Assignment 13

Positional Games, Winter 2009-10
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Solutions are due on Feb 9th (Tuesday) at 16:15PM

Problem 1 Show that the "only if" part of the previous HW2 is practically true for uniform hypergraphs. (See there the definitions.)

Let H be a uniform family of (k,t)-type. If $t > mk\sum_{i=1}^{k-1}\frac{1}{i}$, then Breaker has a winning strategy in the $(m:1)$ Box-game on H. (*Hint:* Try to come up with an argument reminiscent of the proof of the optimal Maker's strategy to build a spanning tree in a biased game.)

Problem 2. Let T be an **arbitrary** tournament on n vertices. Let Mr Red and Mr Blue play on K_N such that in each round they orient one of the unoriented edges of K_N and color it with their own color. Mr Red wins if he occupies a subtournament that is isomorphic to T. Prove that there is an $N_0 = N_0(T)$ such that Mr Red wins the game played on K_N for $N \geq N_0$. (*Hint:* Try to use the biased Weak Win Criterion of Beck.)

Problem 3. Let H be a fixed graph. In the H-game $\mathcal{K}_n(H) = \mathcal{K}(H) = \{E(F) : F \subseteq K_n, F \cong H\}$ the goal of Maker is to build a copy of a graph H in K_n. For the critical bias $b_{\mathcal{K}(H)}$ the following theorem is true:

Theorem (Bednarska-Łuczak) *For every graph H containing at least three nonisolated vertices*

$$b_{\mathcal{K}(H)} = \Theta\left(n^{\frac{1}{m(H)}}\right),$$

where

$$m(H) = \max_{K\subseteq H, |V(K)|\geq 3} \frac{|E(H)|-1}{|V(H)|-2}.$$

In class we prove the lower bound for every graph containing a cycle (using a randomized strategy).

(a) Prove the theorem for forests.
(b) Make the bounds on the leading coefficients as tight as you can. In some cases can you determine $b_{\mathcal{K}(H)}$ asymptotically (that is, the leading coefficient)?

Bednarska and Łuczak conjectures that there is constant $c = c(H)$ for every graph H such that

$$b_{\mathcal{K}(H)} = (c + o(1))\left(n^{\frac{1}{m(H)}}\right).$$