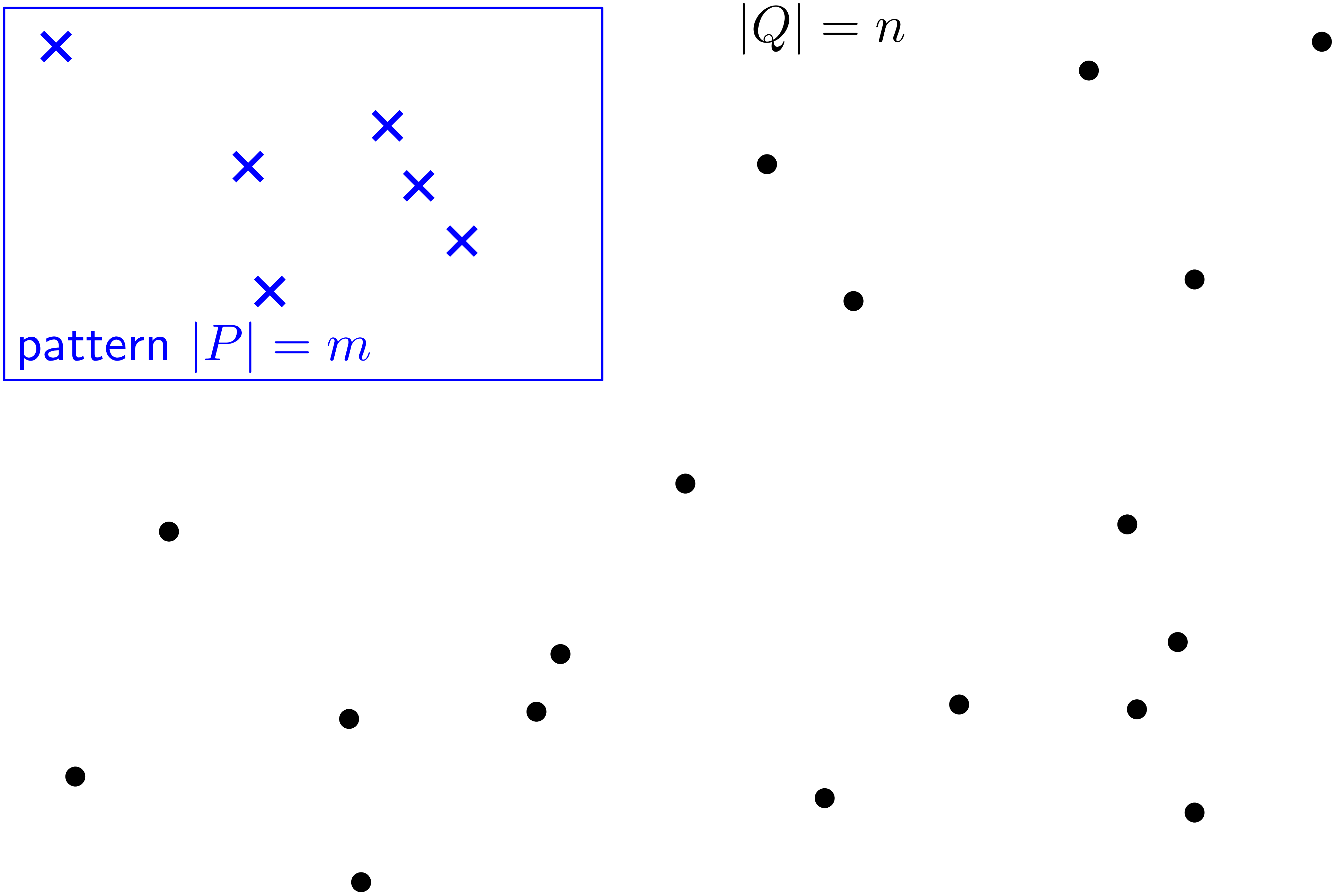
Partial Least-Squares Point Matching under Translations

Günter Rote

Freie Universität Berlin

Partial Matching under Translations



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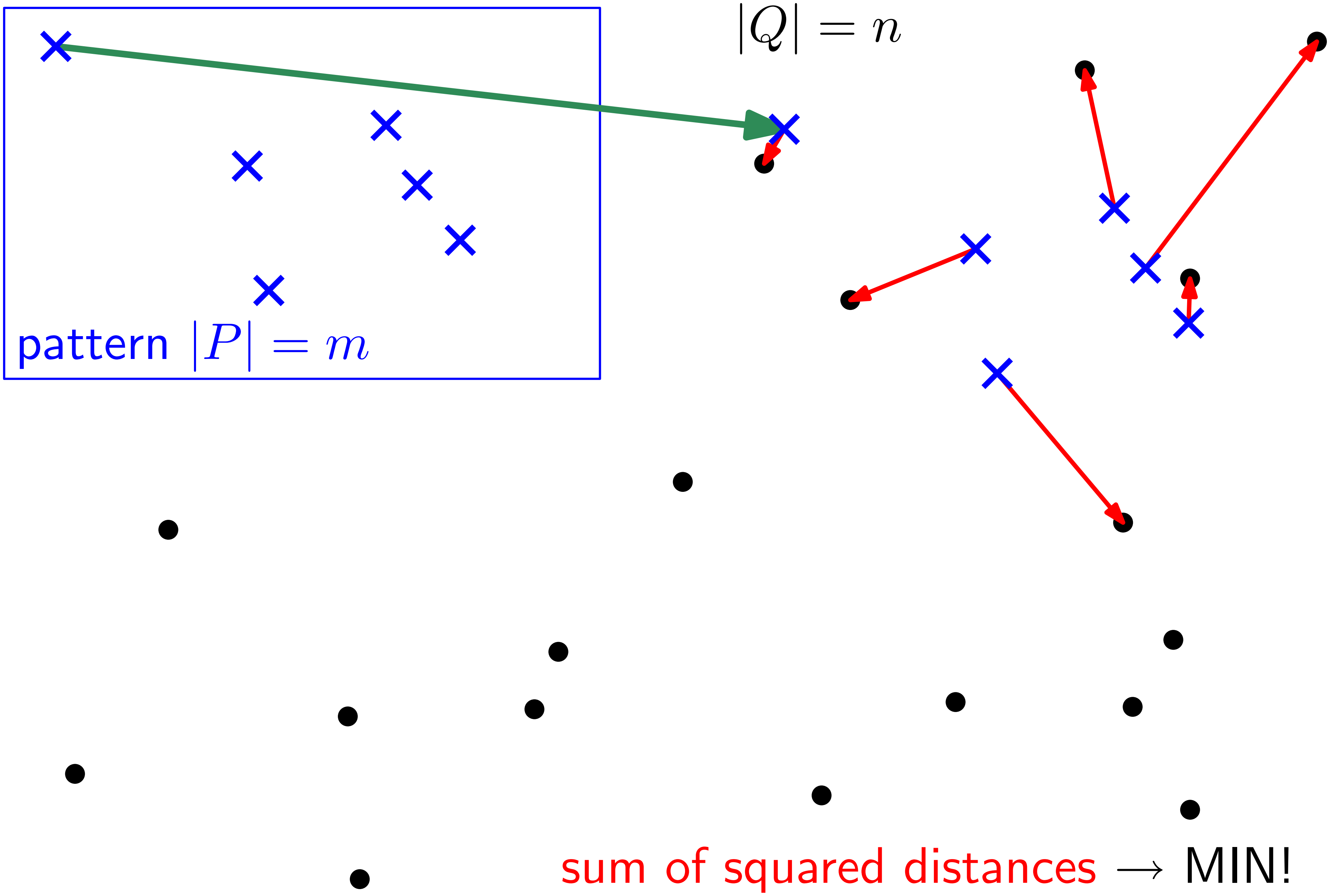

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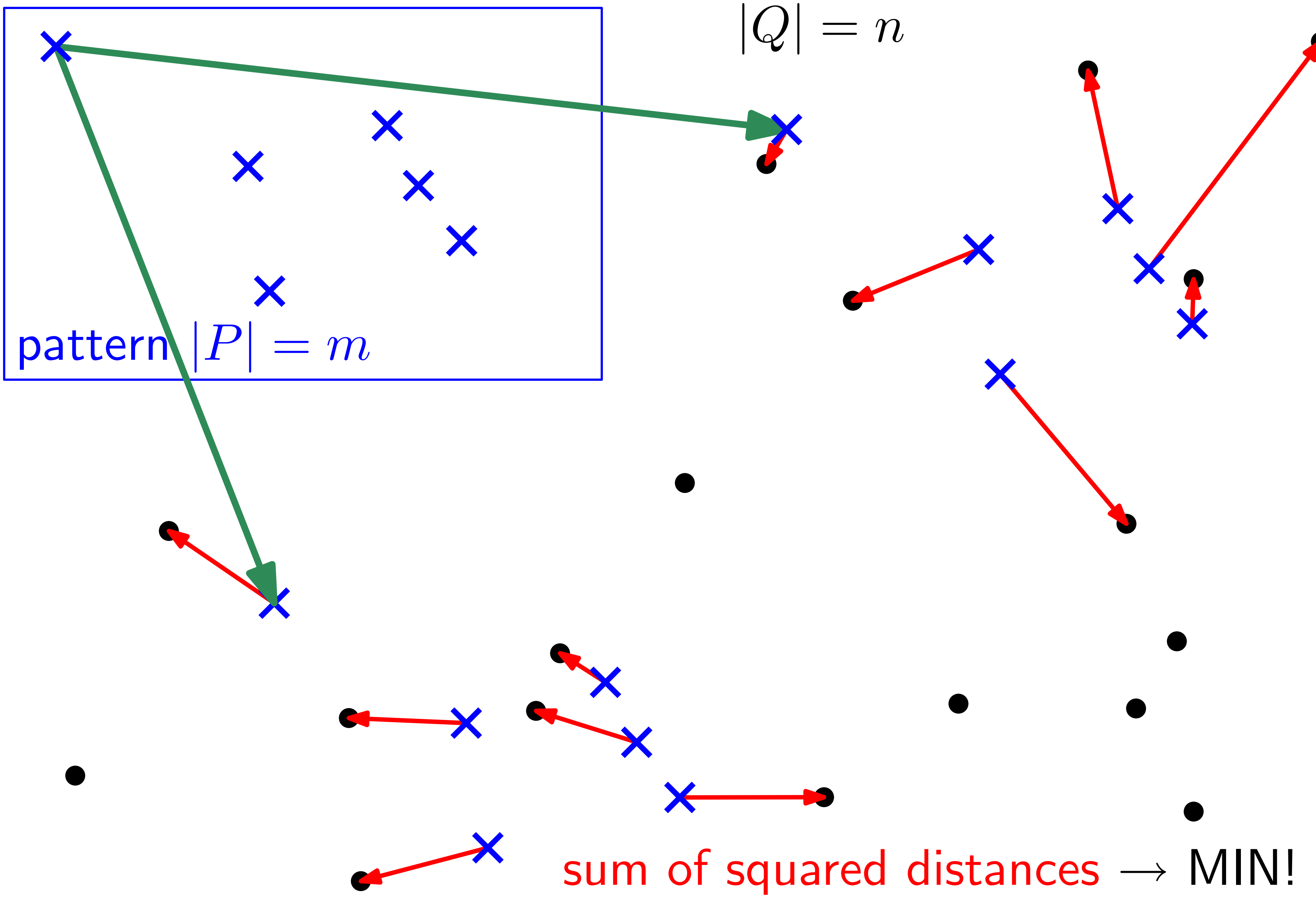
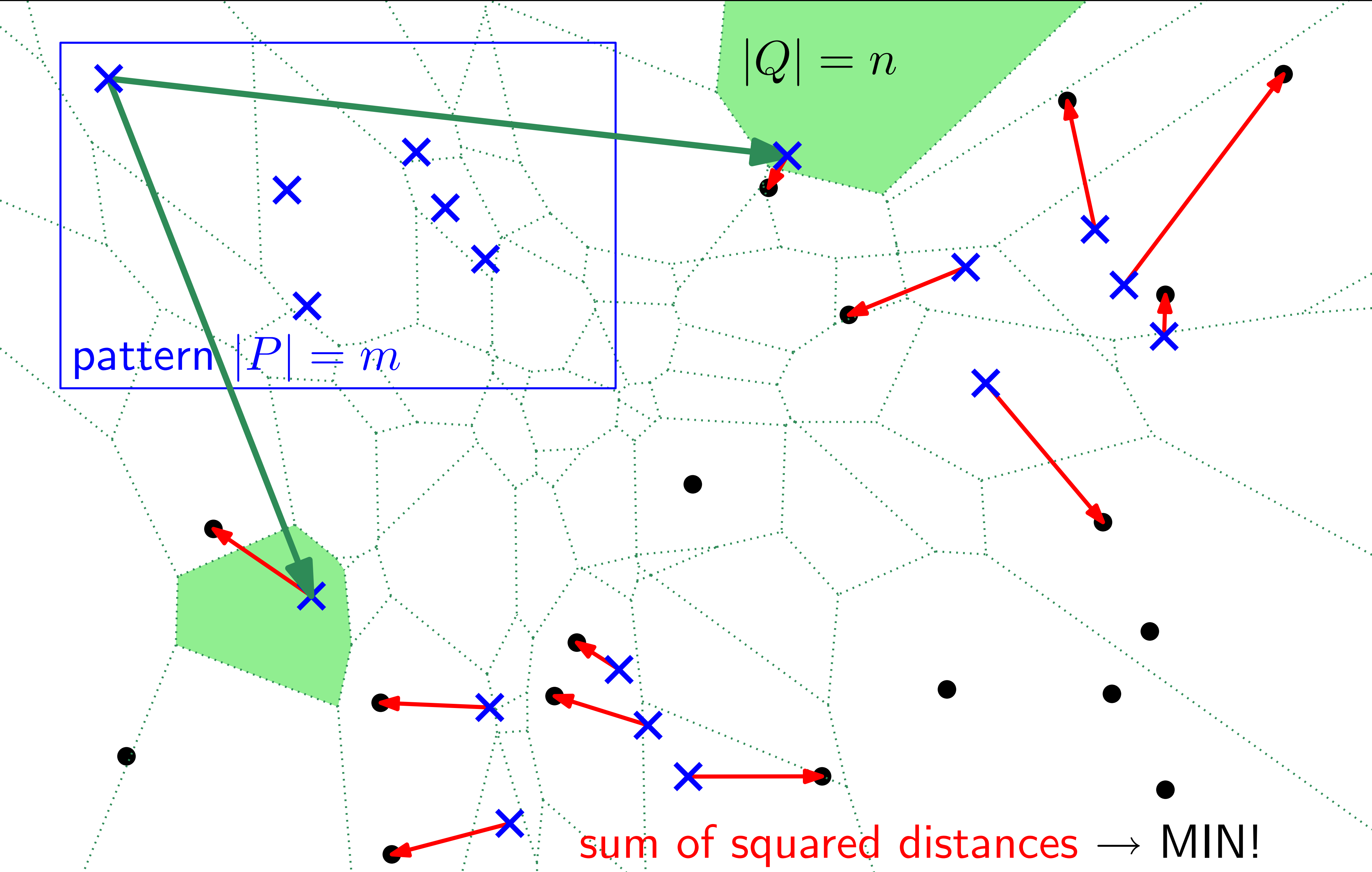


