Freie Universität Berlin

34th EuroCG 18

Booklet of Abstracts

Nachweis Bernd Wannenmacher (auch für die Titelseite)

Nachweis Bernd Wannenmacher

Welcome to EuroCG 2018

The 34th European Workshop on Computational Geometry (EuroCG '18), is held at Freie Universität Berlin, Berlin, Germany, on March 21–23, 2018. EuroCG is an annual workshop that combines a strong scientific tradition with a friendly and informal atmosphere. The workshop is a forum where researchers can meet, discuss their work, present their results, and establish scientific collaborations, in order to promote research in the field of Computational Geometry, within Europe and beyond.

We received 78 submissions, which underwent a limited refereeing process by the program committee in order to ensure some minimal standards and to check for plausibility. We selected 75 submissions for presentation at the workshop. One submission was later withdrawn. This booklet of abstracts contains short abstracts for all lectures, including the three invited lectures by Nina Amenta, Prosenjit Bose, and Raúl Rojas. A book of extended 6-page abstracts is available at the EuroCG '18 web site, `conference.imp.fu-berlin.de/eurocg18`.

Many thanks to all authors, speakers, and invited speakers for their participation, and to the members of the program committee and all external reviewers for their insightful comments. We gratefully thank the supporters of EuroCG '18 for making this event possible and helping to keep the registration fees low: Freie Universität Berlin, keylight GmbH, and the German Research Foundation (DFG grant MU 3501/4-1). Special thanks to all members of the organizing committee and members of the administration at Freie Universität Berlin, for their work that made EuroCG '18 possible.

March 2018

Matias Korman and Wolfgang Mulzer
EuroCG '18 chairs

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Session 1. Invited Talk, Wed 9:00–10:00, main hall

Wed 9:00–10:00 **1**, main hall, #A	**Rigidity and Deformation** *Nina Amenta*, University of California at Davis

Download a triangle-mesh model of a 3D bunny, cut a stick for every edge, and attach them together with a flexible joint at each vertex to re-create the model's one-skeleton. Would it stand up or collapse? Bet on "stand up" — Herman Gluck proved in 1975 that almost all such triangulated one-skeletons are rigid. There are a few examples of non-rigid polyhedra; that is, there is a motion under which the edge lengths remain fixed but the dihedral angles change.

What if, instead of fixing the edge lengths, we fixed the dihedrals? Are there motions which fix the dihedrals but allow the lengths to change? We show an analog of Gluck's theorem, that almost all polyhedra are "dihedral-rigid".

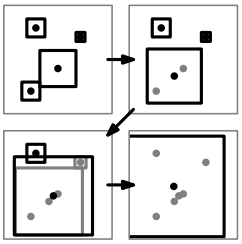
Who cares? Well, deformation is the opposite of rigidity. What can rigidity — and the examples of non-rigidity — tell us about how we can parameterize, measure and control the deformations of a mesh? Parameterizing deformations by edge length turns out to be a bad idea, but we demonstrate that there is reason to be much more hopeful about parameterizing meshes by their dihedrals.

This is work with Carlos Rojas.

Session 2A, Wed 11:05–12:35, main hall

Wed 11:05–11:20 **2A**, main hall, #8	**Agglomerative Clustering of Growing Squares** *Thom Castermans, Bettina Speckmann, Frank Staals, Kevin Verbeek*

We study an agglomerative clustering problem motivated by interactive glyphs in geo-visualization. Consider a set of disjoint square glyphs on an interactive map. When the user zooms out, the glyphs grow in size relative to the map, possibly with different speeds. When two glyphs intersect, we wish to replace them by a new glyph that captures the information of the intersecting glyphs.



We present a fully dynamic kinetic data structure that maintains a set of n disjoint growing squares, allowing us to solve this problem in $\mathcal{O}(n\alpha(n)\log^7 n)$ time.

This presentation has a high potential for nice animations. Join us to see if we deliver!

