PROOF (of Lemma 2.2): Let $\mathcal{T}$ be a binary tree with height d and let

$$w(\mathcal{T}) := \sum_{i=0}^{d} 2^i v_i,$$

where v_i is the number of leaves of $\mathcal{T}$ at level i. At first we change $\mathcal{T}$ by applying the following two transformations, as long as they are possible (d always denotes the height of the current tree $\mathcal{T}$):

Transformation I (cf. Figure 4): If there is a node y at some level $i \leqq d - 2$, which has at most one successor, then we remove an arbitrary node x from level d and make it a successor of y.

By transformation I, w decreases by 2^d by removing x and increases by at most 2^{d-1} by inserting x. Moreover, w may additionally increase by 2^{d-1}, if the predecessor of x becomes a leaf in τ'. Thus we get in any case

$$w(\tau') \leqq w(\tau).$$

Transformation II (cf. Figure 5): If there is a node z at level $d - 1$ with two successors x and y and another node u at level $d - 1$ which is a leaf, then we make y a successor of u.

By transformation II $w(\tau)$ decreases by 2^{d-1}, because u ceases to be a leaf.

When we perform these two transformations as long as they are possible, we arrive finally at a tree $\hat{\tau}$ with the following properties:

- τ and $\hat{\tau}$ have the same number of nodes;
- $w(\hat{\tau}) \leqq w(\tau)$;
- $v_i(\hat{\tau}) = 0$ for $i \leqq d - 2$;
- every node of $\hat{\tau}$ at level i, $i \leqq d - 2$, has exactly two successors;
- there are two cases:
  (a) all nodes at level $d - 1$ have at most one successor, or
  (b) all nodes at level $d - 1$ have one or two successors.

Since we consider from now on only the tree $\hat{\tau}$, we abbreviate $d = d(\hat{\tau})$ and $v_i = v_i(\hat{\tau})$. The nodes in $\hat{\tau}$ from level 0 to $d - 1$ form a complete binary tree with $2^d - 1$ nodes. At level $d - 1$ there are exactly 2^{d-1} nodes and there remain $(n - 1) - (2^d - 1) = n - 2^d$ nodes for level d.

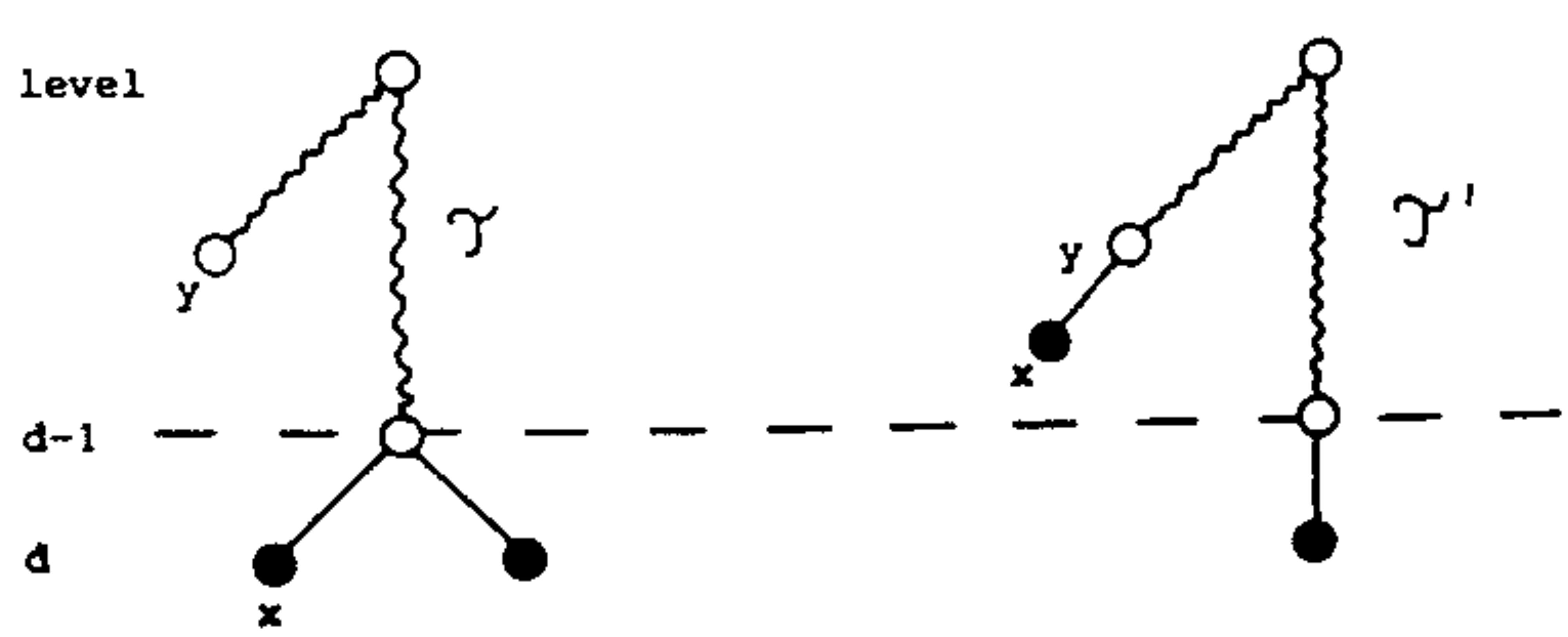

Figure 4. Illustration of transformation I.