sum of squared distances $\longrightarrow$ MIN!

Partial Matching under Translations



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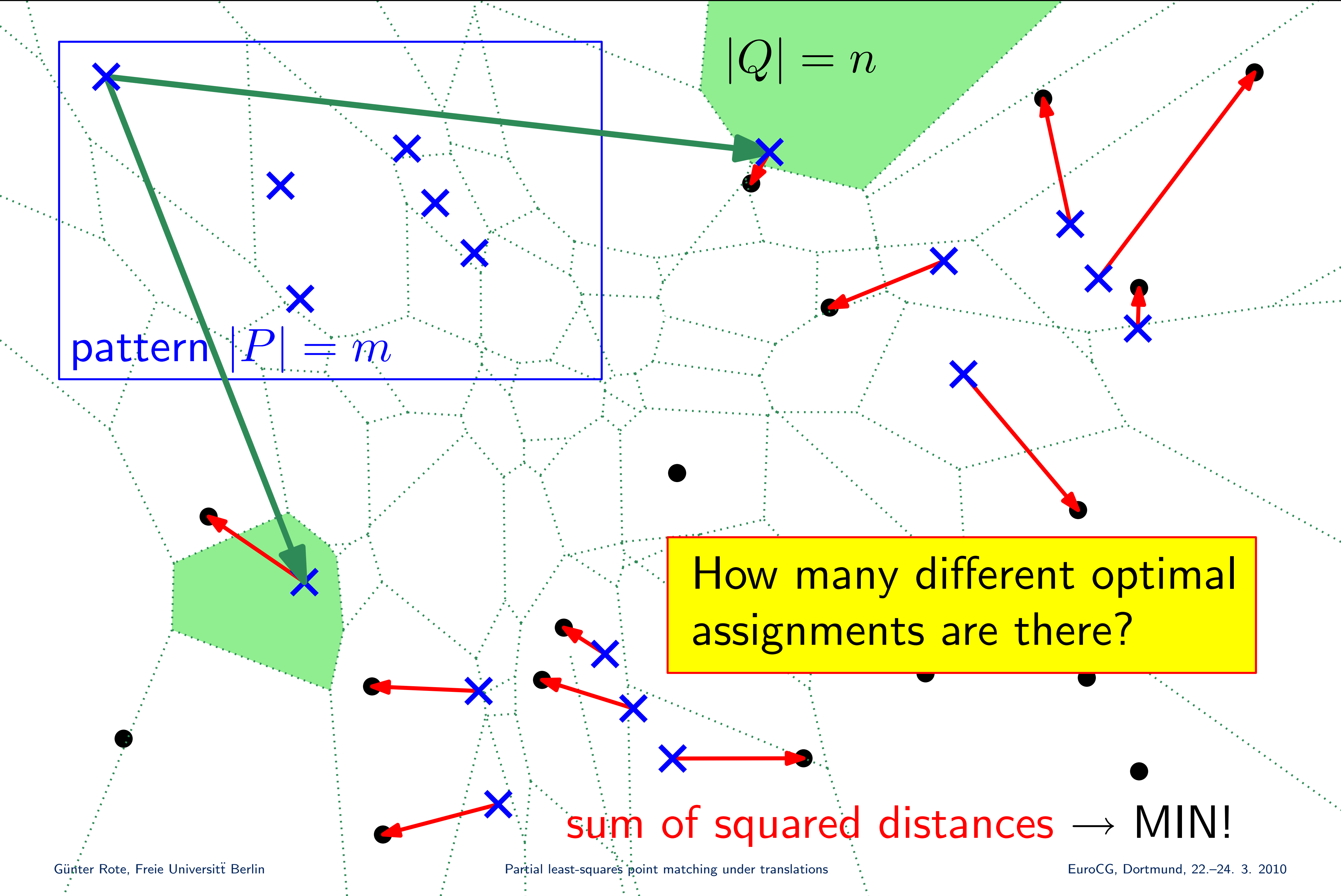

Partial Matching under Translations

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$|P| = m, \ |Q| = n$

How many different optimal (least-squares) assignments $P \to Q$ are there when P is translated (and/or rotated, or scaled) in the plane?

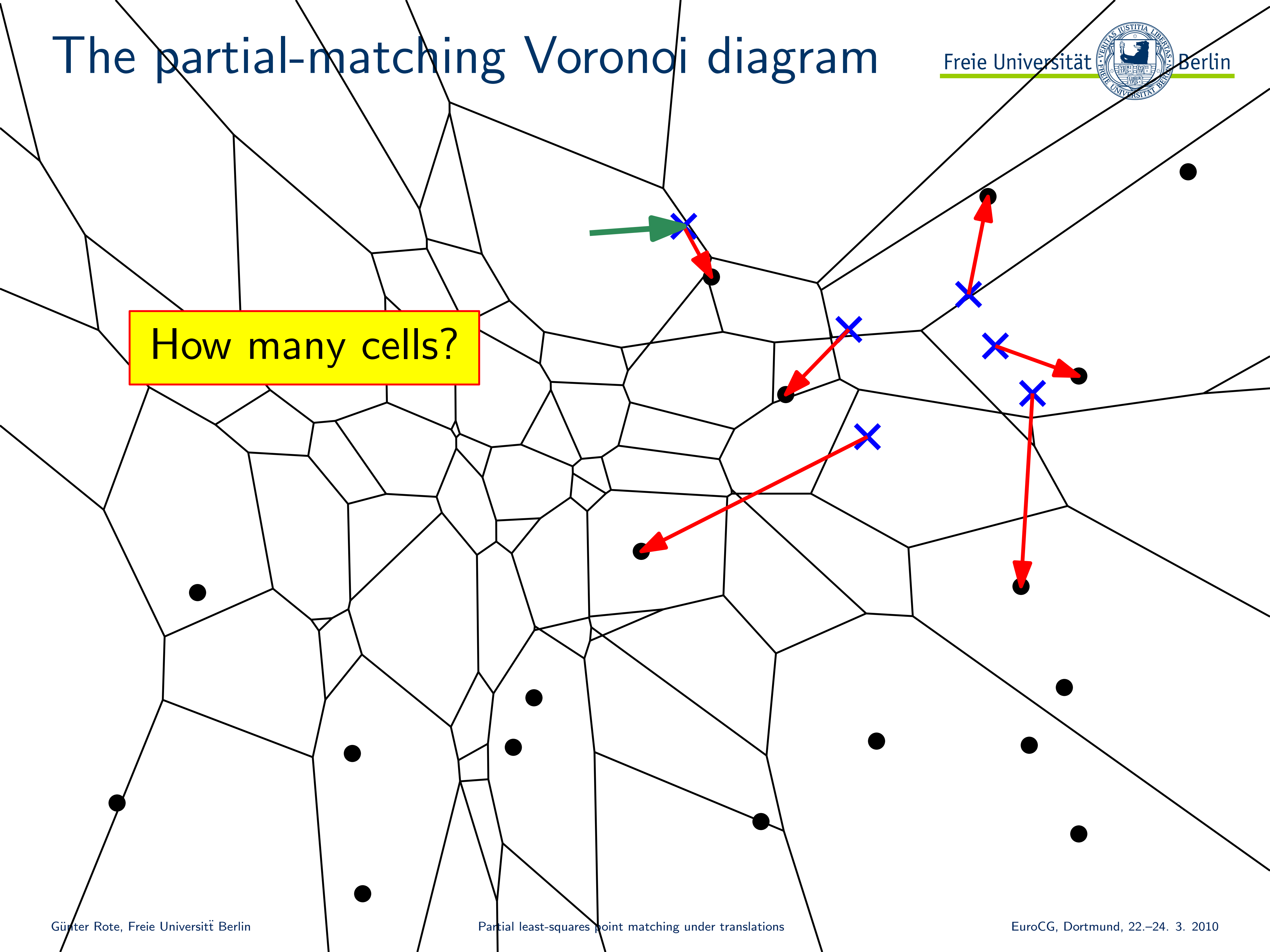
(polynomial or exponential?)

Partial Answer:

Theorem. When P is translated *along a line*, there are at most $m(n-m)+1$ optimal assignments.
This bound is tight.

$\Rightarrow$ polynomial algorithm for finding the optimum translation

The partial-matching Voronoi diagram



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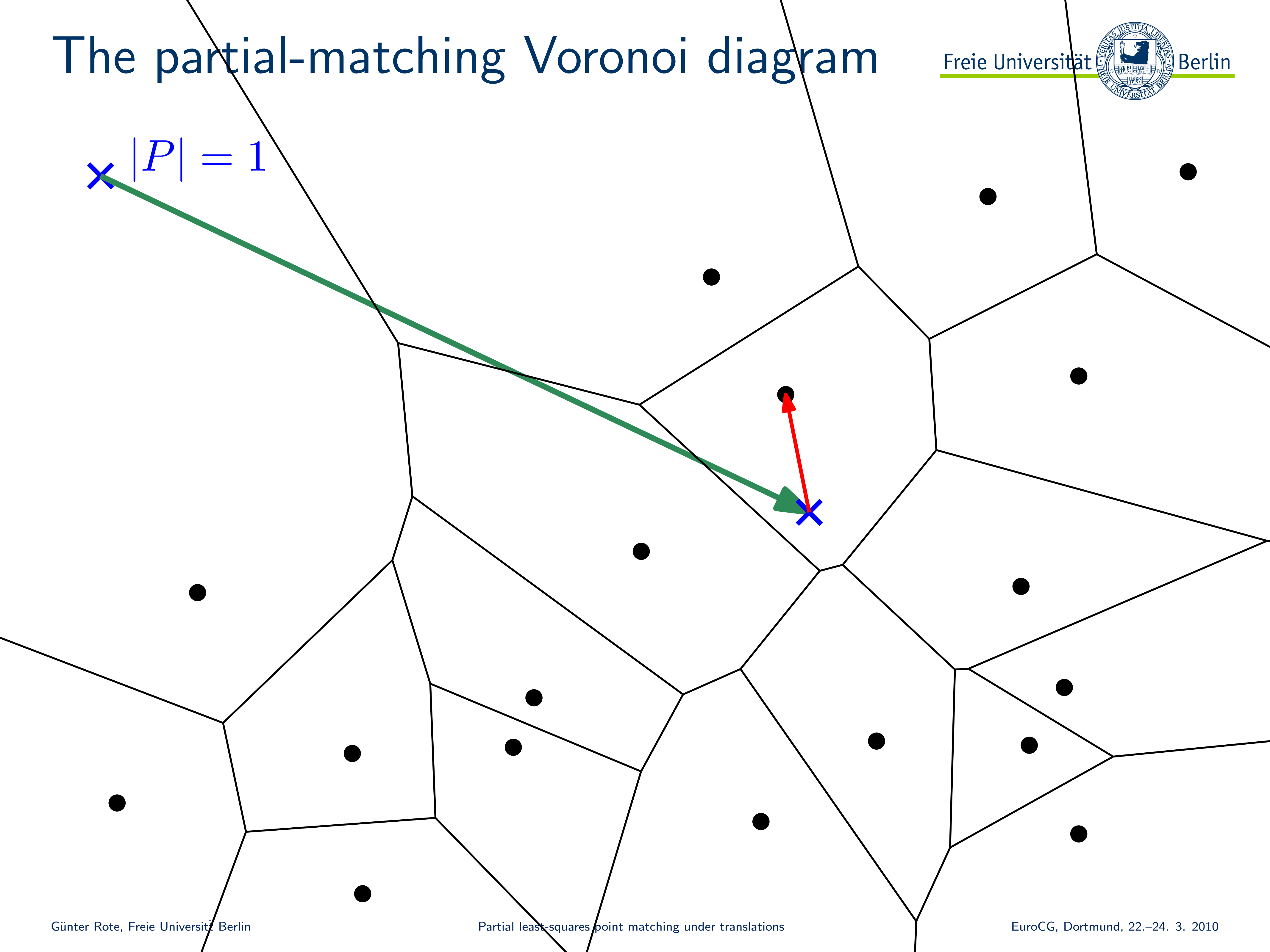