Wed 11:22–11:37 **2A**, main hall, #30	**Combinatorial and Asymptotical Results on the Neighborhood Grid Data Structure** *Martin Skrodzki, Ulrich Reitebuch, Konrad Polthier*

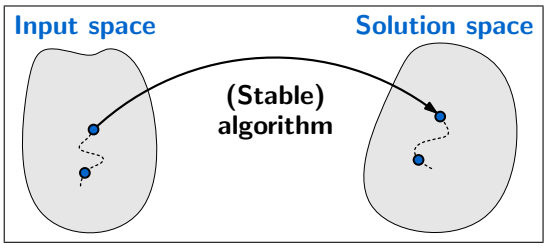
In 2009, Joselli, Passos, Zamith, Clua, Montenegro, and Feijó introduced the Neighborhood Grid data structure for fast computation of neighborhood estimates in point clouds. Even though the data structure has been used in several applications and shown to be practically relevant, it is theoretically not yet well understood. The purpose of this paper is to present a polynomial-time algorithm to build the data structure. Furthermore, current investigations on the optimality of the given algorithm are presented.

Notes:

Wed 11:39–11:54
2A, main hall, #11

A Framework for Algorithm Stability and its Application to Kinetic Euclidean MSTs

Wouter Meulemans, Bettina Speckmann, Kevin Verbeek, Jules Wulms



We say that an algorithm is *stable* if small changes in the input result in small changes in the output. This kind of algorithm stability is particularly relevant when analyzing and visualizing time-varying data. Stability in general plays an important role in a wide variety of areas, such as numerical analysis, machine learning, and topology, but is poorly understood in the context of (combinatorial) algorithms. In this talk we present a framework for analyzing the stability of algorithms. We focus in particular on the tradeoff between the stability of an algorithm and the quality of the solution it computes. Our framework allows for three types of stability analysis with increasing degrees of complexity: event stability, topological stability, and Lipschitz stability. We demonstrate the use of topological stability by applying it to kinetic Euclidean minimum spanning trees.

Wed 11:56–12:11
2A, main hall, #18

Balanced Dynamic Loading and Unloading

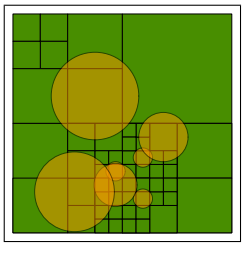
Sándor Fekete, Sven von Höveling, Joseph Mitchell, Christian Rieck, Christian Scheffer, Arne Schmidt, James Zuber

We consider the problem of loading and unloading a container ship with the restriction of maintaining balanced configurations at all times, i.e., we want to minimize the maximal motion of the center of gravity during the entire process. In particular, we consider the one-dimensional case and distinguish between *unloading*, where the positions of the items are fixed, and the *loading* variant where we have to compute the optimal order as well as the positions. In other words: After the talk you might be able to answer your kid's question on how to place/remove containers onto/from its toy ship so that the ship will not capsize in the bathtub.

Wed 12:13–12:28
2A, main hall, #59

Approximate Stabbing Queries with Sub-logarithmic Local Replacement

Ivor Hoog v.d., Maarten Löffler



In this work we present the key ingredients to construct a linear-size data structure that stores a set of spheres or axis-parallel hypercubes in $\mathbb{R}^d$ and supports what we define as 2^m-approximate stabbing queries in logarithmic time and local replacement in sub-logarithmic time, if the regions not overlap "too much". This work uses known techniques such as quadtrees and marked-ancestor trees and introduces a new concept: *key facets* in a d-dimensional quadtree. We show the intuition behind how this new concept can help us perform fast (approximate) stabbing queries with sub-logarithmic local replacement if the dimension d and the approximation variable m are constant.

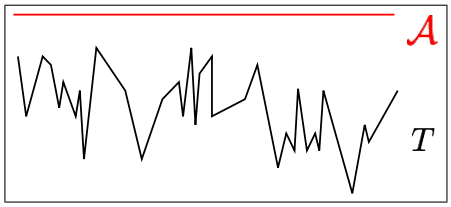
Notes:

