Assignment 8

Positional Games, Winter 2009-10
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Quiz on Dec 15th (Tuesday) at 16:15PM

A k-SAT formula F is called a (k, s)-*SAT formula* if every variable occurs in at most s clauses. Define $f(k)$ to be the largest integer s, such that *every* (k, s)-SAT formula is satisfiable.

Problem 1 Prove that every (k, k)-SAT formula is satisfiable. Conclude that $f(k) \geq k$ for EVERY k.

Recall the definition of (n, d)-tree from the lecture. A rooted tree T is called an (n, d)-tree if

1. every leaf is of distance at least $n - 1$ from the root and
2. for every vertex v the number $l(v)$ of leaf-descendants of v with distance at most $n - 1$ to v is at most d

Problem 2 Prove that if there is a (k, d)-tree, then there is an unsatisfiable $(k, 2d)$-SAT formula.
Conclude that $f(k) = \Theta(2^k/k)$.

Problem 3 (Open ended) Let $\mathcal{F} \subseteq 2^X$ be an n-uniform family of winning sets on the base set X. Give an upper bound (as large as you can) on $MaxEdgeNeigh(\mathcal{F}) := \max |\{B \in \mathcal{F} : B \cap A \neq \emptyset\}|$, that guarantees the existence of a Drawing Terminal Position.