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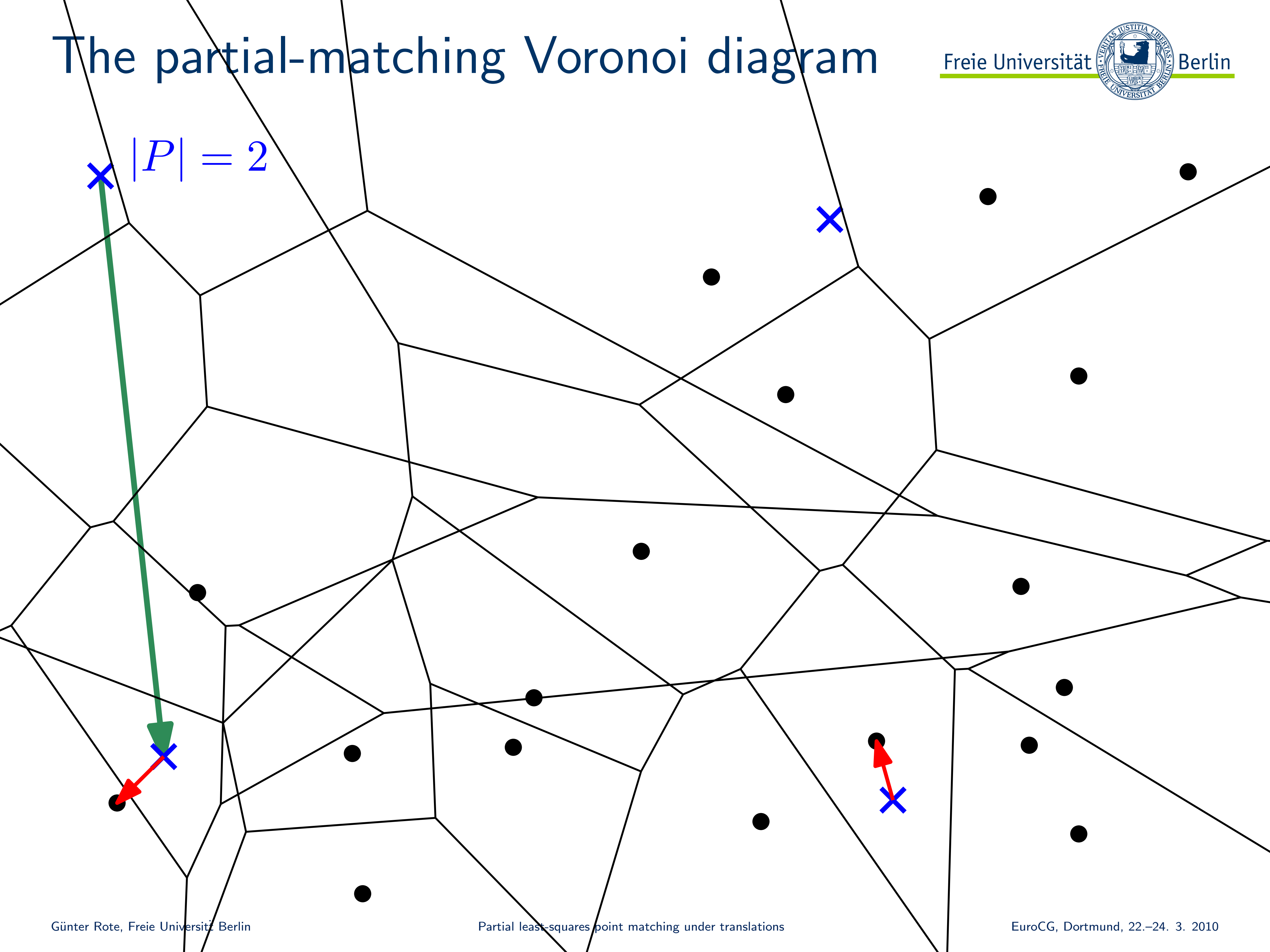

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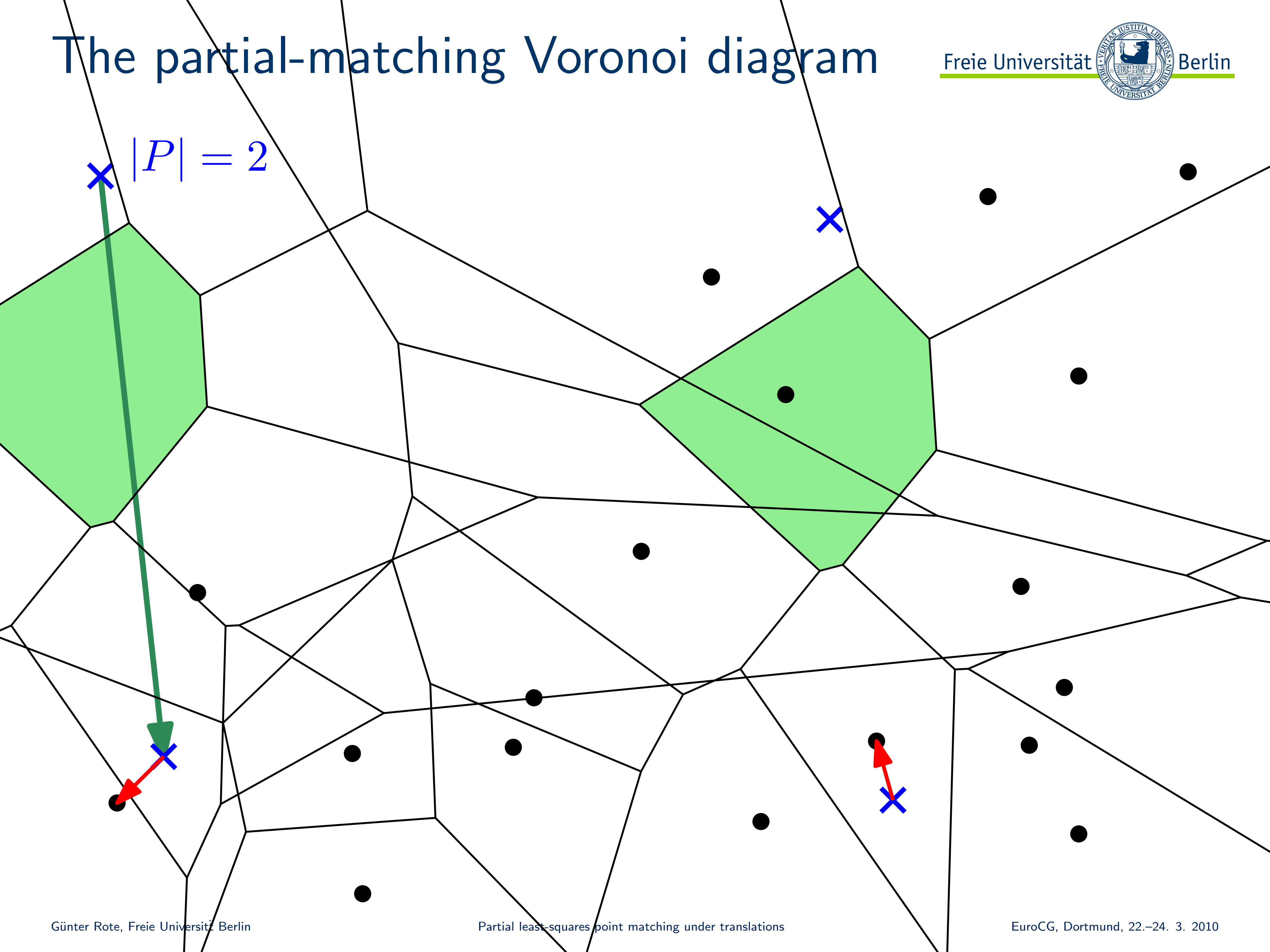