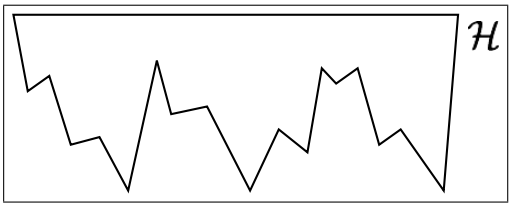
Session 2B, Wed 11:05–12:35, room 005

Wed 11:05–11:20
2B, room 005, #41

Altitude Terrain Guarding and Guarding Uni-Monotone Polygons

Stephan Friedrichs, Valentin Polishchuk, Christiane Schmidt



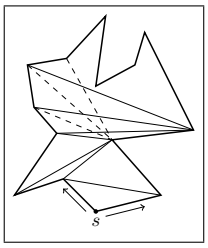


We show that the problem of guarding an x-monotone terrain T from an altitude line $\mathcal{A}$ and the problem of guarding a uni-monotone polygon are equivalent. For uni-monotone polygons we show that there exists an optimal guard set such that all guards are located on its single horizontal segment that constitutes the upper chain, $\mathcal{H}$. We present a polynomial time algorithm for both problems—a simple sweep—, and show that the cardinality of a minimum guard set and the cardinality of a maximum witness set coincide. Thus, uni-monotone polygons are perfect. Hence, we establish the first non-trivial class of perfect polygons under "normal" vision.

Wed 11:22–11:37
2B, room 005, #1

Maximal Two-Guard Walks in a Polygon

Franz Aurenhammer, Michael Steinkogler

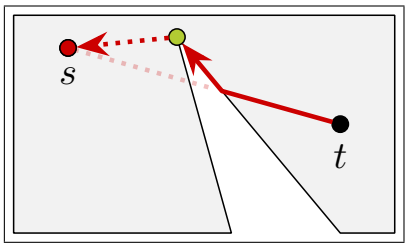


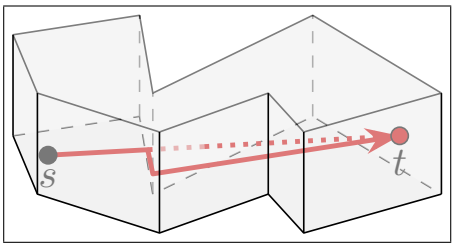
The two guards problem asks whether two guards can walk along the boundary of a simple n-sided polygon from a source vertex to target vertex, one guard in clockwise and one in counterclockwise direction, while maintaining visibility within the polygon. We study the related question of how far the guards can reach from the source vertex s if the polygon is not walkable. There can be $\Theta(n)$ such maximal walks, and we show how to find all of them in $O(n \log n)$ time.

Wed 11:39–11:54
2B, room 005, #48

Combinatorics of Beacon-based Routing in Three Dimensions

Jonas Cleve, Wolfgang Mulzer





Beacons can be imagined as magnets which, when enabled, force all objects in a polytope to try to move in the beacons direction. It is a (non-symmetric) extension of classic visibility and was first introduced for polygons in two dimensions.

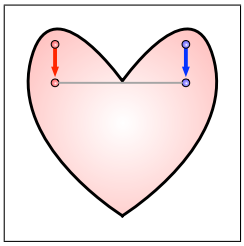
Beacon-based routing means to enable multiple beacons successively (only one at a time) such that an object is moved from a start to designated end point. The goal is to minimize the number of beacons needed to route any given start point to any given end point.

In the talk we introduce you to the model in two dimensions and then show you the first result for routing in three dimensions.

Wed 11:56–12:11
2B, room 005, #16

On Romeo and Juliet Problems: Minimizing Distance-to-Sight

Hee-Kap Ahn, Eunjin Oh, Lena Schlipf, Fabian Stehn, and Darren Strash



Let P be a simple polygon and let r (Romeo's initial location) and j (Juliet's initial location) be two to points inside P (or on its boundary).

We consider the problem of computing locations r' and s' such that Romeo standing at r' and Juliet standing at j' can *see each other* inside P.

We study this problem for two objectives: minimizing the *sum* of the path-lengths from r to r' and j to j' (*min-sum*) or minimizing the length of the *longer* of the two paths (*min-max*).

We show that both variants of the problem can be solved in linear time.

Wed 12:13–12:28
2B, room 005, #12

Guarding Monotone Polygons with Vertex Half-Guards is NP-Hard

Matt Gibson, Erik Krohn, Matthew Rayford

We consider a variant of the art gallery problem where all guards are limited to seeing to the right inside a monotone polygon. We show that the problem is NP-hard if guards are restricted to be at the vertices of the polygon.

