Assignment 7

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Positional Games, Winter 2009-10
Quiz on Dec 8th (Tuesday) at 16:15PM

Problem 1 Prove the Local Lemma by imitating the proof in the lecture: Let $E_1, E_2, \ldots, E_M$ be events in an arbitrary probability space with $Pr[E_i] \le p < 1$ for every i. Suppose that every event E_i is mutually independent from all, but a set of d events. (That is, for every $i \in [M]$ there is subset $K = K(i) \subseteq [M] := \{1, \ldots M\}$ of d indices such that $Pr[E_i|E] := \frac{Pr[E_i \wedge E]}{Pr[E]} = Pr[E_i]$ for any event E of the form $E = \wedge_{k \in J} E_k$, where $J \cap K = \emptyset$.)

Prove that if $4pd \le 1$, then $Pr[\wedge_{i \in [M]} \overline{E}_i] > 0$

Problem 2 Let $\mathcal{F} \subseteq 2^X$ be an n-uniform family of winning sets on the base set X. Suppose that $\Delta(\mathcal{F}) \le 2^{n-4}/n$.

Show that there exists a Drawing Terminal Position. (A Drawing Terminal Position is a two-coloring of X, that could have been produced by a particular play between Maker and Breaker and resulted in a draw. That is, formally, coloring $c : X \to \{M, B\}$ is a DTP if $|c^{-1}(\{M\})| = \lceil |X|/2 \rceil$, $|c^{-1}(\{B\})| = \lfloor |X|/2 \rfloor$ and $|c(A)| = 2$ for every winning set $A \in \mathcal{F}$.)

(*Hint:* Prove something stronger. For even $|X|$ show that for any partition $V_1 \cup \cdots \cup V_{|X|/2}$ of X into subsets V_i of two elements, there is a proper two-coloring of X which uses both colors on every pair V_i.)

Problem 3 Let $F = \wedge_{i=1}^{M} C_i$ be a k-SAT formula with the property that every variable appears in at most $2^{k-2}/k$ clauses C_i. Prove that F is satisfiable.