Sandwiching is a very natural process that has been proposed in different contexts and with different variations by various authors, like Aneja and Nair [1979] or some of the articles cited above. The contribution of our article is mainly theoretical, by giving an error analysis: After presenting the sandwich algorithm (in Section 2), we will show that, for a given bound ε, the number of necessary iterations is bounded by $\frac{3}{2}\sqrt{D(b-a)/2\varepsilon}$, where D is the total increase in slope of h on the interval $[a, b]$. Each iteration amounts essentially to an evaluation of $h(t)$ and two (one-sided) derivatives. A similar approach was developed by Sonnevend [19] in a more general setting. Sonnevend uses an inductive argument for establishing that the number of iterations is $O[\sqrt{D(b-a)/\varepsilon}]$. Our proof is constructive, uses combinatorial arguments, and yields the best possible constant in the iteration bound.

In Section 3, we describe as a simple application the approximative solution of separable convex programs. Concluding remarks and some lines for further research are given in the last section.

2. THE SANDWICH ALGORITHM: APPROXIMATION OF CONVEX FUNCTIONS

2.1. Preliminaries

Let h: $[a, b] \to \mathbf{R}$ be a convex function, defined on a bounded interval $[a, b] \subset \mathbf{R}$. We assume that h is continuous at the endpoints of the interval and that for any $t \in [a, b]$ the left and right derivative of h is available (or can be computed). Moreover, the one-sided derivatives should be finite in the endpoints of $[a, b]$.

We want to compute efficiently two piecewise-linear, convex functions $l(t)$ and $u(t)$ such that

$$l(t) \leqq h(t) \leqq u(t) \quad \text{and} \quad u(t) - l(t) \leqq \varepsilon, \qquad \text{for all } t \in [a, b].$$

The idea for constructing $l(t)$ and $u(t)$ is as follows. Let $a = t_0 < t_1 < t_2 < \cdots < t_n = b$ be a finite partition of the interval $[a, b]$. For any t_i $(i = 0, 1, \ldots, n-1)$ let h_i^+ be the right derivative of h at t_i and let h_i^- be the left derivative of h at t_i $(i = 1, 2, \ldots, n)$. Then $l(t)$ and $u(t)$ are defined as follows:

$$u(t) := h(t_i) + \frac{h(t_{i+1}) - h(t_i)}{t_{i+1} - t_i} \cdot (t - t_i)$$

and

$$l(t) = \max\{h(t_i) + h_i^+ \cdot (t - t_i), h(t_{i+1}) + h_{i+1}^- \cdot (t - t_{i+1})\}$$

$$\text{for} \quad t_i \leqq t \leqq t_{i+1}, \qquad i = 0, 1, \ldots, n-1.$$

The definition of l and u is illustrated in Figure 1. It should be noted that h_i^+ as well as h_i^- can be replaced by any subgradient δ_i of h at t_i. Whereas the worst-case bound developed in the following is independent of the choice of δ_i, h_i^+ and h_i^- are certainly preferable, since they yield the tightest lower bounds.