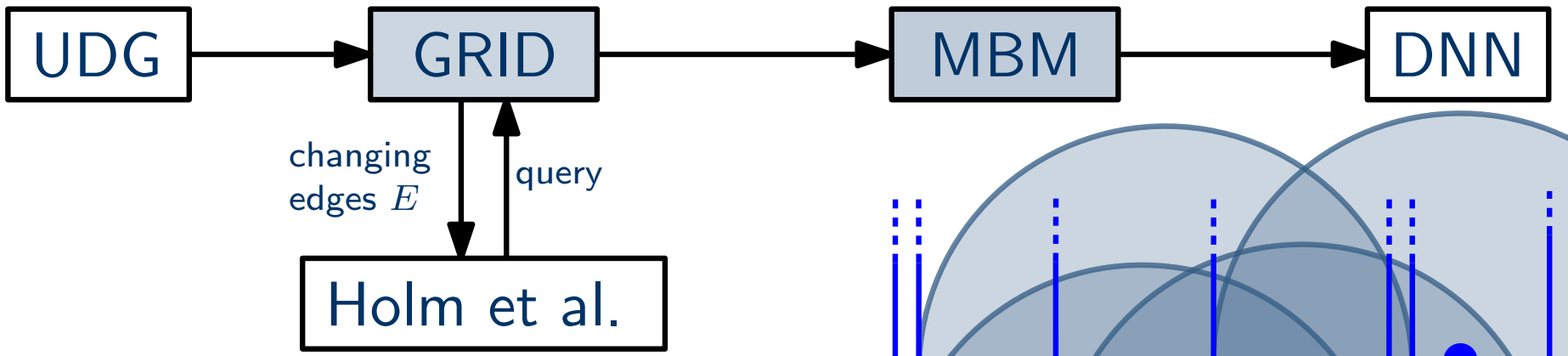
$\Leftrightarrow M \neq \emptyset$

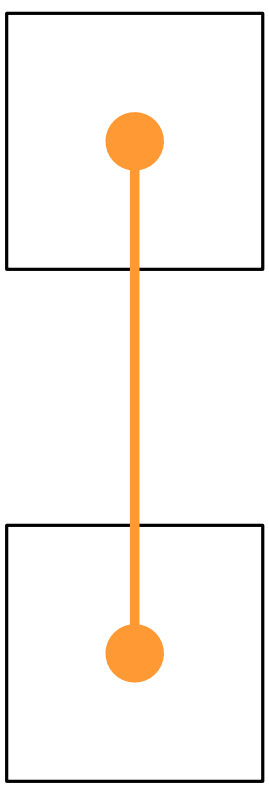




Connectivity in Unit Disk Graphs

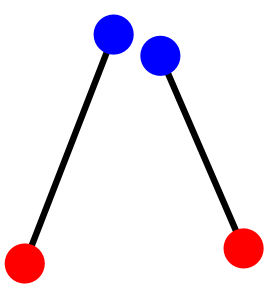
- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ $O(\log^{6} n)$

