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SS2015

Discrete Geometry II

Homework # 1 — due April 22nd

You can write your solution to the homeworks in **pairs**. Please try to solve *all* problems. This will deepen the understanding of the material covered in the lectures. You are welcome to ask (in person or email) for additional **hints** for any exercise. Please think about the exercise before you ask. Please mark **two** of your solutions. Only these will be graded. The problems marked with a ⊞ are **mandatory**. You can earn **20 points** on every homework sheet. You can get extra credit by solving the bonus problems. **State** who wrote up the solution. You have to hand in the solutions **before** the lecture on Wednesday. Homework sheets and additional information (including corrections) will be announced on the mailinglist. If you haven't signed up already, please do that at

`https://lists.fu-berlin.de/listinfo/dg2`

Exercise 1. Let $C \subset \mathbb{R}^d$ be a compact set. A point $s \in C$ is a **star point** if $[s,p] \subseteq C$ for all $p \in C$. Show that the set $\mathrm{St}(C)$ of star points of C is convex.

(10 points)

Exercise 2. i) Prove that the unit ball $B_d = \{x \in \mathbb{R}^d : \|x\| \leq 1\}$ is not a polytope.

ii) For $b, c \in \mathbb{R}$ define the function $f_{b,c}(t) = t^2 + bt + c$. Show that

$$K \; := \; \{(b,c) \in \mathbb{R}^2 : f_{b,c}(t) \geq 0 \text{ for all } t \in \mathbb{R}\}$$

is a convex set. Is it a polytope?

(10 points)

Exercise 3. Let K be a convex set. Show that $K \neq \emptyset$ implies relint$(K) \neq \emptyset$.

(10 points)

Exercise 4. A set $X \subseteq \mathbb{R}^d$ is called **path-connected** if for every two $p_0, p_1 \in X$ there is a continuous function $\gamma : [0,1] \to X$ such that $\gamma(0) = p_0$ and $\gamma(1) = p_1$.

Let $K \subset \mathbb{R}^d$ be a (full-dimensional) convex body. Show that if the set of extreme points $X = \mathrm{ext}(K)$ is path-connected, then every point $p \in K$ is in the convex hull of d points. (This improves Carathéodory's theorem by 1.)

(10 points)