LogiKEy in Education

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Abstract

The LogiKEy logic-pluralistic knowledge representation and reasoning methodology has been used in the education of students in computer science, philosophy and mathematics at various universities for at least a decade. References to related material are provided, many of which also include pointers to source code for reuse.

1 Main LogiKEy sources for use in classroom to date

1.1 Undergraduate Level

The following paper is used to work with LogiKEy at undergraduate level, for example, in courses such as AISE-LKR-B at the University of Bamberg:

- C. Benzmüller. "Faithful Logic Embeddings in HOL — Deep and Shallow". In: *Automated Deduction – CADE-30 – 30th International Conference on Automated Deduction, Stuttgart, Germany, July 28-31, 2025, Proceedings*. Ed. by C. Barrett and U. Waldmann. Vol. 15943. Lecture Notes in Computer Science. (preprint: arXiv:2502.19311). Springer, 2025, pp. 280–302. DOI: 10.1007/978-3-031-99984-0_16. URL: https://people.mpi-inf.mpg.de/~uwe/tmp/CADE-30-Proc.pdf
  - The Isabelle/HOL source code, screenshots and other materials related to this paper can be downloaded from the preprint version on arxiv.org. Download the LaTeX sources, which contain a subdirectory called Isabelle:
    C. Benzmüller. *Faithful Logic Embeddings in HOL – Deep and Shallow (extended preprint)*. arXiv:2502.19311. 2025. DOI: 10.48550/arXiv.2502.19311
  - Related source code is available in the Archive of Formal Proofs:
    C. Benzmüller. "Faithful Logic Embeddings in HOL – Deep and Shallow (Isabelle/HOL dataset)". In: *Archive of Formal Proofs* (2025). URL: https://www.isa-afp.org/entries/FaithfulPMLinHOL.html

1.2 Graduate Level

The following article is used to work with LogiKEy at graduate level, for example, in courses on Computational Metaphysics at the University of Bamberg:

- C. Benzmüller and D. S. Scott. "Notes on Gödel's and Scott's Variants of the Ontological Argument". In: *Monatshefte für Mathematik* 208 (2025), pp. 569–611. DOI: 10.1007/s00605-025-02078-x
  - Related source code is available in the Archive of Formal Proofs:
    C. Benzmüller and D. S. Scott. "Notes on Gödel's and Scott's Variants of the Ontological Argument (Isabelle/HOL dataset)". In: *Archive of Formal Proofs* (2025). URL: https://www.isa-afp.org/entries/Notes_On_Goedels_Ontological_Argument.html
- C. Benzmüller. "A Simplified Variant of Gödel's Ontological Argument". In: *Beyond Babel: Religions and Linguistic Pluralism*. Ed. by A. Vestrucci. Vol. 43. Sophia Studies in Cross-cultural Philosophy of Traditions and Cultures. Cham: Springer, 2023. DOI: 10.1007/978-3-031-42127-3_19. URL: https://www.researchgate.net/publication/358607847
  - Related preprint:
    C. Benzmüller. *A Simplified Variant of Gödel's Ontological Argument*. Preprint. 2022. DOI: 10.48550/ARXIV.2202.06264

  - Related source code is available in the Archive of Formal Proofs:
    C. Benzmüller. "Exploring Simplified Variants of Gödel's Ontological Argument in Isabelle/HOL". in: *Archive of Formal Proofs* (2021). Note: data publication, pp. 1–15. URL: https://www.isa-afp.org/entries/SimplifiedOntologicalArgument.html
  - See also:
    C. Benzmüller. "A (Simplified) Supreme Being Necessarily Exists, says the Computer: Computationally Explored Variants of Gödel's Ontological Argument". In: *Proceedings of the 17th International Conference on Principles of Knowledge Representation and Reasoning, KR 2020*. IJCAI organization, Sept. 2020, pp. 779–789. DOI: 10.24963/kr.2020/80