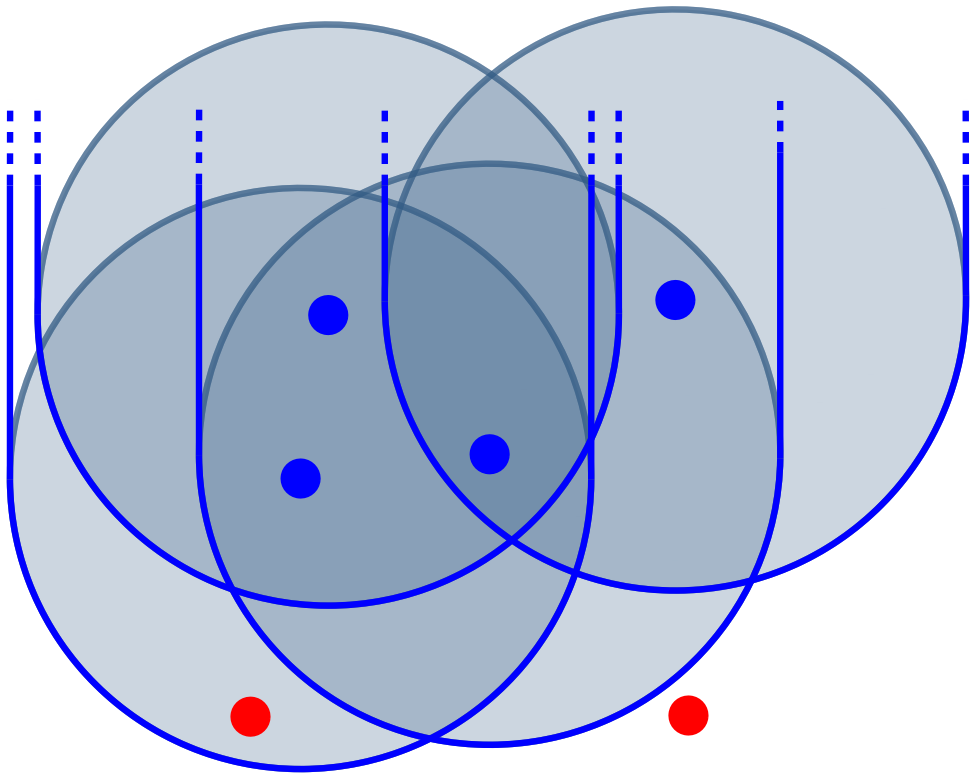




Matching M

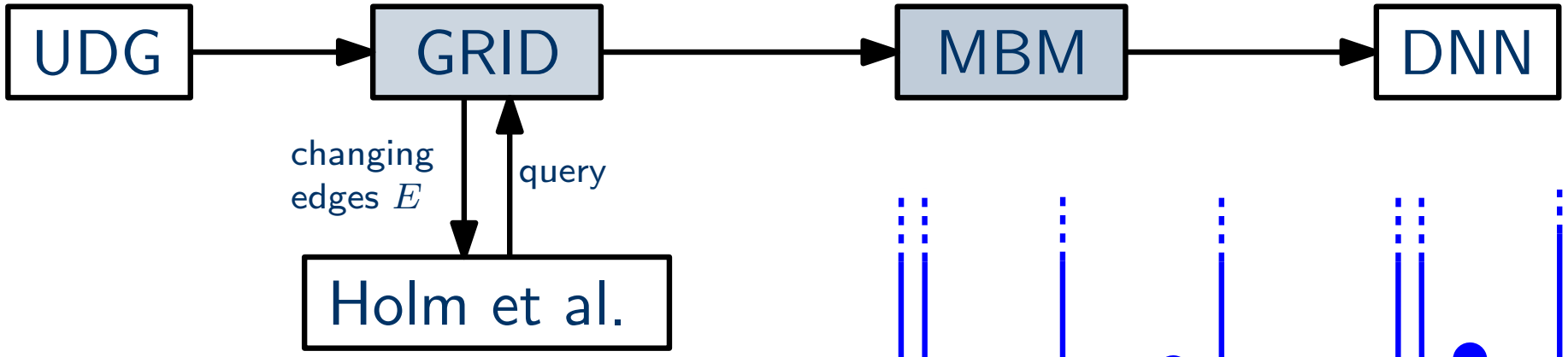
$\Leftrightarrow M \neq \emptyset$

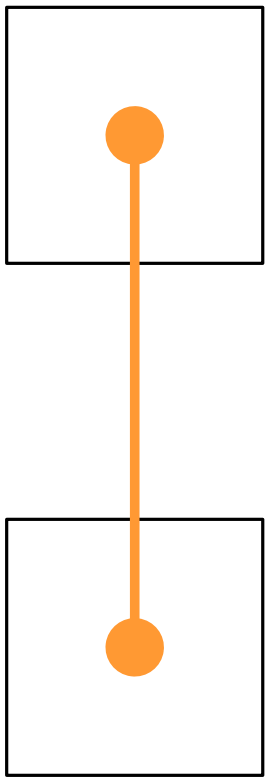




Connectivity in Unit Disk Graphs

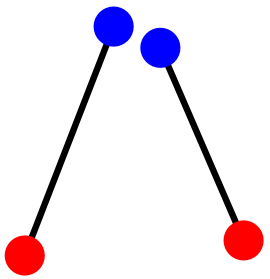
- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ $O(\log^{6} n)$

