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Variational Shape Approximation of Point Set Surfaces

Variational Shape Approximation (VSA)

The VSA procedure [1] partitions a surface $S \subseteq \mathbb{R}^3$ into $k \in \mathbb{N}$ disjoint regions $R_i \subseteq S$, $\sqcup R_i = S$, where each region is associated a linear proxy $P_i = (C_i, N_i) \in \mathbb{R}^3 \times \mathbb{S}^2$, where C_i denotes the center and N_i denotes an associated unit-length normal, i.e. every proxy appears as a plane. The proxies are fitted to the input by minimizing

$$E(\{(R_i, P_i) \mid i = 1, \dots, k\}) = \sum_{i=1}^{k} \mathcal{L}^{2,1}(R_i, P_i), \qquad (1)$$

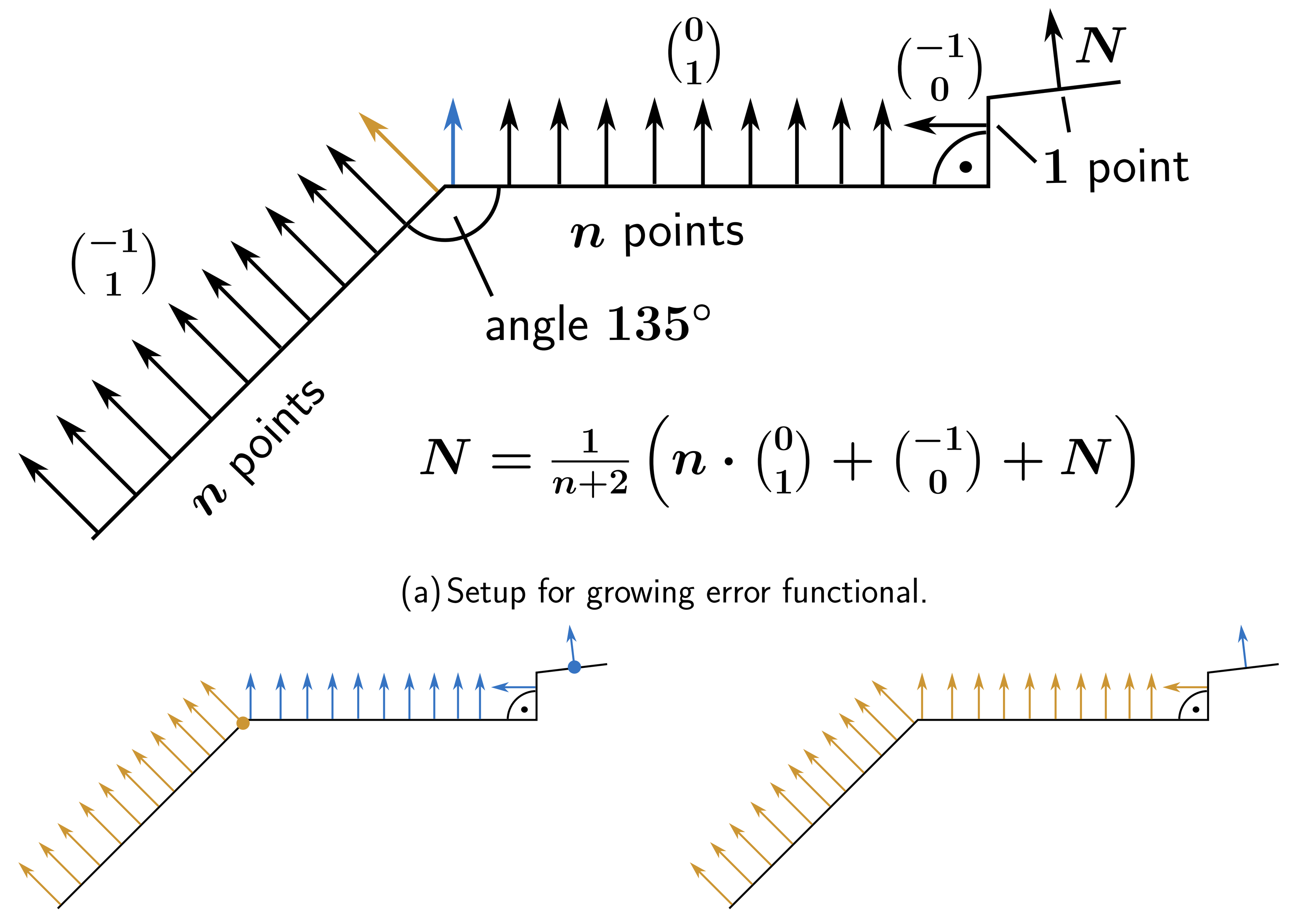
where

$$\mathcal{L}^{2,1}(R_i, P_i) = \sum_{t_j} \|n(t_j) - N_i\|^2 \, |t_j| \, . \qquad (2)$$

For point sets, the area of a triangle $|t_j|$ is not available and the normals are located at the points [2]. Minimization is performed via a k-means clustering approach separated into three steps:

1. Flood: From chosen centers C_i, propagate the normals N_i.
2. Proxy Update: Calculate proxy normals as arithmetic mean of normals in their regions.
3. Seeds: Find new center C_i in the respective region R_i from which to start flooding again.

