Assignment 5

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Positional Games, Winter 2009-10
Quiz on Nov 24th (Tuesday) at 16:15PM

Recall: K_N^p is the the complete p-uniform hypergraph on N vertices:

$$V(K_N^p) = [N], E(K_N^p) = \binom{[N]}{p}.$$

The p-uniform Ramsey number $R^p(k)$ is defined as

$$\begin{aligned} R^p(k) \quad := \quad & \min\{N : \forall c : E(K_N^p) \to \{\text{red}, \text{blue}\}, \exists K \subseteq V(K_N^p), |K| = k, \\ & \text{with } c(P) = \text{red } \forall P \subseteq K, |P| = p \text{ or } c(P) = \text{blue } \forall P \subseteq K, |P| = p\}. \end{aligned}$$

Problem 1 Show

$$R^3(k) \geq 2^{k^2/6}$$

In an Avoider-Enforcer game on a hypergraph $\mathcal{F}$ the players occupy the free vertices of the board alternately, with Avoider going first. The game ends when every vertex is occupied by one of the players. Avoider wins the game, if he does not occupy any winning set fully. Otherwise Forcer wins (that is, if Avoider owns a full winning set at the end.)

Problem 2 Let $\mathcal{F} \subseteq 2^X$ be an n-uniform hypergraph. Suppose $|\mathcal{F}| < 2^n$. Give a strategy for Avoider to win.