- C. Benzmüller and D. Fuenmayor. "Computer-supported Analysis of Positive Properties, Ultrafilters and Modal Collapse in Variants of Gödel's Ontological Argument". In: *Bulletin of the Section of Logic* 49.2 (2020), pp. 127–148. DOI: 10.18778/0138-0680.2020.08. URL: https://www.researchgate.net/publication/336742445
  - Related preprint:
    C. Benzmüller and D. Fuenmayor. *Computer-supported Analysis of Positive Properties, Ultrafilters and Modal Collapse in Variants of Gödel's Ontological Argument*. Preprint. 2019. DOI: 10.48550/ARXIV.1910.08955

- D. Fuenmayor and C. Benzmüller. "Automating Emendations of the Ontological Argument in Intensional Higher-Order Modal Logic". In: *KI 2017: Advances in Artificial Intelligence, 40th Annual German Conference on AI, Dortmund, Germany, September 25-29, 2017, Proceedings*. Vol. 10505. LNAI. Springer, 2017, pp. 114–127. DOI: 10.1007/978-3-319-67190-1_9. URL: http://christoph-benzmueller.de/papers/C65.pdf
  - Related source code is available in the Archive of Formal Proofs:
    D. Fuenmayor and C. Benzmüller. "Types, Tableaus and Gödel's God in Isabelle/HOL". in: *Archive of Formal Proofs* (2017). Note: data publication, pp. 1–34. URL: http://afp.sourceforge.net/entries/Types_Tableaus_and_Goedels_God.shtml

Earlier releated work here includes:

- C. Benzmüller and B. Woltzenlogel Paleo. "Interacting with Modal Logics in the Coq Proof Assistant". In: *Computer Science - Theory and Applications - 10th International Computer Science Symposium in Russia, CSR 2015, Listvyanka, Russia, July 13-17, 2015, Proceedings*. Ed. by L. D. Beklemishev and D. V. Musatov. Vol. 9139. LNCS. Springer, 2015, pp. 398–411. DOI: 10.1007/978-3-319-20297-6_25. URL: https://www.researchgate.net/publication/273201458
- C. Benzmüller and B. Woltzenlogel Paleo. "The Inconsistency in Gödel's Ontological Argument: A Success Story for AI in Metaphysics". In: *IJCAI 2016*. Ed. by S. Kambhampati. Vol. 1-3. AAAI Press, 2016, pp. 936–942. URL: https://www.researchgate.net/publication/301295955
- C. Benzmüller and B. Woltzenlogel Paleo. "Automating Gödel's Ontological Proof of God's Existence with Higher-order Automated Theorem Provers". In: *ECAI 2014*. Ed. by T. Schaub, G. Friedrich, and B. O'Sullivan. Vol. 263. Frontiers in Artificial Intelligence and Applications. (Acceptance rate $\leq 28\%$). IOS Press, 2014, pp. 93–98. DOI: 10.3233/978-1-61499-419-0-93. URL: https://www.researchgate.net/publication/265050231

2 Further LogiKEy sources for use in classroom

2.1 Warm-up example: Cantor's theorem

Section 2 of (Benzmüller and Scott, 2025) from above contains a brief discussion of Cantor's theorem, which is a useful warm-up example. The source code is available as instructed above.

See also this short paper:

- C. Benzmüller and D. Fuenmayor. "Cantor's Theorem without Reductio Ad Absurdum". 2021. DOI: 10.13140/RG.2.2.31069.95201/1. URL: https://dx.doi.org/10.13140/RG.2.2.31069.95201/1

2.2 Logic puzzles: Wise Men Puzzle

Very useful in classroom is the demonstration how logic puzzles can be solved with LogiKEy. The Wise Men Puzzle (aka muddy children puzzle) is a particularly interesting example. It can be solved using an encoding of public announcement logic in LogiKEy.

