Assignment 1

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Positional Games, Winter 2009-10
Due date: Oct 20th (Tuesday) by 12:15PM

Problem 1 (a) Prove that every two-coloring of the 3^3-game board $V = \{0,1,2\}^3$ contains a monochromatic geometric line (a *geometric line* is a sequence $v_1, v_2, v_3 \in V^3$ such that for every i, the three ith coordinates form an increasing sequence $(0,1,2)$ or a decreasing sequence $(2,1,0)$ or a constant sequence.)

(b) Using (a) prove that the 3^3-game is a first player win.

Problem 2 Give an explicit winning strategy for the 3^3-game.

Problem 3 The game of NIM is played with a finite number of stacks of coins. Two players alternately make a move. A move of a player consists of first selecting a stack and then taking away at least one coin from it. The player who takes away the last coin from the table is the winner.

Let there be k stacks, containing $n_1, n_2, \ldots, n_k$ respectively. Let $n_i = a_0^{(1)} + a_1^{(1)}2 + a_2^{(1)}2^2 + \cdots + a_{s_i}^{(1)}2^{s_i}$ be the binary expansion of n_i.

Prove that second player has a winning strategy if and only if $v_j = a_j^{(1)} + \cdots + a_j^{(k)}$ is even for all j

Problem 4 Prove that there is a winning pairing strategy for the first player in the game of Bridge-It. (A *pairing strategy* for second player consists of a partition of the board into two element sets $A_1, \ldots, A_h$, so when first player takes an element z from some set A_i then second player answers with the other element of A_i. A pairing strategy for first player consists of specifying an opening move and a partition of the rest of the board into pairs.)