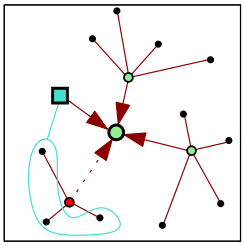
The cats in the images will help us motivate the problem. We will also watch a short video that answers a centuries old question of who wins in a battle between a butterfly and cat.

Session 3A, Wed 13:30–15:00, main hall

Wed 13:30–13:45
3A, main hall, #38

Data Gathering in Faulty Sensor Networks Using a Mule

Stav Ashur



Given a set S of wireless sensors in the plane with unit transmission range, we would like to create a data gathering tree $T \subseteq UDG(S)$ (where $UDG(S)$ denotes the *Unit Disc Graph* on S) in order to efficiently collect the sensed data.

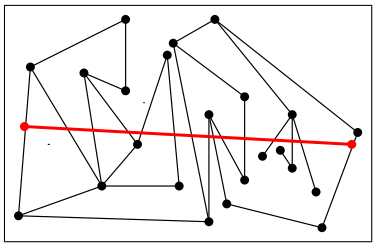
To deal with sensor-failure scenarios that result in significant data loss, we place a special wireless device, a data mule, in one of the network's nodes. When a sensor failure occurs, the mule can collect the data from the disconnected sub-tree by performing an appropriate TSP tour in the plane.

We present a constant-approximation algorithm for this problem, which is faster than the previous algorithm of Yedidsion, Banik, Carmi, Katz, and Segal (2017) by an order of magnitude.

Wed 13:47–14:02
3A, main hall, #45

Computing Optimal Shortcuts for Networks

Delia Garijo, Alberto Márquez, Natalia Rodríguez, Rodrigo I. Silveira

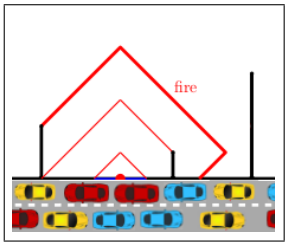


We study augmenting a plane Euclidean network with a segment or *shortcut* to minimize the largest distance between any two points along the edges of the resulting network. Questions of this type have received considerable attention recently, mostly for discrete variants of the problem. We study a fully continuous setting, where all points on the network and the inserted segment must be taken into account. In this setting, two major variants of the problem arise, depending on how the shortcut is inserted into the network: the *highway* model in which the crossings between the shortcut and the network edges do not form new network vertices, and the more general *planar* model where every crossing creates a new vertex. We present the first results on optimal shortcuts in the planar model for general networks, and several improved results for paths. We also highlight important differences between the highway and planar models, the latter resulting in considerably harder problems.

Wed 14:04–14:19
3A, main hall, #54

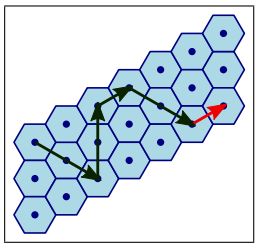
Protecting a Highway from Fire

Rolf Klein, David Kübel, Elmar Langetepe, Barbara Schwarzwald



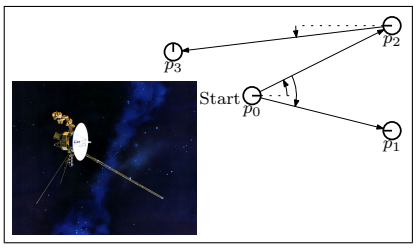
Suppose a fire spreads at speed 1 along a highway given by a horizontal line in the L_1 plane. A fighter is tasked to protect the highway by building barriers along or perpendicular to the highway with a building speed v. The fighter can move without delay or additional costs between different construction sites. Just building walls along the highway in both directions is a naive strategy that requires speed $v > 2$ and a small constant head start. We show that $v > 1.5$ is necessary and present a strategy that requires only $v > \frac{2+\sqrt{5}}{\sqrt{5}} = 1.8944\ldots$.