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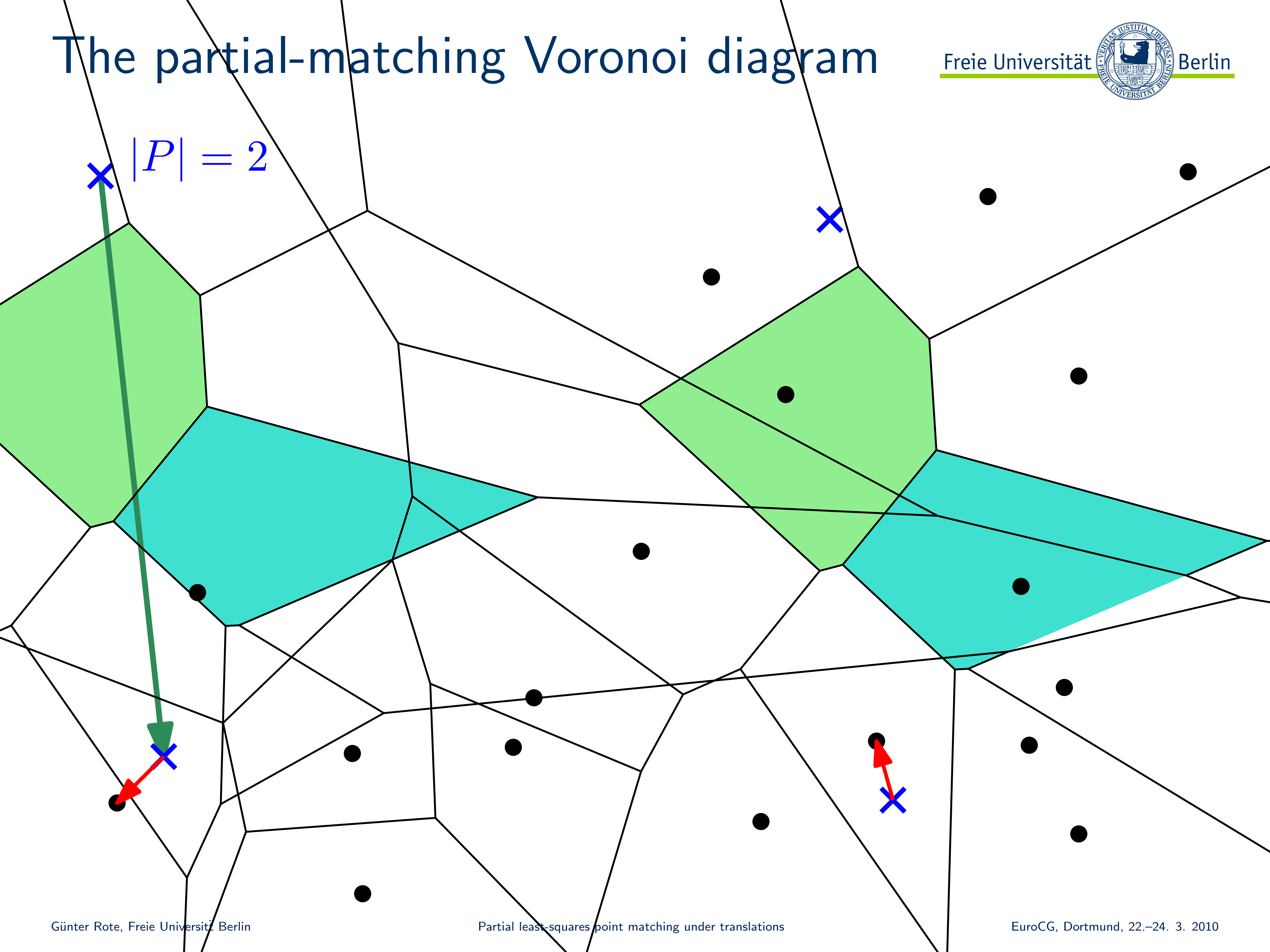
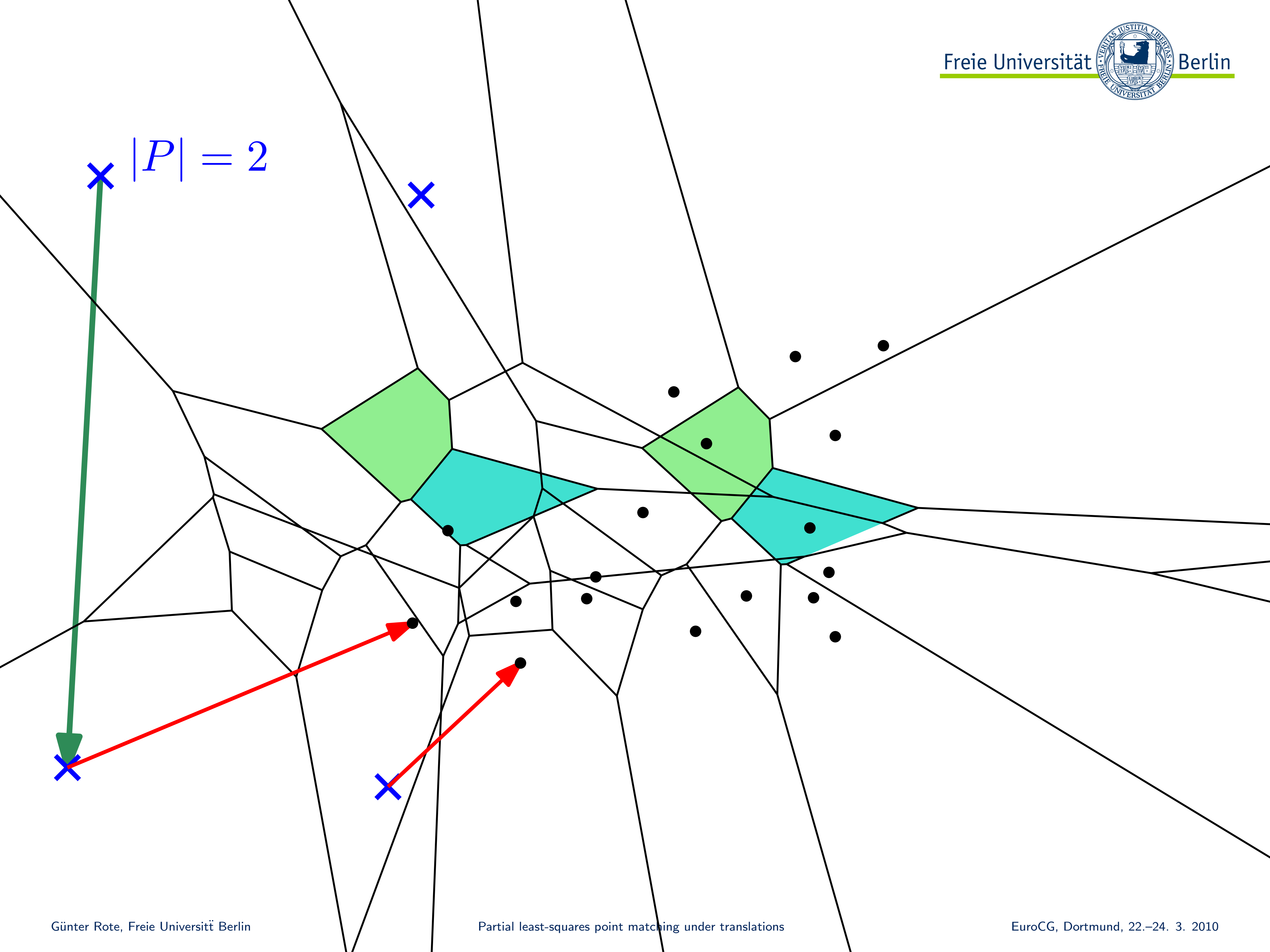
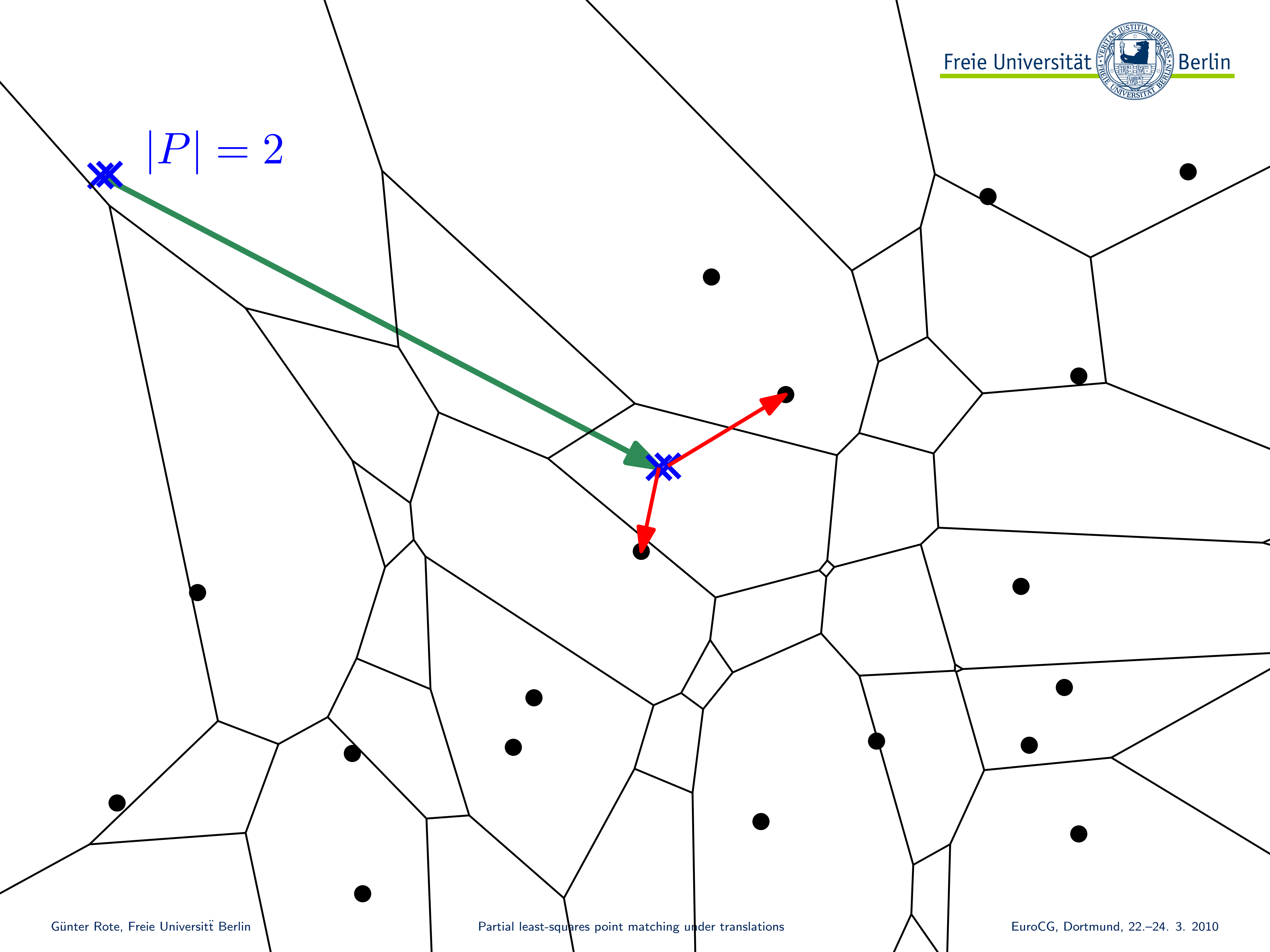



|P| = 2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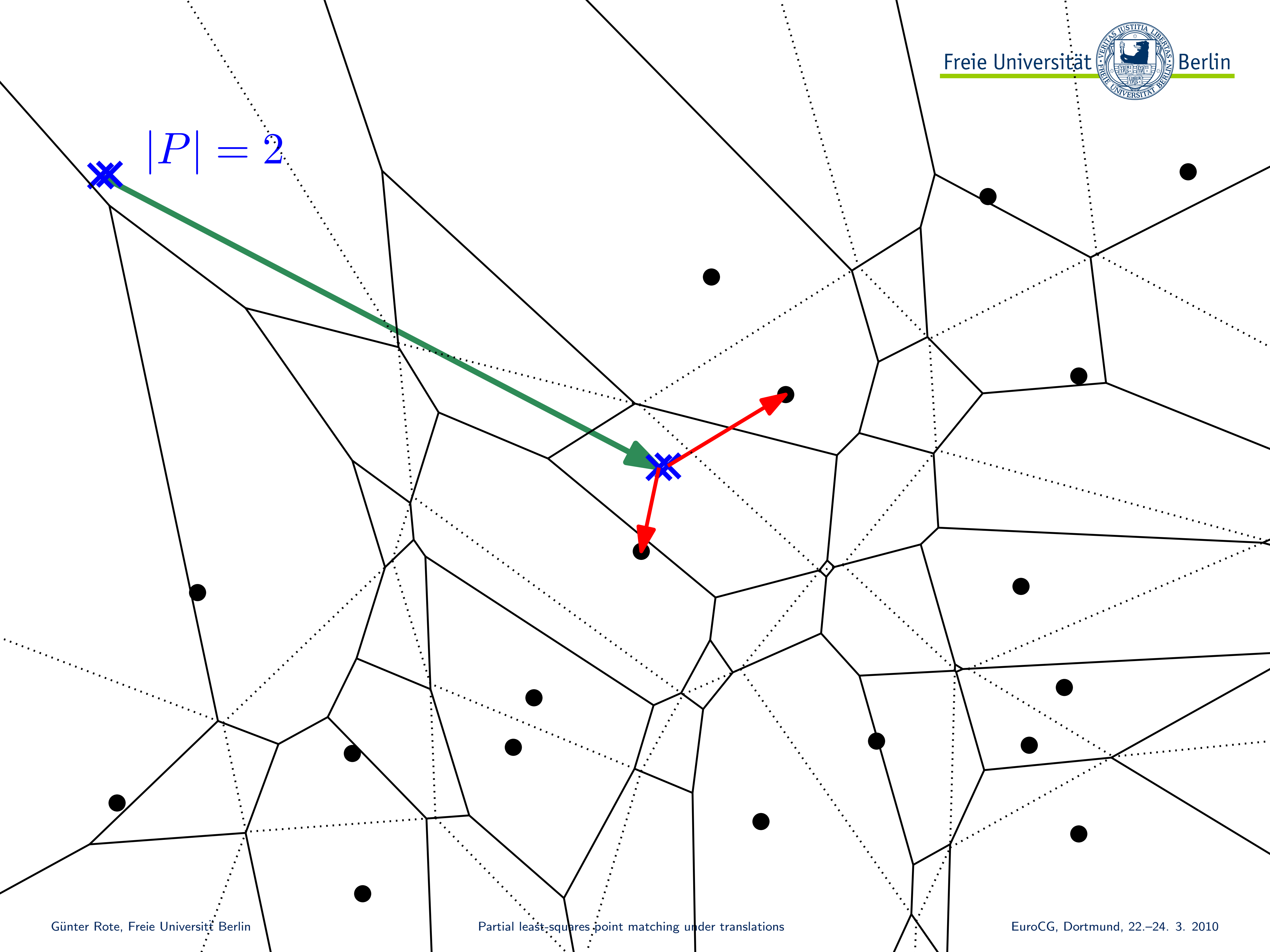
$|P| = 2$

The Main Theorem

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Theorem. When P is translated *along a line*, there are at most $m(n-m)+1$ optimal assignments.

This bound is tight.

Equivalently, every line intersects the LSPM Voronoi diagram at most $m(n-m)$ times.

Proof of the upper bound $O(mn)$

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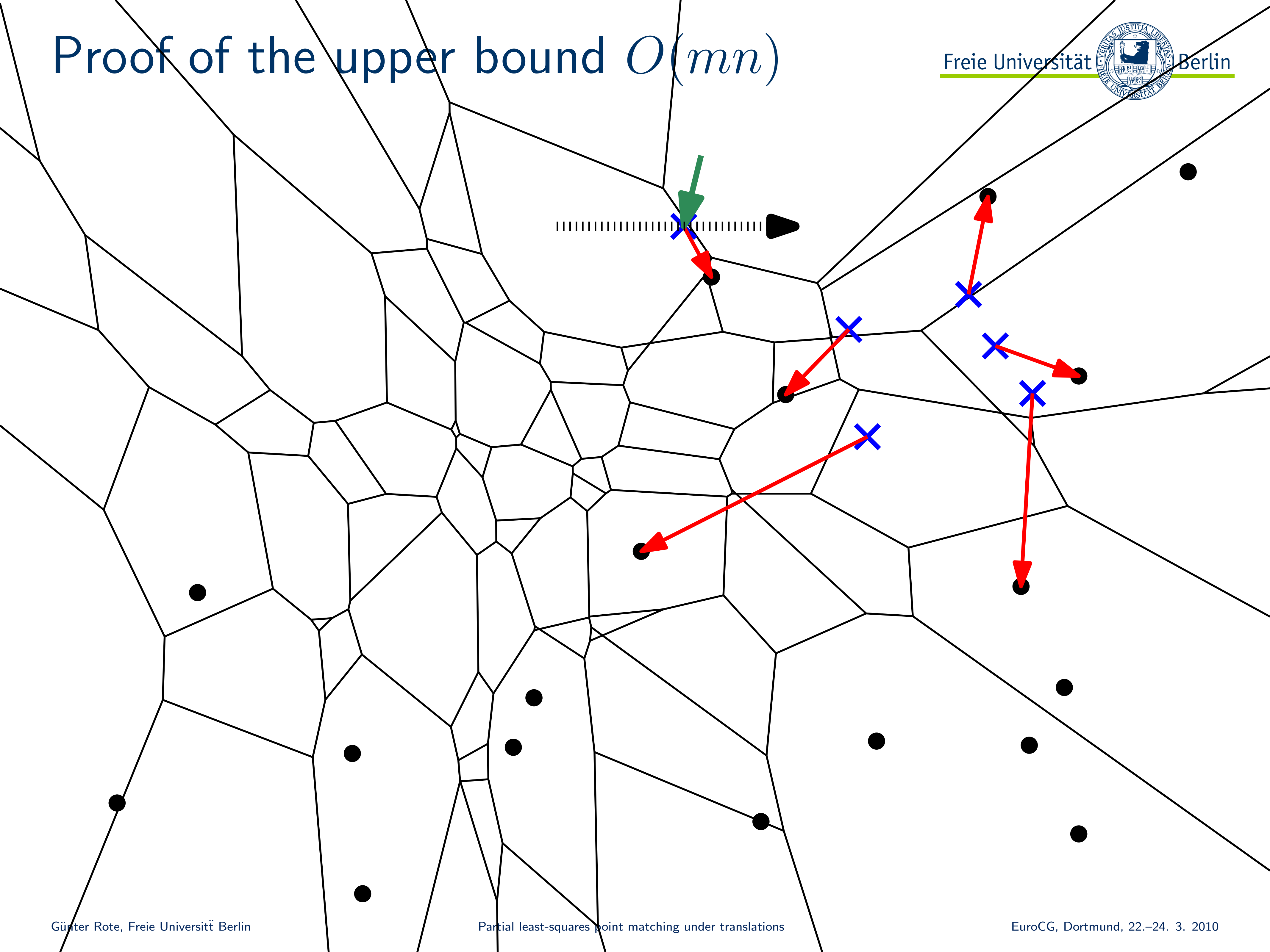