- C. Benzmüller and S. Reiche. "Automating Public Announcement Logic with Relativized Common Knowledge as a Fragment of HOL in LogiKEy". In: *Journal of Logic and Computation* 33.6 (2023), pp. 1243–1269. DOI: 10.1093/logcom/exac029. URL: https://www.researchgate.net/publication/355872829
  - Related preprint:
    C. Benzmüller and S. Reiche. *Automating Public Announcement Logic with Relativized Common Knowledge as a Fragment of HOL in LogiKEy*. Preprint. 2021. DOI: 10.48550/ARXIV.2111.01654
  - Related source code is available in the Archive of Formal Proofs:
    C. Benzmüller and S. Reiche. "Automating Public Announcement Logic and the Wise Men Puzzle in Isabelle/HOL". in: *Archive of Formal Proofs* (2021). Note: data publication, pp. 1–5. URL: https://www.isa-afp.org/entries/PAL.html

2.3 Speed-up of proofs in higher-order logic

This is very useful to illustrate the virtues of higher-order logics in comparison to less expressive logics, since exponentially shorter proofs are enabled in the former.

- C. Benzmüller, D. Fuenmayor, A. Steen, and G. Sutcliffe. "Who Finds the Short Proof?" In: *Logic Journal of the IGPL* 32.3 (2024). (preprint: arXiv.2208.06879), pp. 442–464. DOI: 10.1093/jigpal/jzac082
  - Related preprint:
    C. Benzmüller, D. Fuenmayor, A. Steen, and G. Sutcliffe. *Who Finds the Short Proof? An Exploration of Variants of Boolos' Curious Inference using Higher-order Automated Theorem Provers*. Preprint. 2022. DOI: 10.48550/arXiv.2208.06879
  - Related source code is available in the Archive of Formal Proofs:
    C. Benzmüller, D. Fuenmayor, A. Steen, and G. Sutcliffe. "Automation of Boolos' Curious Inference in Isabelle/HOL". in: *Archive of Formal Proofs* 2022 (2022). URL: https://www.isa-afp.org/entries/Boolos_Curious_Inference_Automated.html
  - See also:
    C. Benzmüller and C. Brown. "The curious inference of Boolos in MIZAR and OMEGA". in: *From Insight to Proof – Festschrift in Honour of Andrzej Trybulec*. Ed. by R. Matuszewski and A. Zalewska. Vol. 10(23). Studies in Logic, Grammar, and Rhetoric. The University of Bialystok, Polen, 2007, pp. 299–388. URL: http://christoph-benzmueller.de/papers/B6.pdf

2.4 Free logic and foundational theories (here: category theory)

- C. Benzmüller and D. S. Scott. "Automating Free Logic in HOL, with an Experimental Application in Category Theory". In: *Journal of Automated Reasoning* 64.1 (2020), pp. 53–72. DOI: 10.1007/s10817-018-09507-7. URL: http://doi.org/10.13140/RG.2.2.11432.83202
  - Related preprint:
    C. Benzmüller and D. S. Scott. *Axiomatizing Category Theory in Free Logic*. Preprint. 2016. DOI: 10.48550/ARXIV.1609.01493
  - Related source code is available in the Archive of Formal Proofs:
    C. Benzmüller and D. S. Scott. "Axiom Systems for Category Theory in Free Logic". In: *Archive of Formal Proofs* (2018). Note: data publication, pp. 1–12. URL: https://www.isa-afp.org/entries/AxiomaticCategoryTheory.html
  - See also:
    J. Bayer, A. Gonus, C. Benzmüller, and D. S. Scott. "Category Theory in Isabelle/HOL as a Basis for Meta-logical Investigation". In: *Intelligent Computer Mathematics - International Conference, CICM 2023, September 4–8, 2023, Cambridge, UK, Proceedings*. Ed. by C. Dubois and M. Kerber. Vol. 14101. Lecture Notes in Computer Science. Springer, 2023, pp. 69–83. DOI: 10.1007/978-3-031-42753-4_5 (and J. Bayer, A. Gonus, C. Benzmüller, and D. S. Scott. *Category Theory in Isabelle/HOL as a Basis for Meta-logical Investigation – Preprint*. Preprint. 2023. URL: https://arxiv.org/abs/2306.09074)