Wed 14:21–14:36 **3A**, main hall, #73	**Shape Recognition by a Finite Automaton Robot** *Robert Gmyr, Kristian Hinnenthal, Irina Kostitsyna, Fabian Kuhn, Dorian Rudolph, Christian Scheideler*



We investigate the problem of recognizing the ratio of the sides of a parallelogram by a finite-state automaton robot with pebbles operating on a triangular grid. Our goal is to determine the computational power of a single finite automaton robot in this setting with and without the help of pebbles. As a first simple example, we demonstrate that a robot without pebbles can determine whether a given shape is a parallelogram whose sides have a ratio of h to $ah + b$ for constant integers a and b. On the other side, a robot cannot detect whether the ratio is h to $f(h)$, where $f(x) = \omega(x)$ is a superlinear function. However, as we will show in this talk, having a single pebble enables the robot to decide whether the sides ratio is h to $p(h)$ for a given polynomial $p(\cdot)$ of constant degree. Finally, we briefly discuss how to decide more complex functions, such as exponential functions, using multiple pebbles.

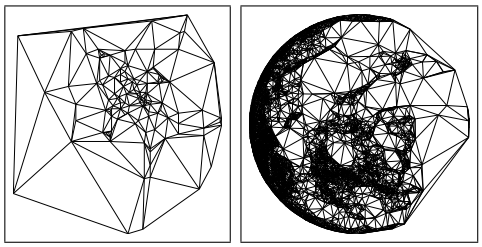
Wed 14:38–14:53 **3A**, main hall, #22	**Beam It Up, Scotty: Angular Freeze-Tag with Directional Antennas** *Sándor P. Fekete, Dominik Krupke*



We consider distributing mission data among the members of a satellite swarm. In this process, spacecraft cannot be reached all at once by a single broadcast, because transmission requires the use of highly focused directional antennas. As a consequence, a spacecraft can transmit data to another satellite only if its antenna is aiming right at the recipient; this may require adjusting the orientation of the transmitter, incurring a time cost proportional to the required angle of rotation. The task is to minimize the total distribution time. This makes the problem similar in nature to the *Freeze-Tag Problem* of waking up a set of sleeping robots, but with angular cost at vertices, instead of distance cost along the edges of a graph. We prove that approximating the minimum length of a schedule for this *Angular Free-Tag Problem* within a factor of less than 5/3 is NP-complete, and provide a 9-approximation for the 2-dimensional case that works even in online settings with incomplete information. Furthermore, we develop an exact method based on Mixed Integer Programming that works in arbitrary dimensions and can compute provably optimal solutions for benchmark instances with about a dozen satellites.

Session 3B, Wed 13:30–15:00, room 005

Wed 13:30–13:45 **3B**, room 005, #20	**Solving Large-Scale Minimum-Weight Triangulation Instances to Provable Optimality** *Andreas Haas*



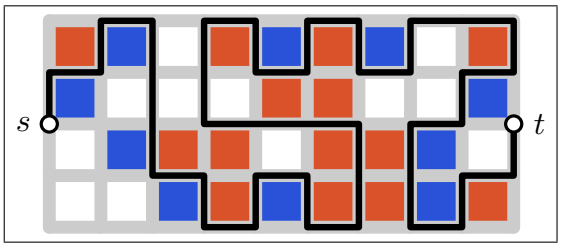
We present practical implementations of well-known techniques for the problem of finding a minimum-weight triangulation (MWT) of a planar point set. We show that these techniques can be refined and extended to solve large MWT instances of various types. As a result, we are able to solve MWT instances with up to 30,000,000 uniformly distributed points in less than 4 minutes. Moreover, we can compute optimal solutions for a vast array of other benchmark instances that are *not* uniformly distributed, including normally distributed instances (up to 30,000,000 points), all point sets in the TSPLIB (up to 85,900 points), and VLSI instances with up to 744,710 points. This demonstrates that from a practical point of view, MWT instances can be handled quite well, despite their theoretical difficulty.