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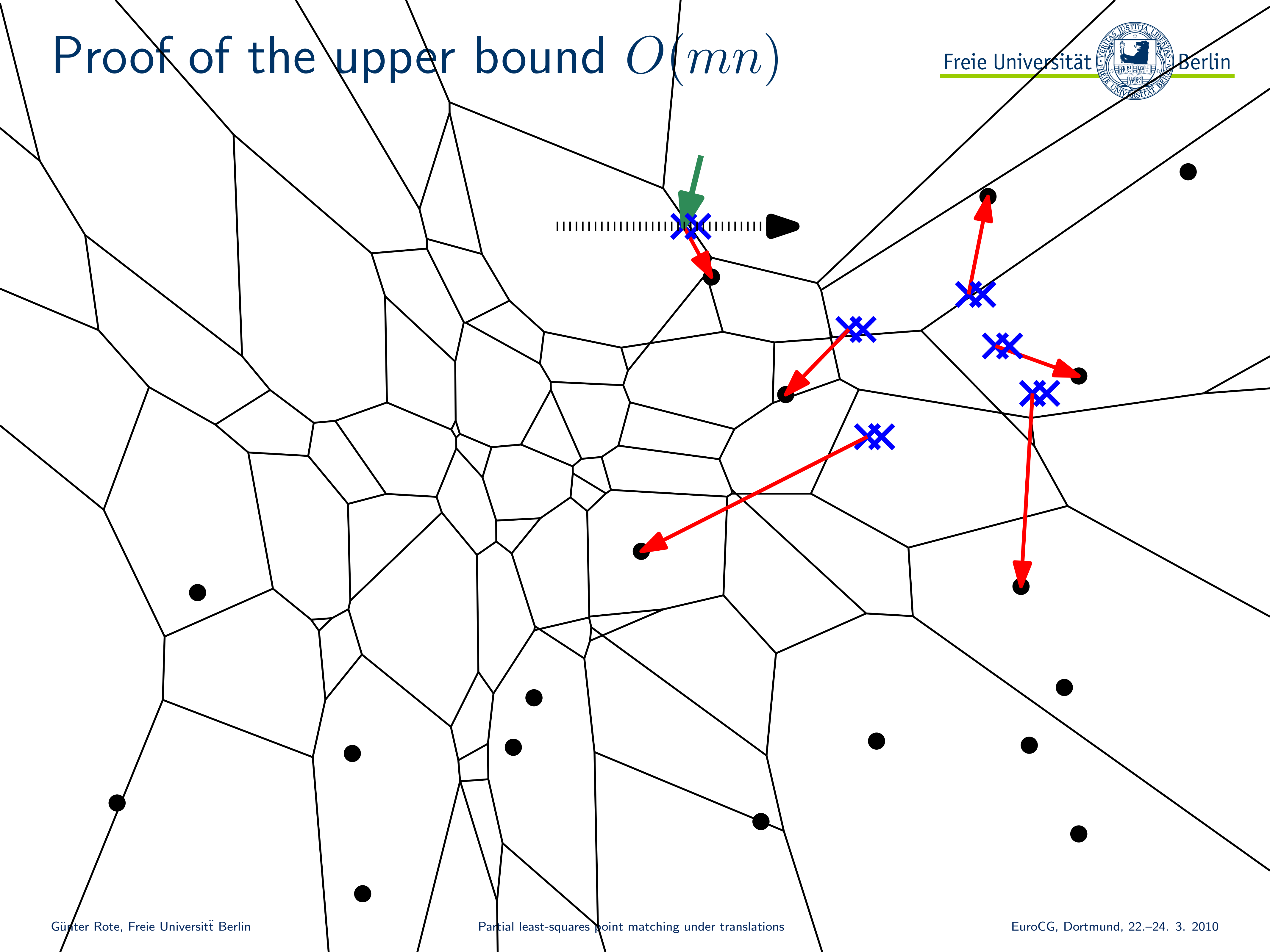

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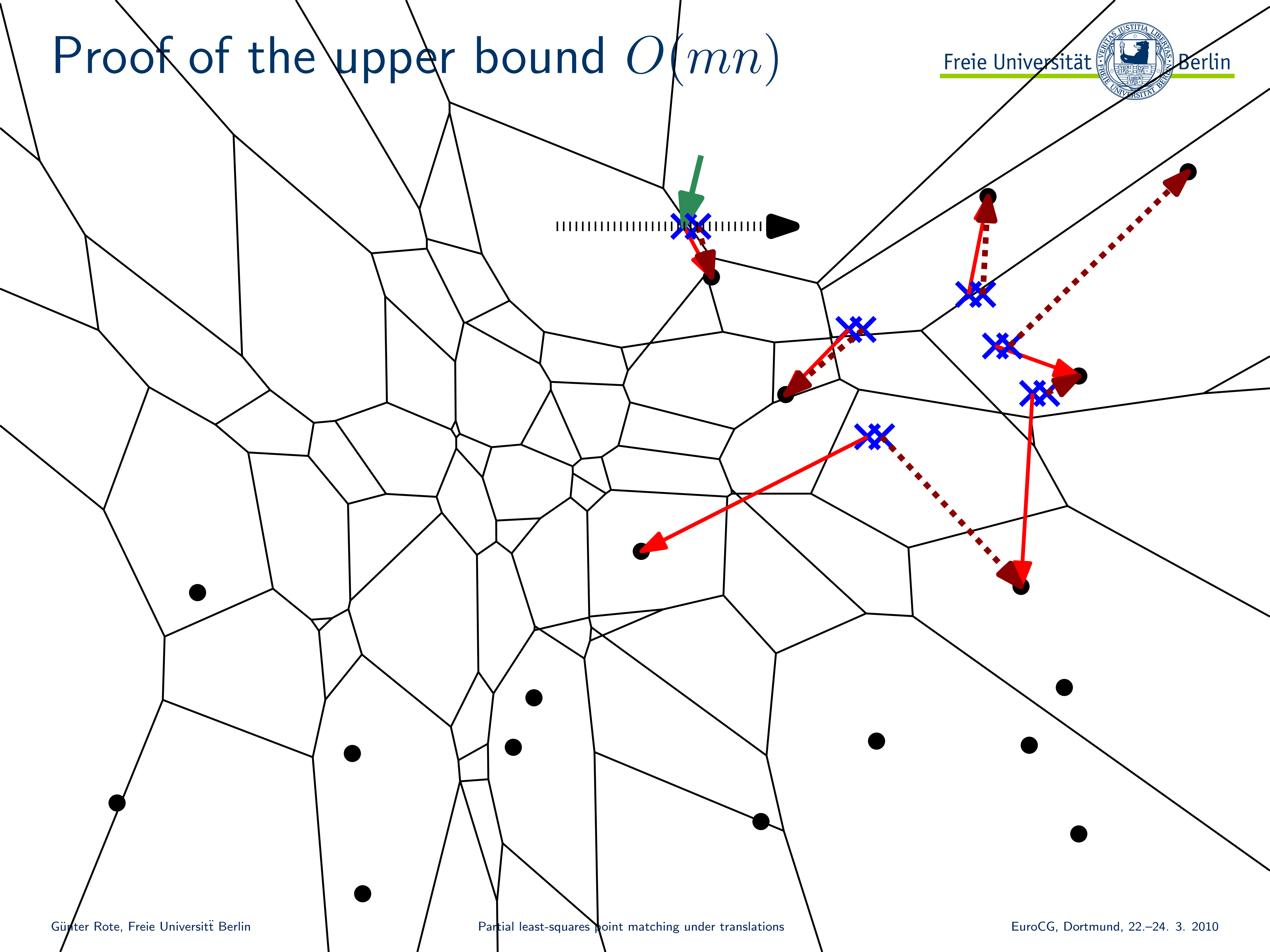

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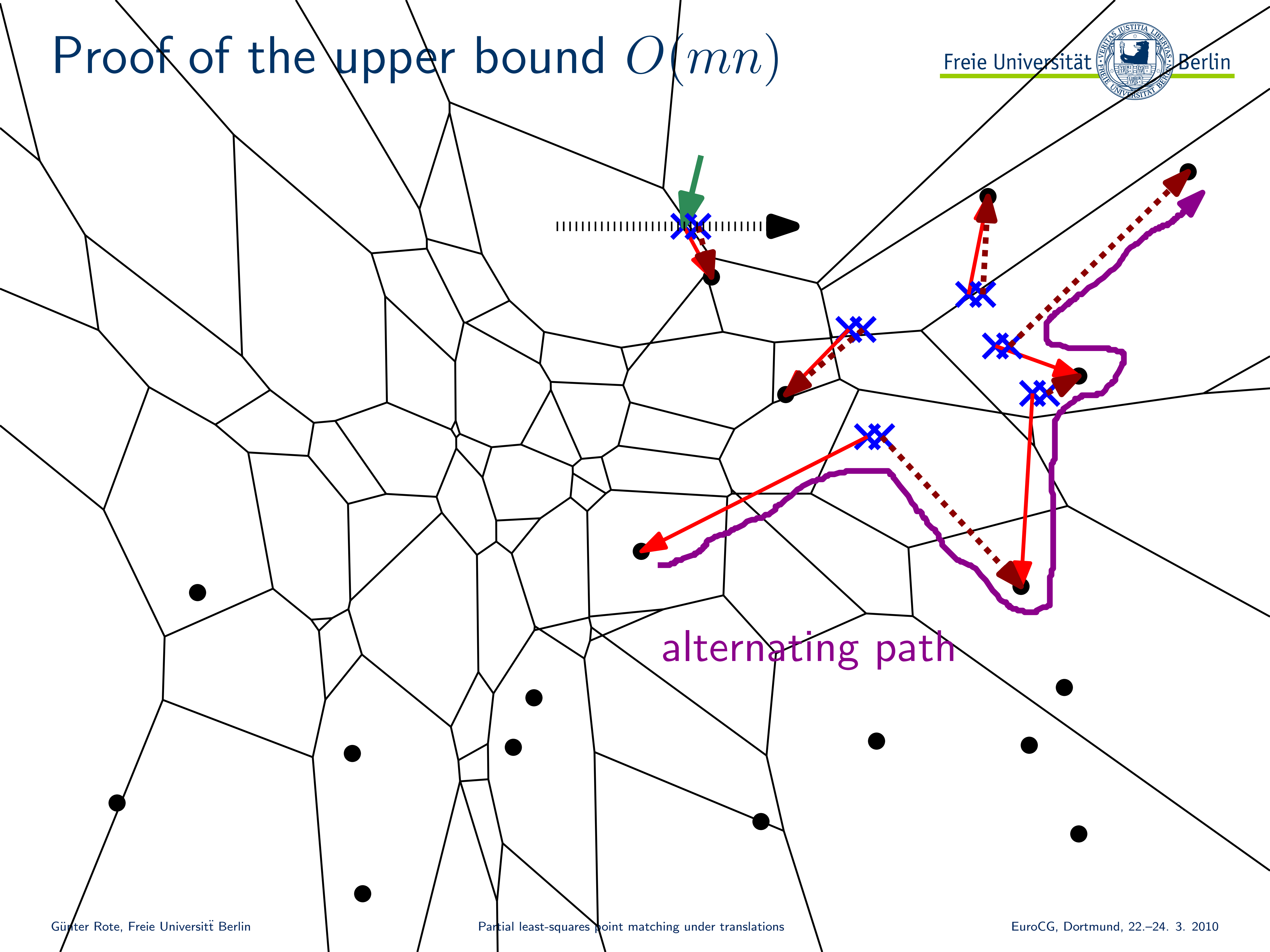

