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# Discrete Geometry III

## Homework # 6 — due February 10th

Please do all the problems. You can get extra credit by solving the bonus problems. **State** who wrote up the solution. You have to hand in the solutions **before** the recitation on Wednesday.

**Exercise 1.** Let $U = (u_1, \dots, u_k)$ be a $k$-frame and $P \subset \mathbb{R}^d$ a polytope.

i) Show that for $\varepsilon > 0$ sufficiently small

$$P^U \;:=\; (P^{u_1})^{u_2} \cdots)^{u_k} \;=\; P^{u_\varepsilon},$$

where $u_\varepsilon := u_1 + \varepsilon u_2 + \cdots + \varepsilon^{k-1} u_k$.

ii) Let $U'$ be a frame such that $P^U = P^{U'}$. Show that

$$\mathrm{F}_U(f) \;=\; \mathrm{F}_{U'}(f)$$

for all $f \in \Pi(P)$.

**(10 points)**

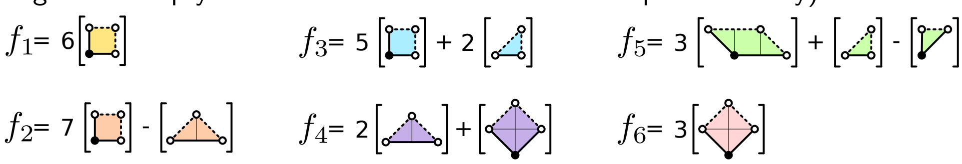
**Exercise 2.** Let $Q \subset \mathbb{R}^d$ be a polytope.

i) Show that there is a simple polytope $P$ such that $Q \preceq P$. [Hint: Represent $Q = \{x : Ax \le b\}$.]

ii) Show that there is always a zonotope $Z$ (i.e. Minkowski sum of segments) such that $Q \preceq Z$. [Hint: What are the edges of a zonotope?]

**(10 points)**

**Exercise 3.** Consider the following six elements $f_1, \dots, f_6$ in the polytope algebra $\Pi^2$ (dashed edges and empty circles are used to indicate a half-open boundary).




Here, the small cube represents the unit square $[0,1]^2$.

i) Find a polygon $P$ such that $f_i \in \Pi(P)$ for $1 \le i \le 6$.

ii) For each pair $1 \le i < j \le 6$, decide whether $f_i = f_j$. You have to argue your answer but you don't have to write the full computation.

**(10 points)**

**Exercise 4.** Let $C_n^\triangle = \mathrm{conv}(\pm e_1, \pm e_2, \pm e_n)$ be the $n$-dimensional cross-polytope. For every $k = 0, 1, \dots, n$, determine the dimension of $\Omega_k(C_n^\triangle)$.

**(10 points)**