Notes:

Wed 13:47–14:02
3B, room 005, #67

The Hardness of Witness Puzzles

Irina Kostitsyna, Maarten Löffler, Max Sondag, Willem Sonke and Jules Wulms



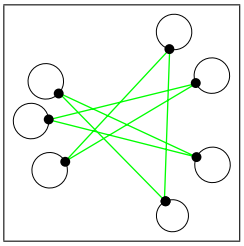
We resolve the complexity of two types of puzzles in the game The Witness, namely the Black and White Squares and the Multicolor Squares puzzles. In these puzzles a grid with colored squares is given, along with two vertices s and t on the outer boundary. The goal is to find a simple path from s to t that partitions the grid into pieces, where each piece contains cells of only a single color. We show that in a restricted setting these types of puzzles can be solved in polynomial running time, but that in general they are NP-complete.

Wed 14:04–14:19
3B, room 005, #55

A Fully Polynomial Time Approximation Scheme for the Smallest Diameter of Imprecise Points

Vahideh Keikha, Maarten Löffler, Ali Mohades

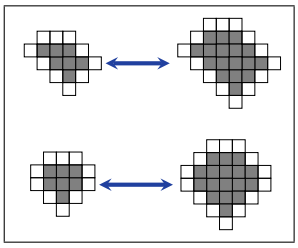
Given a set $D = \{d_1, ..., d_n\}$ of imprecise points modeled as disks, the minimum diameter problem is to locate a set $P = \{p_1, ..., p_n\}$ of fixed points, where $p_i \in d_i$, such that the furthest distance between any pair of points in P is as small as possible. This introduces a tight lower bound on the size of the diameter of any instance P. In this paper, we present a fully polynomial time approximation scheme (FPTAS) for this problem that runs in $O(n^3\epsilon^{-2})$ time, where the input is a set of disjoint disks.



Wed 14:21–14:36
3B, room 005, #24

Properties of Minimal-Perimeter Polyominoes

Gill Barequet, Gil Ben-Shachar



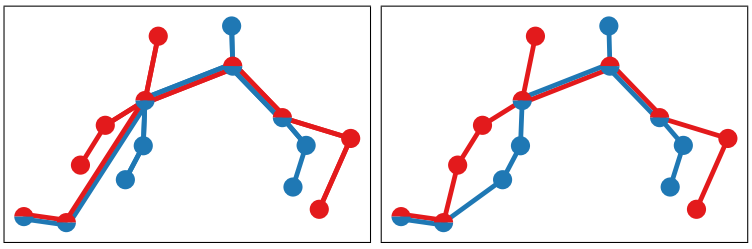
A polyomino is a set of connected squares on a grid. The area of a polyomino is the number of cells it contains, and the perimeter of a polyomino is the set of cells adjacent to some polyomino cell.

In this work we address the class of polyominoes with minimal perimeter for their area. First, we establish a connection between the geometric structure of a polyomino border (which we define to be the outer layer of polyomino cells) and its perimeter. Second, we define the operation of "*inflating*" a polyomino, which is adding the polyomino perimeter cells to itself. We use our first result to show that inflating a minimal perimeter polyomino results in a minimal-perimeter polyomino as well (inflation is one-to-one). Then, we show that if some of the polyominoes of a certain area are created by an inflation of a minimal-perimeter polyomino, then all the minimal perimeter polyominoes of the same size are also created by an inflation of another minimal-perimeter polyomino (inflation is onto). This leads to our main result, which states that the inflation operation induce a bijection between (infinitely many) sets of minimal perimeter polyominoes of certain areas.

Wed 14:38–14:53
3B, room 005, #35

Short Plane Support Trees for Hypergraphs

Thom Castermans, Mereke van Garderen, Wouter Meulemans, Martin Nöllenburg, Xiaoru Yuan



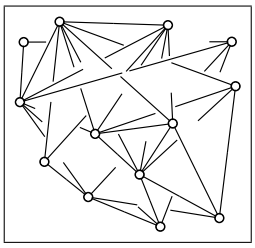
We study *support graphs*, a notion for drawing hypergraphs commonly used in the context of set visualization. A support graph is a graph spanning the same vertices (elements) as its hypergraph, in which each hyperedge (set) induces a connected subgraph. We present results on existence and hardness of finding a support graph of a hypergraph with fixed vertex locations. The support graph must minimize total edge length. We focus on these constraints: (i) enforcing planarity using a straight-line embedding; and (ii) requiring the support graph to be acyclic. Spoiler alert: it's hard; we have an ILP.