- See also:
  L. Tiemens, D. S. Scott, C. Benzmüller, and M. Benda. "Computer-supported Exploration of a Categorical Axiomatization of Modeloids". In: *Relational and Algebraic Methods in Computer Science – 18th International Conference, RAMiCS 2020, Palaiseau, France, April 8–11, 2020, Proceedings*. Vol. 12062. Lecture Notes in Computer Science. Springer, 2020, pp. 302–317. DOI: 10.1007/978-3-030-43520-2_19. URL: https://www.researchgate.net/publication/336838722
- See also the code of Jonas Bayer at https://gitlab.com/jonas.bayer/bachelor-thesis

2.5 Further examples in computational metaphysics

- D. Fuenmayor and C. Benzmüller. "A Case Study On Computational Hermeneutics: E. J. Lowe's Modal Ontological Argument". In: *Journal of Applied Logic - IfCoLoG Journal of Logics and their Applications (special issue on Formal Approaches to the Ontological Argument)* 5.7 (2018). Published also as chapter in the book 'Beyond Faith and Rationality: Essays on Logic, Religion and Philosophy' printed in the Springer book series 'Sophia Studies in Cross-cultural Philosophy of Traditions and Cultures', pp. 1567–1603. URL: https://www.researchgate.net/publication/333804824
  - Related source code is available in the Archive of Formal Proofs:
    D. Fuenmayor and C. Benzmüller. "Computer-assisted Reconstruction and Assessment of E. J. Lowe's Modal Ontological Argument". In: *Archive of Formal Proofs* (2017). Note: data publication, pp. 1–19. URL: https://www.isa-afp.org/entries/Lowe_Ontological_Argument.html
  - See also:
    D. Fuenmayor and C. Benzmüller. "A Case Study On Computational Hermeneutics: E. J. Lowe's Modal Ontological Argument". In: *Beyond Faith and Rationality: Essays on Logic, Religion and Philosophy*. Ed. by R. Silvestre, B. Göcke, J.-Y. Béziau, and P. Bilimoria. Vol. 34. Sophia Studies in Cross-cultural Philosophy of Traditions and Cultures. Springer Nature Switzerland AG, 2020. Chap. 12. DOI: 10.1007/978-3-030-43535-6_12

2.6 Ambitious examples in computational metaphysics

There is a special course offered at U Bamberg (and before at FU Berlin) by the authors of the following article, in which the related highly non-trivial encoding work is demonstrated and discussed:

- D. Kirchner, C. Benzmüller, and E. N. Zalta. "Mechanizing Principia Logico-Metaphysica in Functional Type Theory". In: *Review of Symbolic Logic* 13.1 (2020), pp. 206–218. DOI: 10.1017/S1755020319000297. URL: https://www.researchgate.net/publication/321160582
  - Related source code is available in the Archive of Formal Proofs:
    D. Kirchner. "Abstract Object Theory". In: *Archive of Formal Proofs* (Nov. 2022). https://isa-afp.org/entries/AOT.html, Formal proof development
  - See also:
    D. Kirchner. "Computer-Verified Foundations of Metaphysics and an Ontology of Natural Numbers in Isabelle/HOL". PhD thesis. Free University of Berlin, Germany, 2022. URL: https://refubium.fu-berlin.de/handle/fub188/35426
  - See also:
    D. Kirchner, C. Benzmüller, and E. N. Zalta. "Computer Science and Metaphysics: A Cross-Fertilization". In: *Open Philosophy* 2.1 (2019). Ed. by P. Grim, pp. 230–251. DOI: 10.1515/opphil-2019-0015. URL: http://doi.org/10.13140/RG.2.2.25229.18403

