Digital Models and Computer Assisted Proofs

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The first collection of reviewed electronic geometry models is available online at the new Internet server `http://www.eg-models.de`. This archive is open for any geometer to publish new geometric models, or to browse this site for material to be used in education and research. Access to the server is free of charge.

The geometry models in this archive cover a broad range of mathematical topics from geometry, topology, and, to some extent, from numerics. Examples are geometric surfaces, algebraic surfaces, topological knots, simplicial complexes, vector fields, curves on surfaces, convex polytopes, and, in some cases, experimental data from finite element simulations.

All models of this archive are reviewed by an international team of editors. The criteria for acceptance follow the basic rules of mathematical journals and are based on the formal correctness of the data set, the technical quality, and the mathematical relevance. This strict reviewing process ensures that users of the EG-Models archive obtain reliable and enduring geometry models. For example, the availability of certified geometry models allows for the validation of numerical experiments by third parties. All models are accompanied by a suitable mathematical description. The most important models will be reviewed by the Zentralblatt für Mathematik.

We are advocating the construction and submission of digital geometric models from various areas of mathematics. The advantages of these digital models go beyond those of the classical plaster shapes and dynamic steel models of earlier days. At the end of the 19th century several mathematicians felt the need to physically handle the geometric objects they thought about. In particular, Felix Klein and Hermann Amandus Schwarz in Göttingen built many models of curves, surfaces and mechanical devices for teaching and other educational purposes.

What are the main reasons for today's mathematicians to construct digital models of geometric shapes and make them available via the EG-Models server? There are obvious educational aspects, like for the historical models. And the means of interactive visualization are definitively useful for scientific purposes, too.

But, the focus of this article is another, somewhat different, view. Nowadays computer generated or assisted proofs enter virtually all areas of mathematics, and still the majority of the mathematicians are reluctant to accept the validity of such results. On the one hand, it seems somewhat strange to completely abstain from using tools, such as the computer, for doing mathematics; disregarding, maybe, aesthetic arguments. On the other hand, the inherent property of a proof is its verifiability; that is, verifiable by someone who is sufficiently trained. But, this very property of a proof might be challenged in individual

cases, where a computer is involved to solve a task too arduous or too tiring for any human. We are not going to raise the general question about the development of the mathematical culture, but we do believe that the installation of a server for mathematical models can help to improve the transparency of computer assisted proofs. For instance, think of a proof which is established by a computer construction of some complicated geometric shape. A standardized description, independently checked by experts and available to everyone, would provide an enormous potential for validation.

Using the digital models, interested mathematicians can verify the claims on their own, using appropriate software of their choice. Moreover, once there is a model available, it is possible to perform one's own computational experiments on this data set. This could be a numerical evaluation as well as a search for another property yet to be analyzed for this model.

Each model comes with a detailed description which identifies the author, explains the mathematical purpose, and includes references to other sources of information. Each model has a unique identification number for unambiguous citation. Each model is equipped with qualified meta data information; therefore, the archive can be searched via specialized search engines such as those from EMIS (`http://www.emis.de`) and MathNet (`http://www.math-net.de/search/germany/`). Each model itself is represented by a master file which is from a fixed set of file formats, including XML formats specified by DTDs. By restricting the data formats we want to ensure that the server's information can be kept up to date on a technical level. Additional files in arbitrary formats are welcome for explanatory purposes.

The Electronic Geometry Models Server has opened November 2000.

References

[1] *Electronic Geometry Models*, `http://www.eg-models.de`.

[2] Udo Hertrich-Jeromin: *Isothermic cmc-1 Cylinder*, Electronic Geometry Models, No. 2000.09.038, DarbouxSphere_Master.jvx.

[3] Michael Joswig and Günter M. Ziegler: *A neighborly cubical* 4*-polytope*, Electronic Geometry Models, No. 2000.05.003, C45_Master.poly.

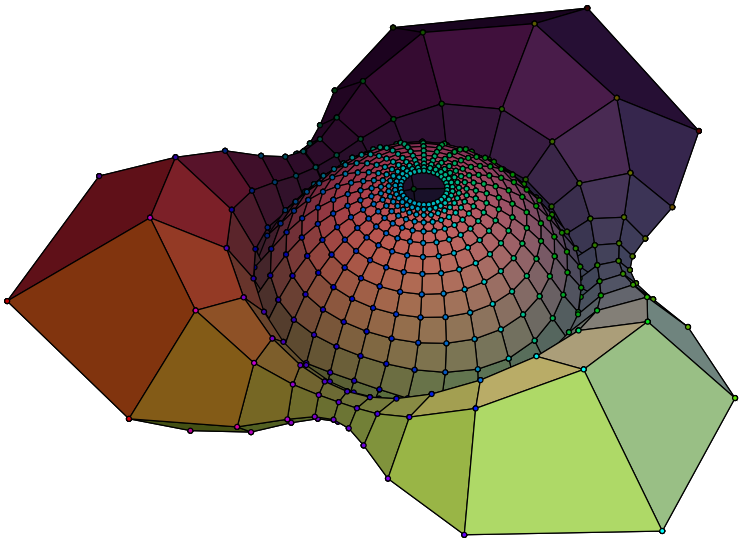

Figure 1: Darboux transform of a spherical discrete isothermic net [2]. Given the data it is easy to verify that this describes an isothermic surface. Additionally, it can be checked that this surface has discrete constant mean curvature.

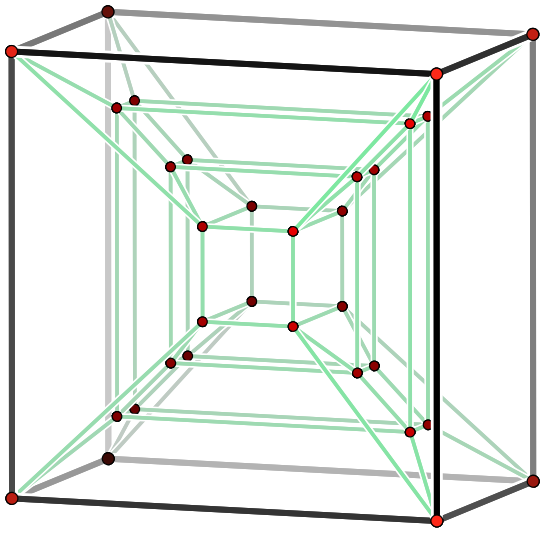

Figure 2: Schlegel diagram of a cubical 4-polytope whose graph is isomorphic to the graph of the 5-dimensional cube [3].