Assignment 4

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Positional Games, Winter 2009-10
Quiz on Nov 17th (Tuesday) at 16:15PM

The Ramsey number $R(k,l)$ is defined as

$$\begin{aligned} R(k,l) \;\; := \;\; & \min\{N : \forall c : E(K_N) \to \{\text{red}, \text{blue}\}, \\ & \qquad \exists K \subseteq V(K_N), |K| = k, \text{with } c(xy) = \text{red } \forall x, y \in K \\ & \qquad \text{or } \exists L \subseteq V(K_N), |L| = l, \text{with } c(xy) = \text{blue } \forall x, y \in L\}. \end{aligned}$$

Problem 1 Prove $R(3,4) = 9$

Problem 2 Prove $R(4,4) \leq 18$

Problem 3 Prove $R(4,4) \geq 18$. (Hint: Consider the Paley graph P_{17}, defined on the vertex set $V(P_{17}) = F_{17}$, where F_{17} is the 17 element field of residues modulo 17. Vertices a and b are defined to be adjacent in P_{17} if $a - b$ is a quadratic residue, that is, there exists a $z \in F_{17} \setminus \{0\}$ such that $a - b = z^2$.)

Problem 4 The r-color Ramsey number $R(k_1, \ldots, k_r)$ is defined to be the smallest integer N, such that for every r-coloring of the edges of the clique K_N, there exists an index i, $1 \leq i \leq r$, and a clique $K \subseteq K_N$ of order k_i whose edges are all colored with color i. (For example, $R(3,3) = 6$ and one can prove (you don't have to) that $R(3,3,3) = 17$.)

Prove that for the r-color Ramsey number for triangles we have

$$R_r(3) := R(3, \ldots, 3) < er! + 1.$$

(Hint: Prove the recurrence $R_r(3) - 1 \leq 1 + r(R_{r-1}(3) - 1)$ and proceed by induction.)

Note that you just proved $R(3,3,3) \leq 17$.