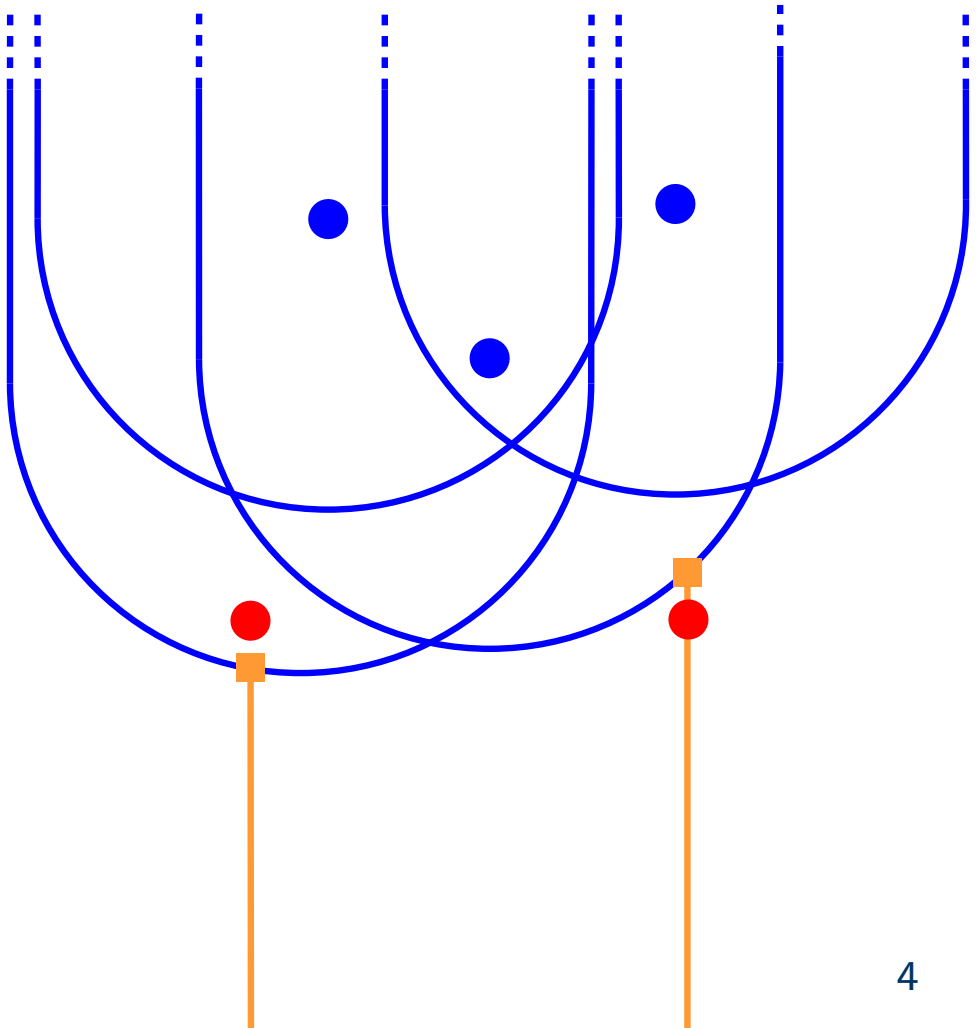




Matching M

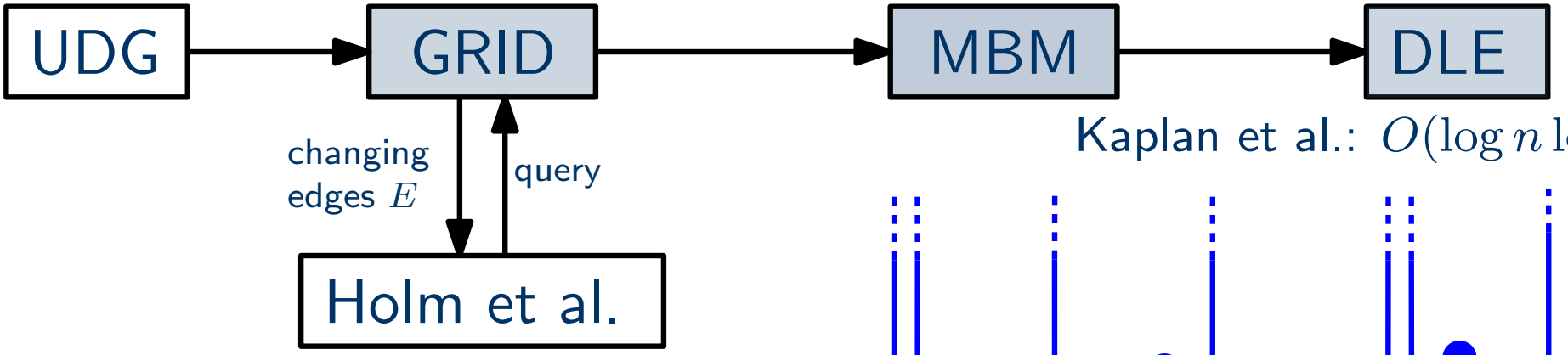
$\Leftrightarrow M \neq \emptyset$



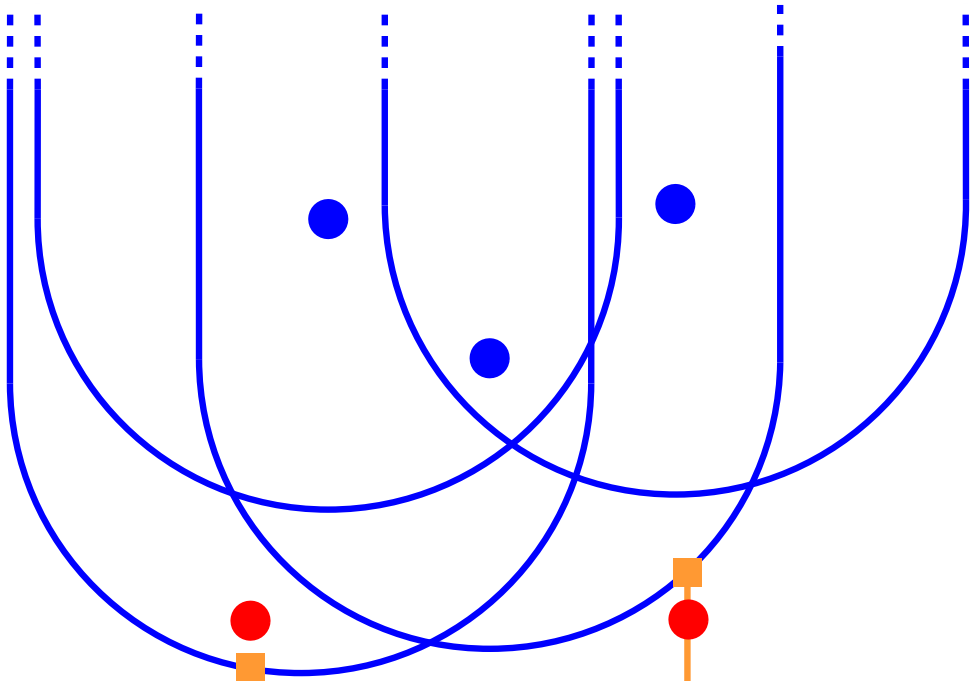


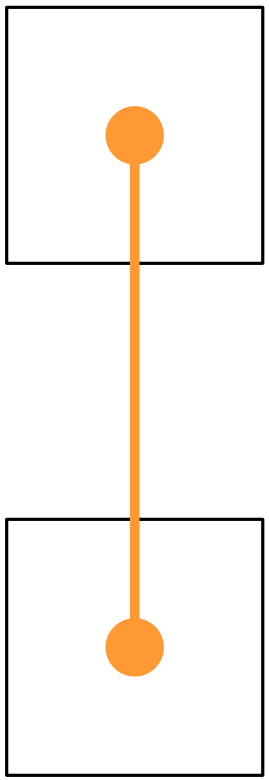
Connectivity in Unit Disk Graphs

- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ ~~$O(\log^{6} n)$~~ $O(\log^2 n)$