Example: Growth in Error Functional



(a) Setup for growing error functional.

(b) Segmentation after first flood.

(c) Segmentation after second flood.

Example for a growth in the error measure after a flood and proxy update.

Evaluating (1) for the situation in Figures 1(b) and 1(c) gives an error of $E_1 \approx 1.9802$ and $E_2 \approx 39.395$ respectively, when choosing $n = 100$ points.

Goals

Obtain a version of VSA for both meshes and point sets with guaranteed convergence. Also, reduce the dependency on the a priori chosen number of seeds and their positions. To achieve this, we introduce a new user-given parameter $\kappa \in \mathbb{R}_{\geq 0}$ to measure a proxy's flatness.

New Operation: Split

For any proxy P_i with $\mathcal{L}^{2,1}(R_i, P_i) > \kappa$, use weighted principal component analysis [3] to compute the most spread direction of R_i and split it into two new regions R_i^1, R_i^2 thus increasing the number of regions k.

New Operation: Merge

Consider a pair P_i, P_j of neighboring proxies with their respective normals N_i, N_j. If the region $R' = R_i \sqcup R_j$ with normal $N' = \frac{N_i + N_j}{2}$ achieves an error measure (2) strictly less than κ, replace them by their union, decreasing k.

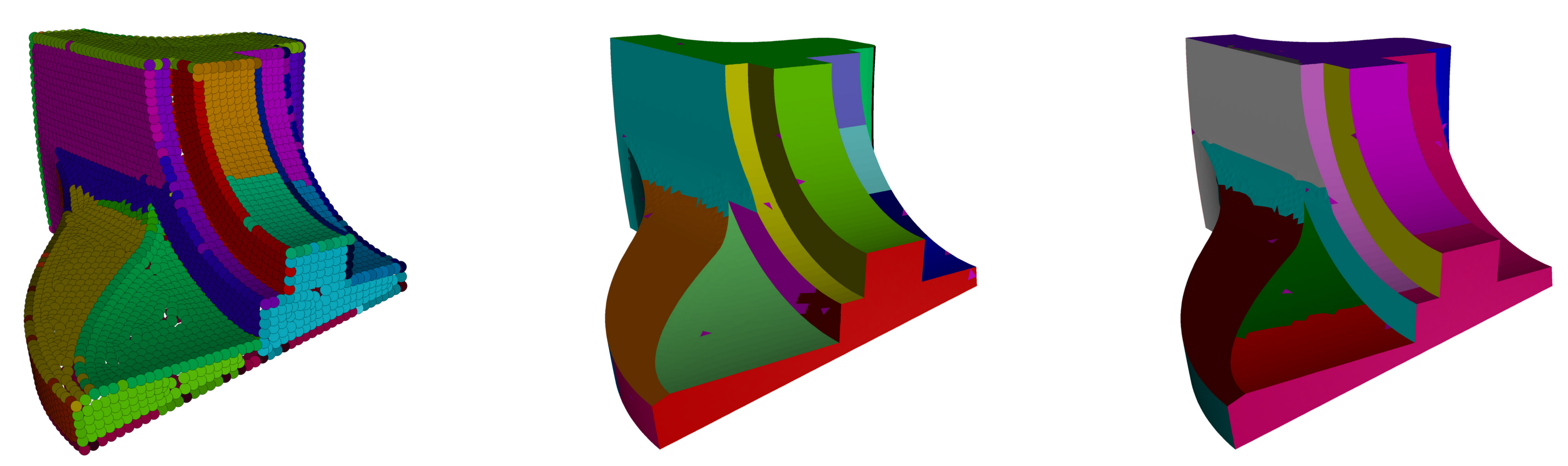
New Operation: Switch

For all elements, consider their nearest neighbors. Assume a neighbor is assigned to a different proxy, compute the change of (1) resulting from switching this assignment. Reassign the element such that the error measure is reduced maximally.

Experimental Results: Bunny

From left to right: The Bunny model segmented by our method, by automatic VSA, and by VSA with manual seed placement.

Experimental Results: Fandisk



From left to right: The Fandisk model segmented by our method, by automatic VSA, and by VSA with manual seed placement.

Future Work

- Optimize runtime of the switch operation
- Find a general framework for patch reconstruction
- Compare to other segmentation approaches

References

[1] D. Cohen-Steiner, P. Alliez, and M. Desbrun. Variational Shape Approximation. In *ACM Transactions on Graphics* (TOG), 2004.

[2] K. Lee and P. Bo. Feature curve extraction from point clouds via developable strip intersection. *Journal of Computational Design and Engineering*, 2016.

[3] P. Harris, C. Brunsdon, and M. Charlton. Geographically weighted principal components analysis. *International Journal of Geographical Information Science*, 2011.