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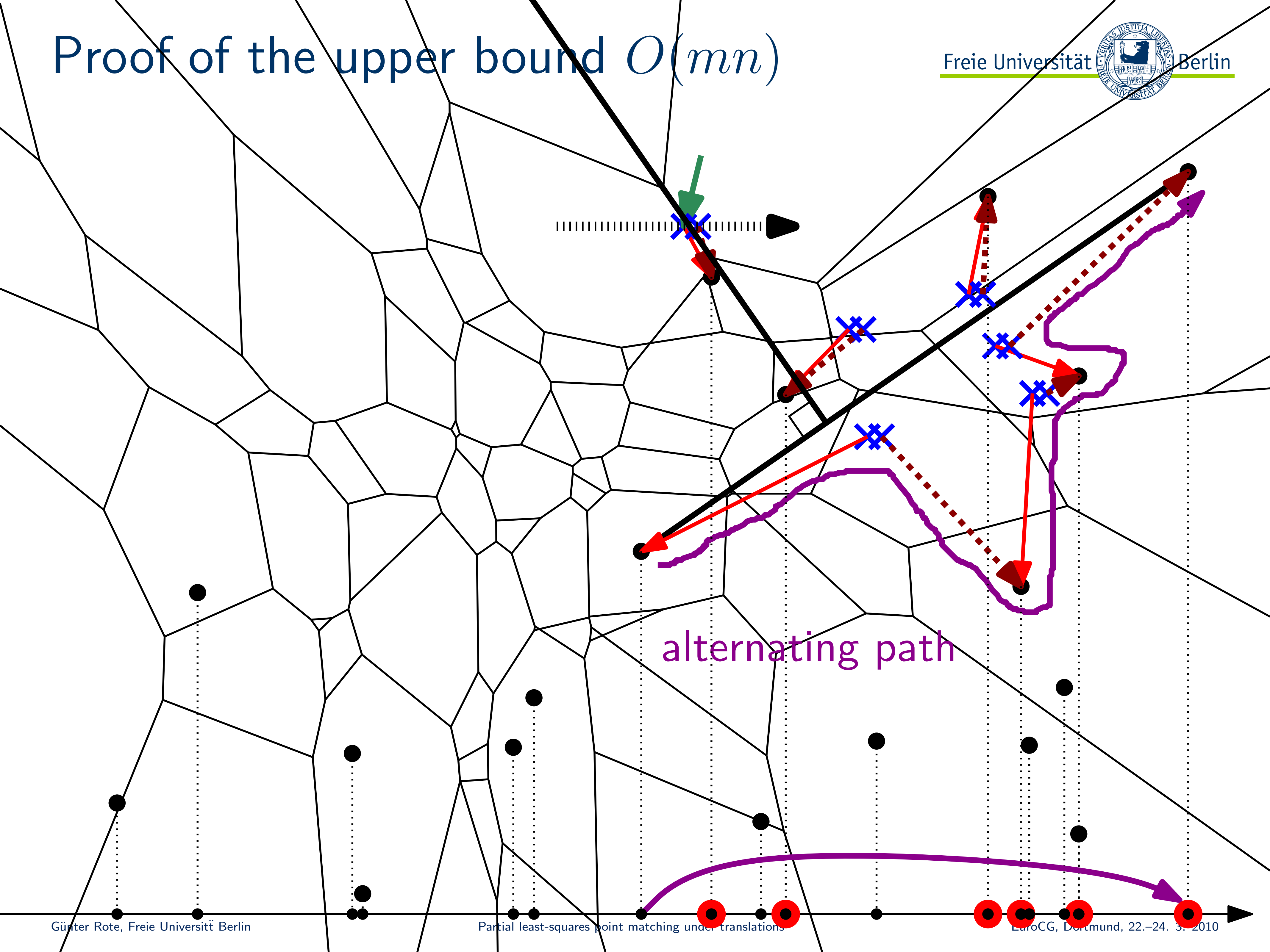
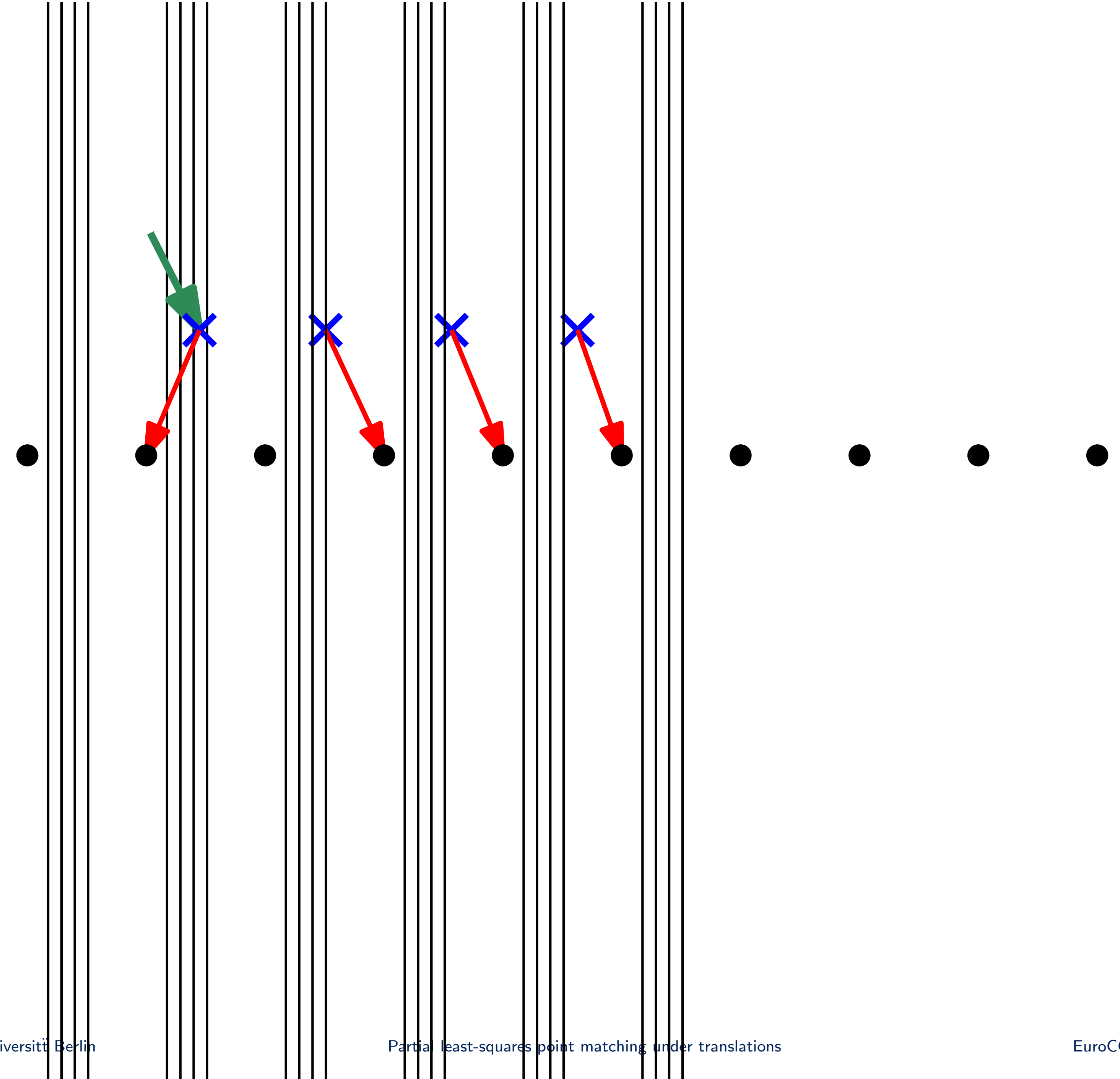
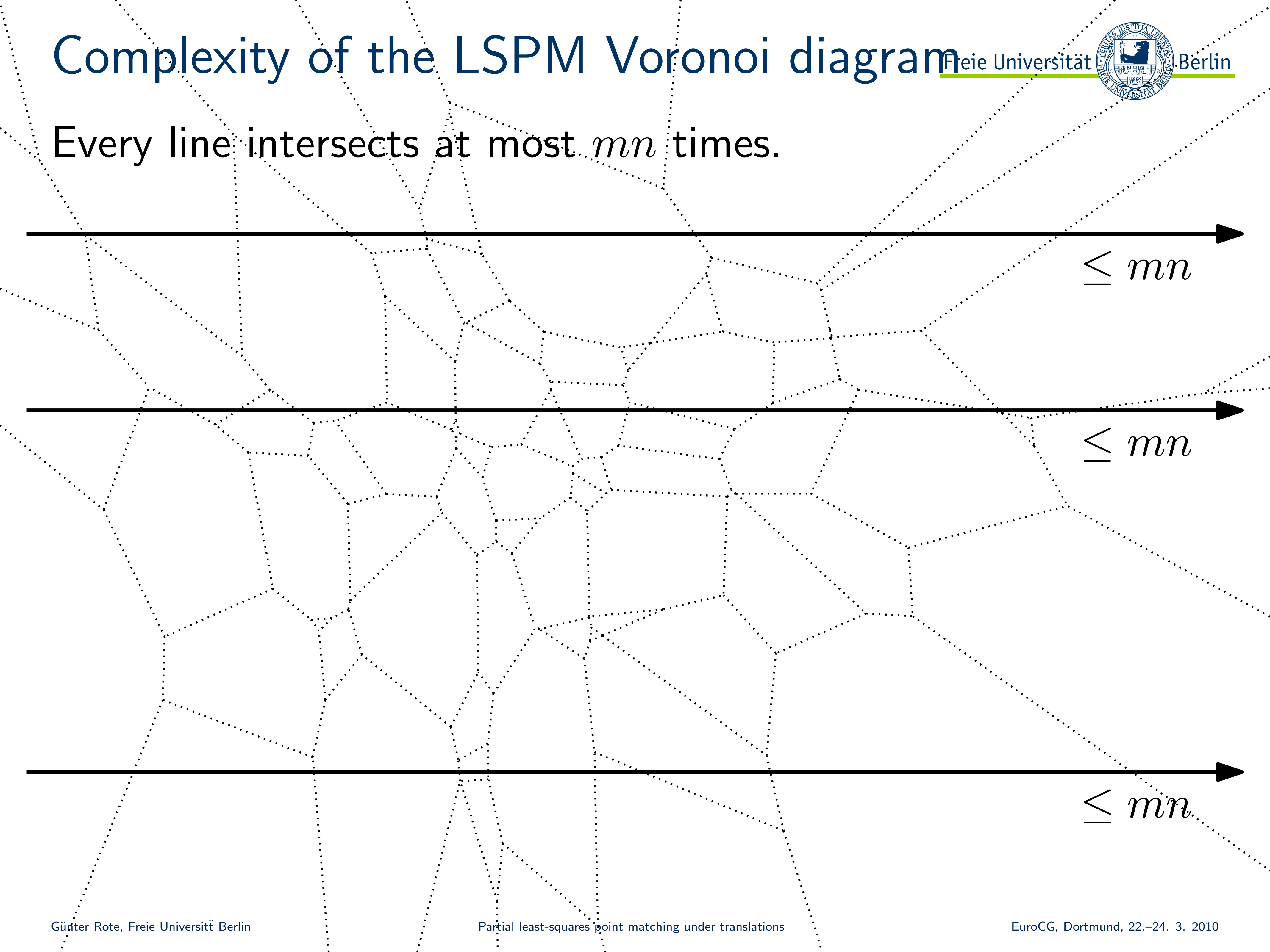


The precise lower bound: $m(n-m)+1$



Complexity of the LSPM Voronoi diagram

Every line intersects at most mn times.



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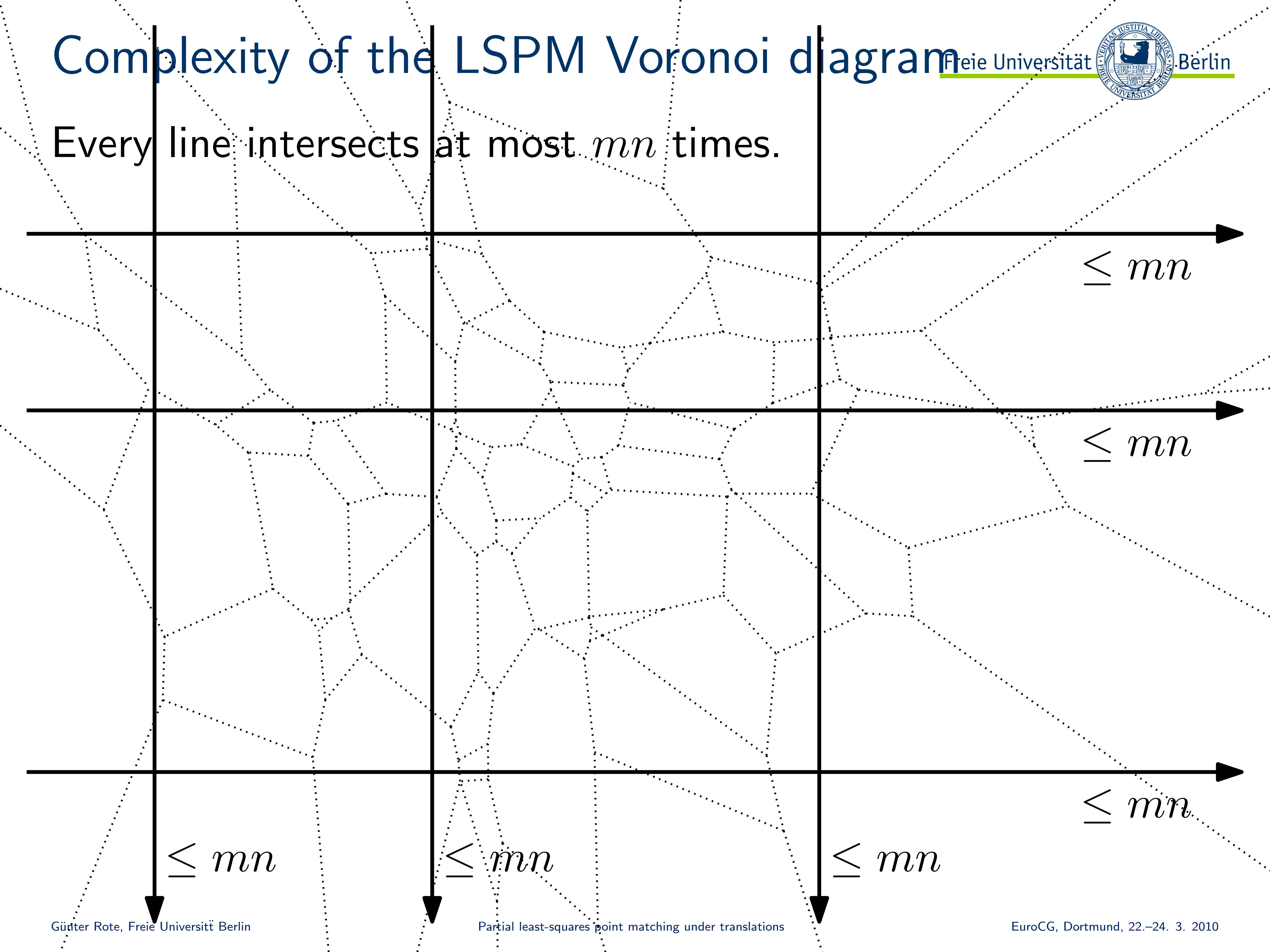

