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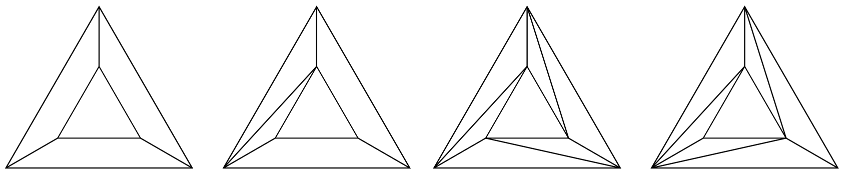
Discrete Geometry II

Homework # 6 — due May 27th

Please try to solve *all* problems. This will deepen the understanding of the material covered in the lectures. You are welcome to ask (in person or email) for additional **hints** for any exercise. Please think about the exercise before you ask. Please mark **two** of your solutions. Only these will be graded. **State** who wrote up the solution. You have to hand in the solutions **before** the lecture on Wednesday. Please write different exercises in different sheets.

Exercise 1. Let $P = \text{conv}(V)$ with $V = \{(0,0),(4,0),(0,4),(1,1),(2,1),(1,2)\}$. Consider the four subdivisions of the triangle P



For every subdivision $\mathcal{S}$ prove that either $\mathcal{S}$ is not regular or determine the set

$$W(\mathcal{S}) := \{\omega : \mathcal{S} = \mathcal{S}^\omega \text{ and } \omega(1,1) = \omega(1,2) = \omega(2,1) = 0\}.$$

(10 points)

Exercise 2. Let $\sigma = \text{conv}(v_0,\dots,v_d) \subset \mathbb{R}^d$ be a d-simplex and let $a_0,\dots,a_d,b_0,\dots,b_d \in R$ such that $a_i < b_i$. The polytope $P = \text{conv}((v_i,a_i),(v_i,b_i) : i = 0,\dots,d)$ is a combinatorially isomorphic to a prism over σ.

i) For $j = 0,\dots,d$ define

$$P_j := \text{conv}\left(\begin{pmatrix} v_0 \\ a_0 \end{pmatrix},\dots,\begin{pmatrix} v_j \\ a_j \end{pmatrix},\begin{pmatrix} v_j \\ b_j \end{pmatrix},\begin{pmatrix} v_{j+1} \\ b_{j+1} \end{pmatrix},\dots,\begin{pmatrix} v_d \\ b_d \end{pmatrix}\right)$$

Prove that $P = P_0 \cup \cdots \cup P_d$ is a dissection of P into simplices.

ii) Show that

$$V_{d+1}(P) \;=\; \frac{V_d(\sigma)}{d+1}\sum_{i=0}^{d}(b_i - a_i).$$

(10 points)

Exercise 3. i) Let $P = \text{conv}(V)$ be a polygon with vertex set $V \subset \mathbb{R}^2$. Show that every triangulation of P that uses only vertices in V is regular.

ii) Let $V_n = \{0,1,\dots,n\} \subset \mathbb{R}$. Show that the secondary polytope $\Sigma(V_n)$ is isomorphic to a cube of dimension $(n-2)$.

(10 points)