2.7 Normative and legal reasoning

2.7.1 Deontic logics and Chisholm's Paradox

- C. Benzmüller, X. Parent, and L. van der Torre. "Designing Normative Theories for Ethical and Legal Reasoning: LogiKEy Framework, Methodology, and Tool Support". In: *Artificial Intelligence* 287 (2020), p. 103348. DOI: 10.1016/j.artint.2020.103348. URL: https://www.researchgate.net/publication/342146653
  - Related source code is available as:
    C. Benzmüller, A. Farjami, D. Fuenmayor, P. Meder, X. Parent, A. Steen, L. van der Torre, and V. Zahoransky. "LogiKEy Workbench: Deontic Logics, Logic Combinations and Expressive Ethical and Legal Reasoning (Isabelle/HOL Dataset)". In: *Data in Brief* 33.106409 (2020), pp. 1–10. DOI: 10.1016/j.dib.2020.106409

2.7.2 Logic of the right to know

- Lawniczak, L. Pasetto, C. Benzmüller, X. Li, and R. Markovich. "Reasoning with Epistemic Rights and Duties: Automating a Dynamic Logic of the Right to Know in LogiKEy". In: *ECAI 2025 - 28th European Conference on Artificial Intelligence*. Ed. by I. Lynce, N. Murano, M. Vallati, S. Villata, F. Chesani, M. Milano, A. Omicini, and M. Dastani. Vol. 413. IOS Press, 2025, pp. 1623–1630. DOI: 10.3233/FAIA250988

  – The sources of the encoding can be found at https://github.com/cbenzmueller/LogiKEy/tree/master/LRK

2.7.3 Value-oriented legal reasoning

- C. Benzmüller, D. Fuenmayor, and B. Lomfeld. "Modelling Value-oriented Legal Reasoning in LogiKEy". In: *Logics* 2.1 (2024), pp. 31–78. DOI: 10.3390/logics2010003

  – Related preprint:
  C. Benzmüller, D. Fuenmayor, and B. Lomfeld. *Modelling Value-oriented Legal Reasoning in LogiKEy*. Preprint. 2020. DOI: 10.48550/ARXIV.2006.12789

  – See also:
  C. Benzmüller and D. Fuenmayor. "Value-oriented Legal Argumentation in Isabelle/HOL". in: *International Conference on Interactive Theorem Proving (ITP), Proceedings*. Ed. by L. Cohen and C. Kaliszyk. Vol. 193. LIPIcs 23. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2021, 23:1–23:18. DOI: 10.4230/LIPIcs.ITP.2021.7. URL: https://dx.doi.org/10.13140/RG.2.2.21514.80320

2.7.4 Conditional normative reasoning

- X. Parent and C. Benzmüller. "Conditional Normative Reasoning as a Fragment of HOL". in: *Journal of Applied Non-Classical Logics* 34.4 (2024), pp. 1–32. DOI: 10.1080/11663081.2024.2386917

  – Related preprint:
  X. Parent and C. Benzmüller. *Normative Conditional Reasoning as a Fragment of HOL*. Preprint. 2023. DOI: 10.48550/arXiv.2308.10686

  – Related source code is available in the Archive of Formal Proofs:
  X. Parent and C. Benzmüller. "Conditional Normative Reasoning as a Fragment of HOL (Isabelle/HOL dataset)". In: *Archive of Formal Proofs* (2024). URL: https://www.isa-afp.org/entries/CondNormReasHOL.html

2.8 Abstract argumentation

- D. Fuenmayor and C. Benzmüller. "Computer-supported Analysis of Arguments in Climate Engineering". In: *Logic and Argumentation, Fourth International Conference, CLAR 2021, Hangzhou, China, April 6-9, 2020, Proceedings*. Ed. by M. Dastani, H. Dong, and L. van der Torre. Vol. 12061. Lecture Notes in Computer Science. Springer, Cham, 2020, pp. 104–115. DOI: 10.1007/978-3-030-44638-3_7. URL: https://www.researchgate.net/publication/338829452

  – Source code: https://github.com/davfuenmayor/CE-Debate

- C. Benzmüller, M. Claus, and N. Sultana. "Systematic Verification of the Modal Logic Cube in Isabelle/HOL". in: *PxTP 2015*. Ed. by C. Kaliszyk and A. Paskevich. Vol. 186. Berlin, Germany: EPTCS, 2015, pp. 27–41. DOI: 10.4204/EPTCS.186.5. URL: http://christoph-benzmueller.de/papers/C47.pdf