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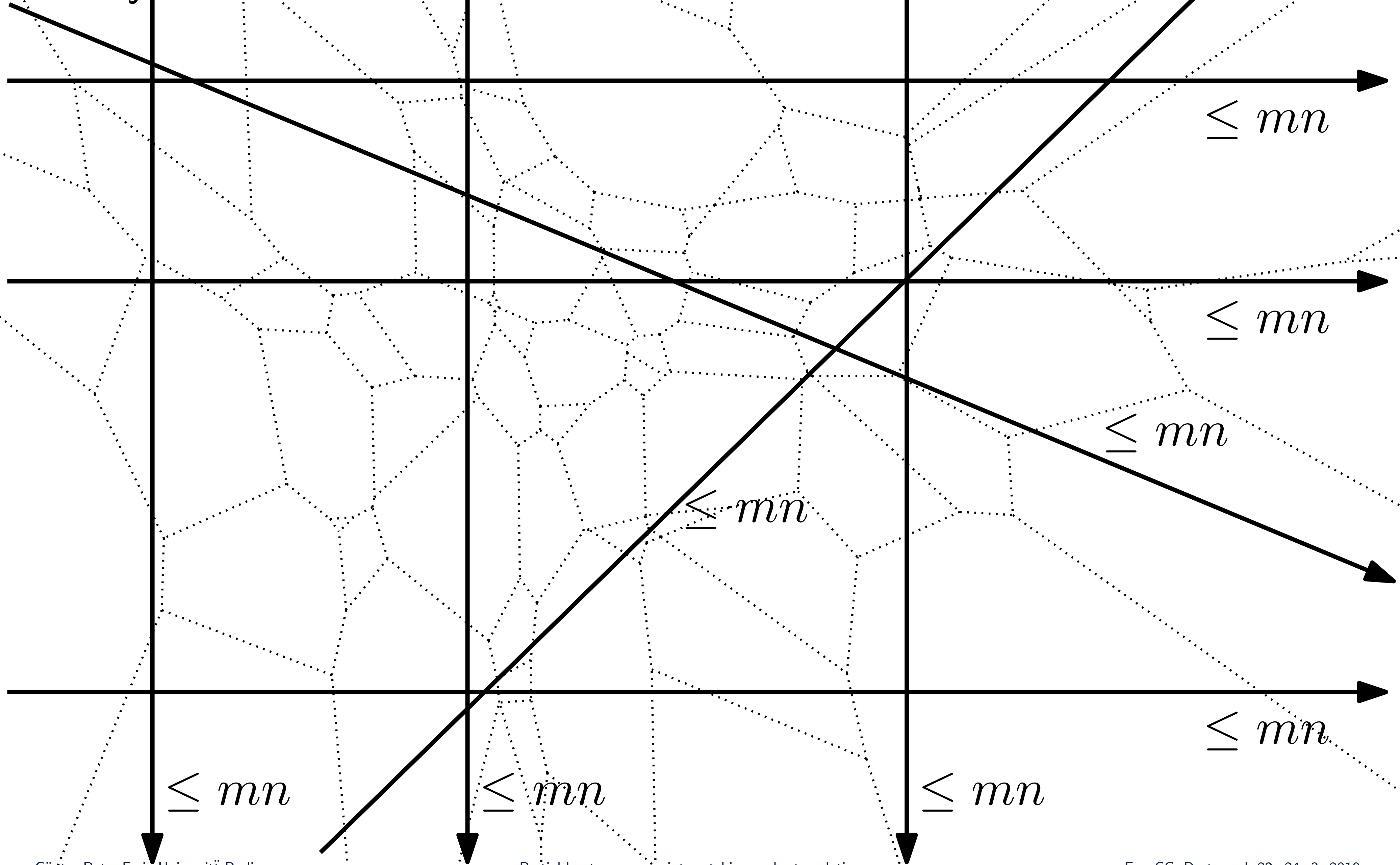

Complexity of the LSPM Voronoi diagram

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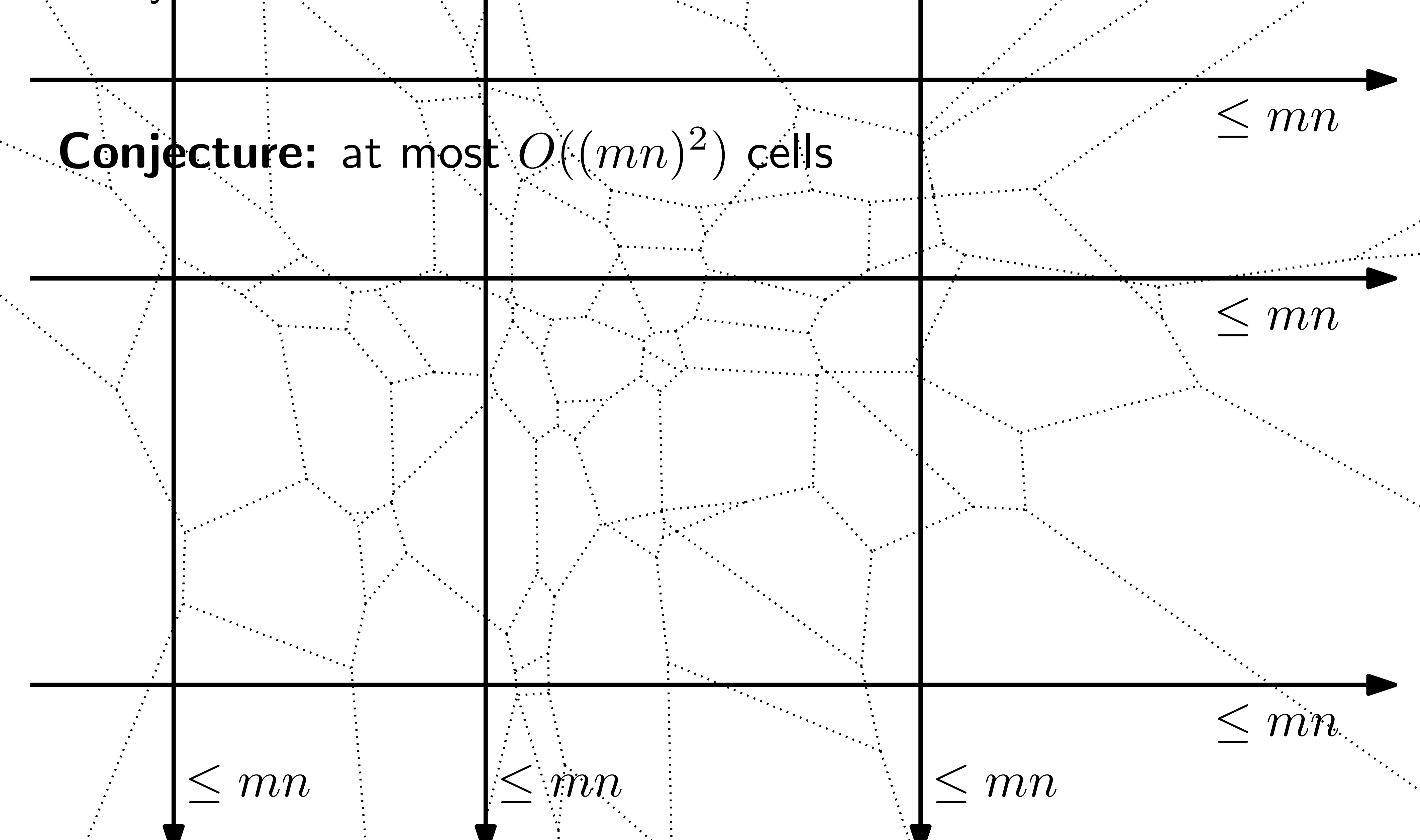
Conjecture: at most $O((mn)^2)$ cells



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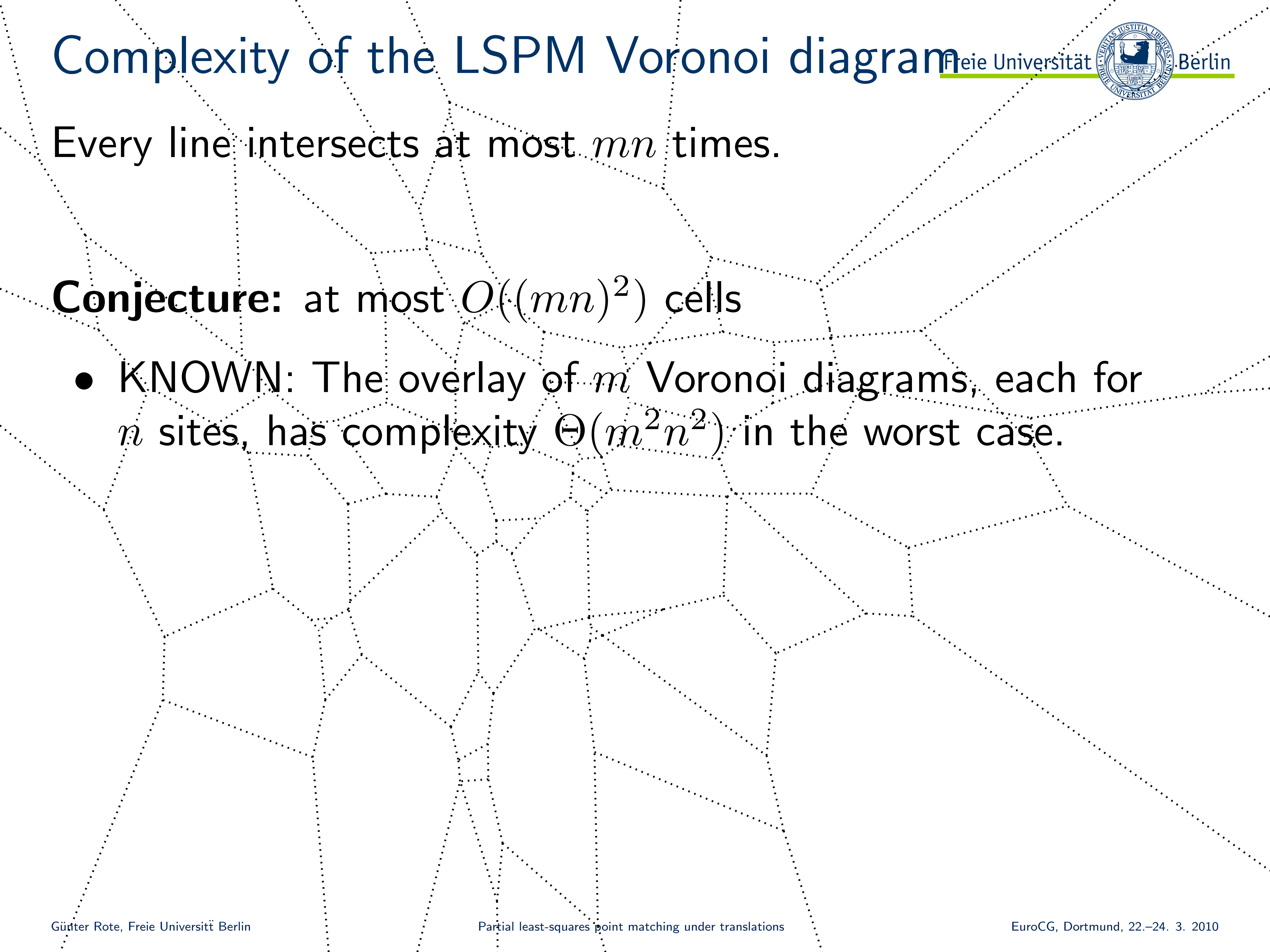
- KNOWN: The overlay of m Voronoi diagrams, each for n sites, has complexity $\Theta(m^2n^2)$ in the worst case.



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This alone is not sufficient to bound the number of cells.

