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SS2013

Discrete Geometry II

Homework # 2— due April 24th

Exercise 1. ■ Let $K_1, K_2, L \subset \mathbb{R}^d$ be convex bodies.

i) Show that $h_{K_1+L}(c) = h_{K_1}(c) + h_L(c)$ for all $c \in \mathbb{R}^d \setminus \{0\}$.

ii) Show that if $K_1 + L = K_2 + L$, then $K_1 = K_2$.

iii) (Bonus) Show that if $K_1 \cup K_2$ is convex, then

$$(K_1 \cup K_2) + (K_1 \cap K_2) \;=\; K_1 + K_2.$$

(10+3 points)

Exercise 2. Let $A, B \subset \mathbb{R}^d$ be convex sets (not necessarily closed or bounded). Show that A and B can be separated if and only if $A + (-B)$ can be separated from the origin 0.

(10 points)

Exercise 3. Let $K \subset \mathbb{R}^d$ be a convex body with $0 \in \text{int}(K)$.

i) Show that for $(\alpha K)^\circ = \frac{1}{\alpha} K^\circ$ for all $\alpha \neq 0$.

ii) If $K = B_d(0)$ is the unit ball centered at the origin, then $K = K^\circ$.

iii) Show that if $K = K^\circ$, then $K = B_d(0)$.

(10 points)

Exercise 4. i) Show that the support function h_K is linear if and only if K is a point.

ii) Let $K \subset \mathbb{R}^d$ be a convex body, $v \in \mathbb{R}^d$ be a point. What is the support function of $K + v$?

(10 points)

Exercise 5. Let $K \subset \mathbb{R}^d$ be a compact subset. The (outer) parallel body K_r of K at distance $r > 0$ is defined as the set of all points at distance $\leq r$ from K.

i) Show that if K is convex, then so is K_r.

ii) What is the support function of K_r?

(10 points)

Supplementary problems

Exercise 6. Call a d-dimensional convex body $K \subset \mathbb{R}^d$ *strictly convex*, if ∂K does not contain a line segment, and *differentiable*, if there exists only one supporting hyperplane in each $x \in \partial K$. Show that the following three properties are equivalent:

(a) K is strictly convex;

(b) every point of ∂K is an extreme point of K;

(c) the polar dual K° is differentiable.