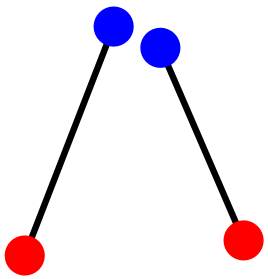
Kaplan et al.: $O(\log n \log\log n)$





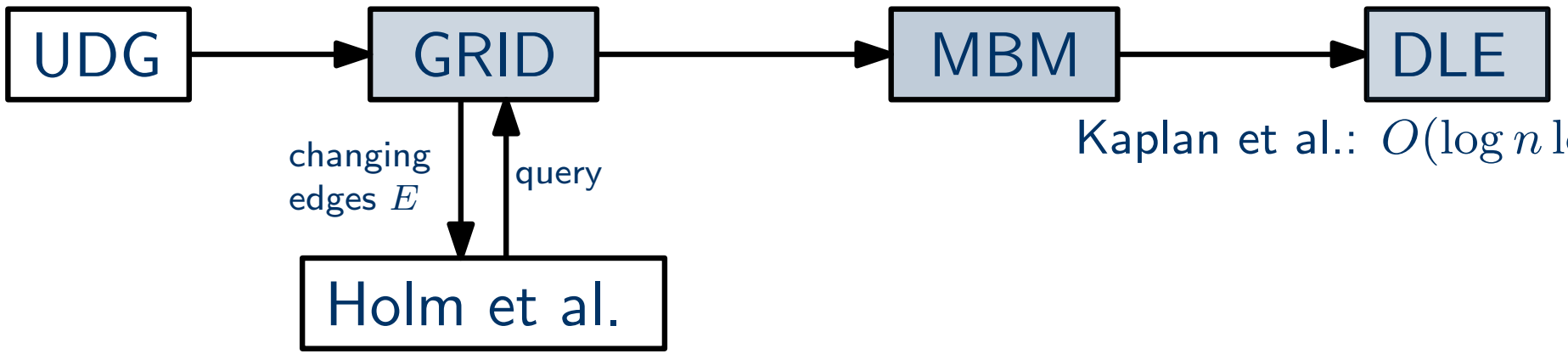
Matching M

$\Leftrightarrow M \neq \emptyset$



Connectivity in Unit Disk Graphs

- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ ~~$O(\log^{6} n)$~~ $O(\log^2 n)$



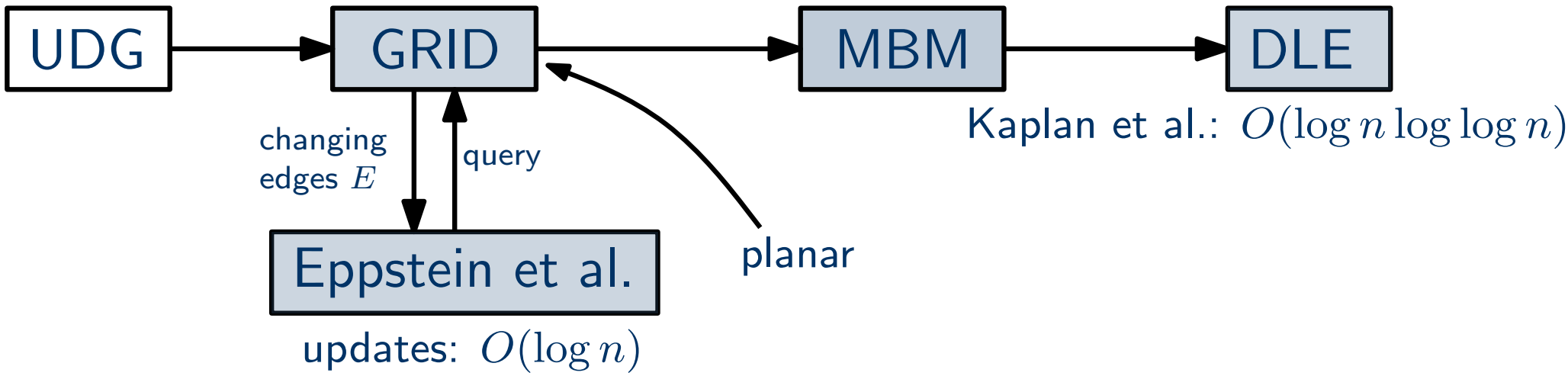
Kaplan et al.: $O(\log n \log\log n)$

Thm 1: We can dynamically maintain UD(P) with

- update time $O(\log^2 n)$
- query time $O(\log n / \log\log n)$

Connectivity in Unit Disk Graphs

- update time ~~$O(\log^{10} n)$~~ ~~$O(\log^{8} n)$~~ ~~$O(\log^{6} n)$~~ ~~$O(\log^{2} n)$~~



Thm 1: We can dynamically maintain UD(P) with
- update time $O(\log^2 n)$
- query time $O(\log n / \log\log n)$

Thm 2: ... or with
- update time $O(\log n \log\log n)$
- query time $O(\log n)$