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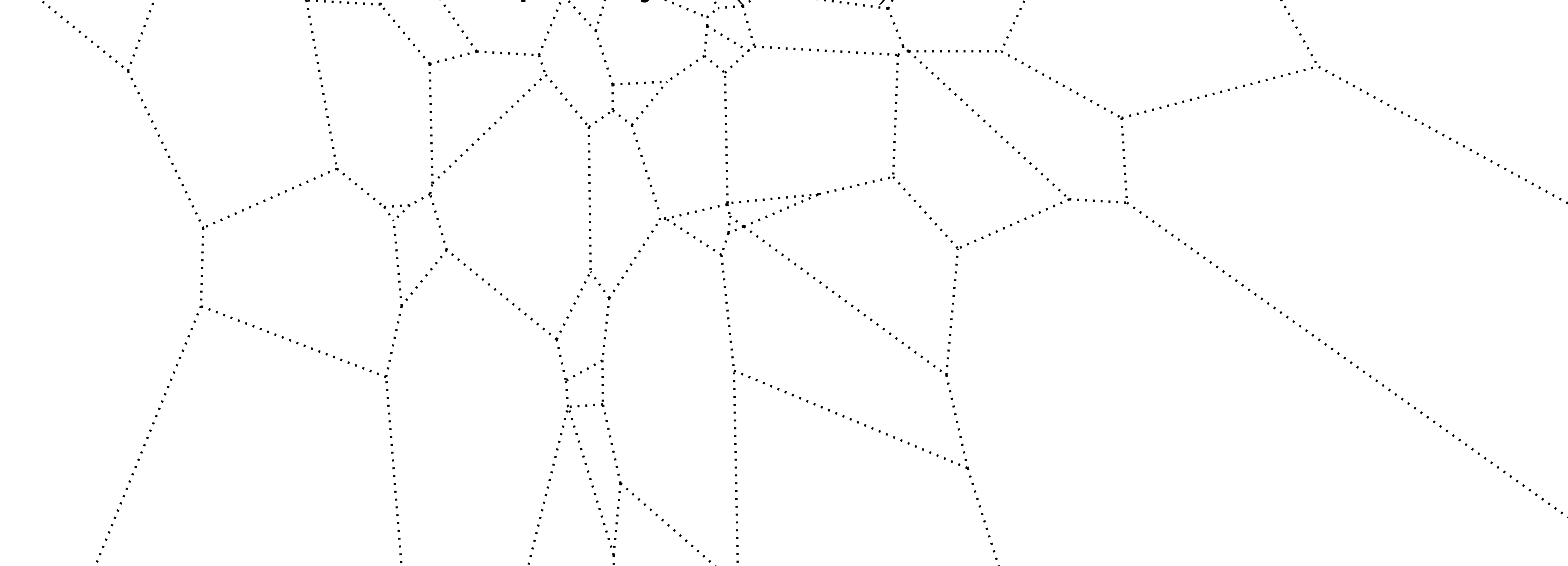

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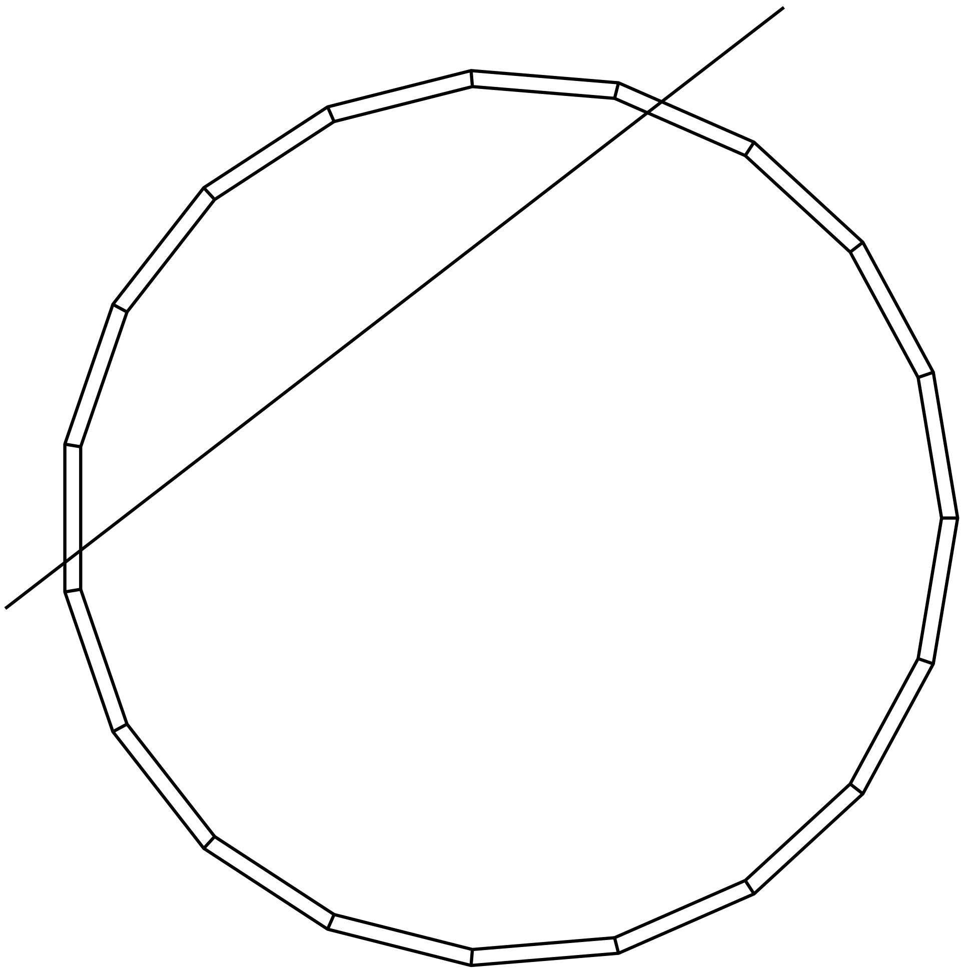

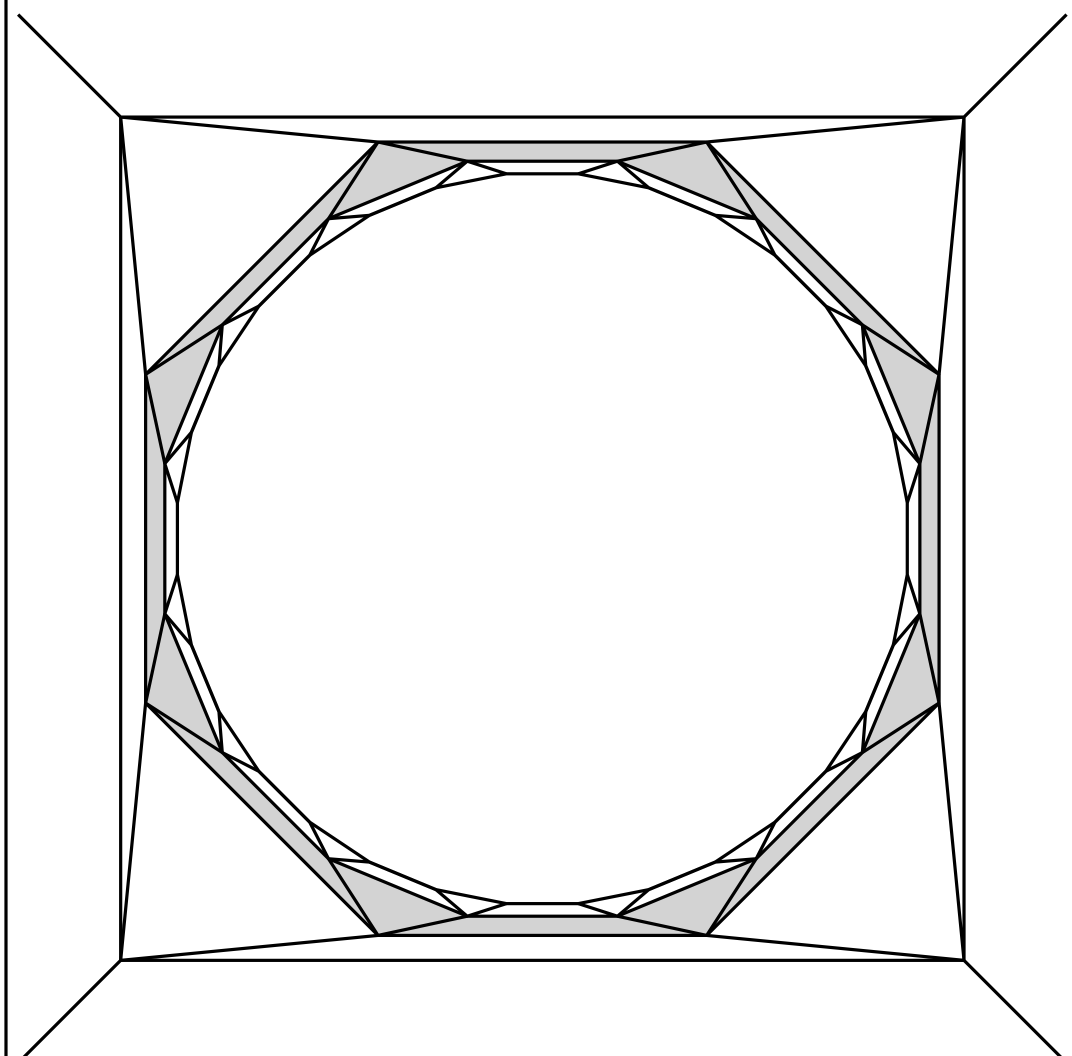

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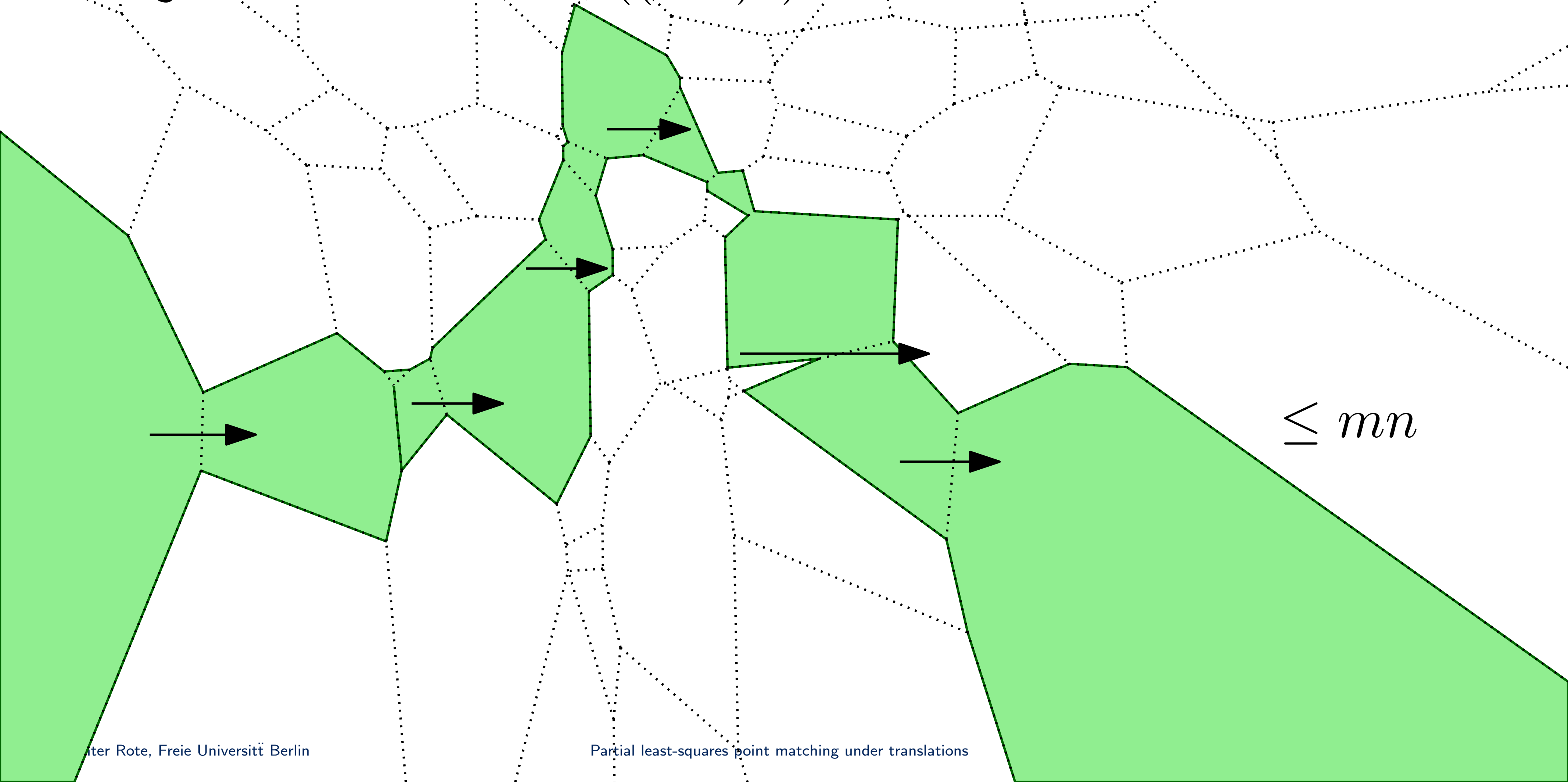

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The following would be sufficient (by polarity):

Conjecture: Every 3D polytope with n vertices has a monotone path of length $\Omega(\sqrt{n})$ in *some* direction.