  – See also:
  C. Benzmüller. "Verifying the Modal Logic Cube is an Easy Task (for Higher-Order Automated Reasoners)". In: *Verification, Induction, Termination Analysis - Festschrift for Christoph Walther on the Occasion of His 60th Birthday*. Ed. by S. Siegler and N. Wasser. Vol. 6463. LNCS. (Superseded by PxTP-2015 paper). Springer, 2010, pp. 117–128. DOI: 10.1007/978-3-642-17172-7_7. URL: http://christoph-benzmueller.de/papers/B12.pdf

2.9 Logic explorations and verification

- C. Benzmüller, M. Claus, and N. Sultana. "Systematic Verification of the Modal Logic Cube in Isabelle/HOL". in: *PxTP 2015*. Ed. by C. Kaliszyk and A. Paskevich. Vol. 186. Berlin, Germany: EPTCS, 2015, pp. 27–41. DOI: 10.4204/EPTCS.186.5. URL: http://christoph-benzmueller.de/papers/C47.pdf

  - See also:
    C. Benzmüller. "Verifying the Modal Logic Cube is an Easy Task (for Higher-Order Automated Reasoners)". In: *Verification, Induction, Termination Analysis - Festschrift for Christoph Walther on the Occasion of His 60th Birthday*. Ed. by S. Siegler and N. Wasser. Vol. 6463. LNCS. (Superseded by PxTP-2015 paper). Springer, 2010, pp. 117–128. DOI: 10.1007/978-3-642-17172-7_7. URL: http://christoph-benzmueller.de/papers/B12.pdf

- C. Benzmüller, H. Lachnitt, and M. Claus. *Systematic Verification of the Intuitionistic Modal Logic Cube in Isabelle/HOL*. tech. rep. Freie Universität Berlin, 2017. DOI: 10.13140/RG.2.2.26232.56325

  - See also:
    C. Benzmüller and L. C. Paulson. "Multimodal and Intuitionistic Logics in Simple Type Theory". In: *The Logic Journal of the IGPL* 18.6 (2010), pp. 881–892. DOI: 10.1093/jigpal/jzp080. URL: http://christoph-benzmueller.de/papers/J21.pdf

2.10 Other material

- D. Fuenmayor and C. Benzmüller. "Formalisation and Evaluation of Alan Gewirth's Proof for the Principle of Generic Consistency in Isabelle/HOL". in: *Archive of Formal Proofs* (2018). Note: data publication, pp. 1–15. URL: https://www.isa-afp.org/entries/GewirthPGCProof.html

3 Papers on the use of theorem provers and proof assistants in classroom (including very early papers)

- C. Benzmüller and D. Fuenmayor. "Mathematical Proof Assistants for Teaching Logic: The LogiKEy Methodology". In: *Book of Abstracts — V Congress Tools for Teaching Logic*. Madrid, Spain, March 23-24 2023, 2023. DOI: 10.13140/RG.2.2.24708.74888

- M. Wisniewski, A. Steen, and C. Benzmüller. "Einsatz von Theorembeweisern in der Lehre". In: *Hochschuldidaktik der Informatik: 7. Fachtagung des GI-Fachbereichs Informatik und Ausbildung/Didaktik der Informatik; 13.-14. September 2016 an der Universität Potsdam*. Ed. by A. Schwill and U. Lucke. Commentarii informaticae didacticae (CID). Potsdam, Germany: Universitätsverlag Potsdam, 2016, pp. 81–92. URL: https://publishup.uni-potsdam.de/opus4-ubp/frontdoor/index/index/docId/9485

- C. Benzmüller, M. Wisniewski, and A. Steen. "Computational Metaphysics". This lecture course proposal received the 2015 central teaching award of FU Berlin. 2015. DOI: 10.13140/RG.2.1.3535.2568. URL: https://dx.doi.org/10.13140/RG.2.1.3535.2568

- M. Schiller and C. Benzmüller. "Human-Oriented Proof Techniques are Relevant for Proof Tutoring". In: *Workshop on Mathematically Intelligent Proof Search (MIPS 2010, affiliated with CICM 2010)*. Paris, France, 2010. URL: http://christoph-benzmueller.de/papers/W45.pdf

- M. Schiller, D. Dietrich, and C. Benzmüller. "Proof Step Analysis for Proof Tutoring – A Learning Approach to Granularity". In: *Teaching Mathematics and Computer Science* 6.2 (2008), pp. 325–343. URL: http://tmcs.math.unideb.hu/load_doc.php?p=142&t=doc

- C. Benzmüller, D. Dietrich, M. Schiller, and S. Autexier. "Deep Inference for Automated Proof Tutoring?" In: *KI 2007: Advances in Artificial Intelligence, 30th Annual German Conference on AI, KI 2007, Osnabrück, Germany, September 10-13, 2007, Proceedings*. Ed. by J. Hertzberg, M. Beetz, and R. Englert. Vol. 4667. Springer, 2007, pp. 435–439. DOI: 10.1007/978-3-540-74565-5_34. URL: http://christoph-benzmueller.de/papers/C24.pdf

- C. Benzmüller, A. Fiedler, M. Gabsdil, H. Horacek, I. Kruijff-Korbayova, D. Tsovaltzi, B. Q. Vo, and M. Wolska. "Towards a Principled Approach to Tutoring Mathematical Proofs". In: *Proceedings of the Workshop on Expressive Media and Intelligent Tools for Learning, German Conference on AI (KI 2003)*. Hamburg, Germany, 2003. URL: http://christoph-benzmueller.de/papers/W26.pdf

- J. Siekmann, C. Benzmüller, A. Fiedler, A. Franke, G. Goguadze, H. Horacek, M. Kohlhase, P. Libbrecht, A. Meier, E. Melis, M. Pollet, V. Sorge, C. Ullrich, and J. Zimmer. "Adaptive Course Generation and Presentation". In: *Proceedings of the Fifth International Conference on Intelligent Tutoring Systems—Workshop W2: Adaptive and Intelligent Web-Based Education Systems*. Ed. by P. Brusilovski. Montreal, 2000, pp. 54–61. URL: http://christoph-benzmueller.de/papers/W5.pdf

See also the following papers for a demonstration of how an interactive proof for the irrationality for the square root of 2 was developed interactively in the OMEGA proof assistant system we developed at Saarbrücken until about 2008. This included natural language translations of automatically found (sub-)proofs.

- J. Siekmann, C. Benzmüller, A. Fiedler, A. Meier, I. Normann, and M. Pollet. "Proof Development in OMEGA: The Irrationality of Square Root of 2". In: *Thirty Five Years of Automating Mathematics*. Ed. by F. Kamareddine. Applied Logic series (28). Kluwer Academic Publishers, 2003, pp. 271–314. DOI: 10.1007/978-94-017-0253-9_11. URL: http://christoph-benzmueller.de/papers/B1.pdf
- J. Siekmann, C. Benzmüller, A. Fiedler, A. Meier, and M. Pollet. "Proof Development with OMEGA: Sqrt(2) is irrational". In: *Logic for Programming, Artificial Intelligence, and Reasoning, 9th International Conference, LPAR 2002*. Ed. by M. Baaz and A. Voronkov. LNCS 2514. Springer, 2002, pp. 367–387. URL: http://christoph-benzmueller.de/papers/C12.pdf

The full details can be found in this techreport:

- C. Benzmüller, A. Fiedler, A. Meier, and M. Pollet. *Irrationality of square root of 2 – A case study in OMEGA*. SEKI Report SR-02-03. Saarland University, SEKI Publications (ISSN 1437-4447), 2002, pp. 1–103. DOI: 10.22028/D291-42121. URL: http://christoph-benzmueller.de/papers/R15.pdf