Session 4A, Wed 15:20–16:50, main hall

Wed 15:20–15:35
4A, main hall, #50

Maximizing Ink in Symmetric Partial Edge Drawings of k-plane Graphs

Michael Höller, Fabian Klute, Soeren Nickel, Martin Nöllenburg, Birgit Schreiber

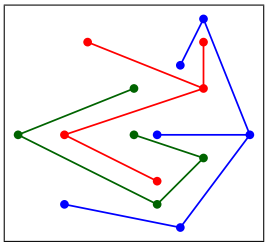


We study symmetric partial edge drawings (SPEDs) for non-planar graphs, in which edges are drawn only partially as pairs of opposing stubs of equal length on the respective end-vertices. It is known that maximizing the ink (or the total stub length) when transforming a straight-line drawing with crossings into a SPED is tractable for 2-plane input drawings, but generally NP-hard. We show that the problem remains NP-hard even for 3-plane input drawings. We also present efficient algorithms for ink maximization of k-plane input drawings whose edge intersection graph forms a collection of trees or cacti.

Wed 15:37–15:52
4A, main hall, #53

The Partition Spanning Forest Problem

Philipp Kindermann, Boris Klemz, Ignaz Rutter, Patrick Schnider, André Schulz

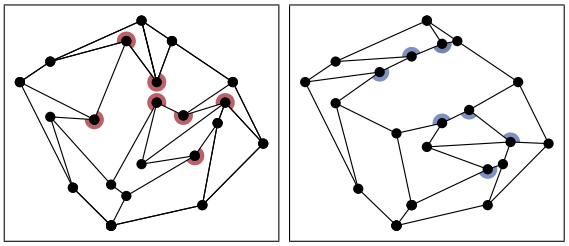


Given a set of colored points in the plane, we ask if there exists a crossing-free straight-line drawing of a spanning forest, such that every tree in the forest contains exactly the points of one color class. We show that the problem is NP-complete, even if every color class contains at most five points, but it is solvable in $O(n^2)$ time when each color class contains at most three points. If we require that the spanning forest is a linear forest, then the problem becomes NP-complete even if every color class contains at most four points.

Wed 15:54–16:09
4A, main hall, #65

Convexity-Increasing Morphs of Planar Graphs

Linda Kleist, Boris Klemz, Anna Lubiw, Lena Schlipf, Frank Staals, Darren Strash



A *morph* between two straight-line planar drawings Γ_1, Γ_2 of the same plane graph G is a continuous deformation that transforms Γ_1 into Γ_2 while preserving straight-line planarity at all times.

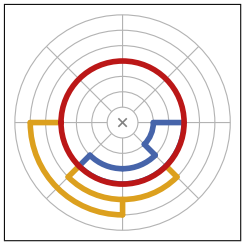
We want to find a morph from a given drawing Γ of G to *some* drawing of G in which all faces are convex. Further, we want the morph to be *convexity-increasing*, meaning that inner angles never change from convex to reflex. In the example, vertices are moved along vertical lines to convexify the red angles without changing the convexity-status of the remaining angles.

For the case of 3-connected planar graphs, we give an efficient algorithm that constructs such a morph as a composition of a linear number of steps which move vertices along horizontal or vertical lines.

Wed 16:11–16:26
4A, main hall, #71

Efficient Algorithms for Ortho-Radial Graph Drawing

Benjamin Niedermann, Ignaz Rutter, Matthias Wolf



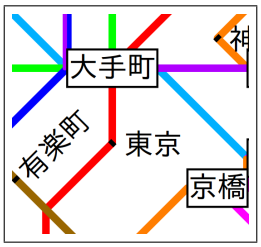
An ortho-radial drawing is an embedding of a graph into an ortho-radial grid, whose gridlines are concentric circles around the origin and straight-line spokes emanating from the origin but excluding the origin itself. Recently, Barth et al. showed that such drawings can be described combinatorially by so-called *valid ortho-radial representations*, which only specify angles at vertices and bends on the edges. Their result is existential, but does not provide an efficient procedure for drawing valid ortho-radial representations. In this talk we present an efficient algorithm for testing whether a given ortho-radial representation is valid. Based on this result we show how to draw a valid ortho-radial representation in quadratic time.

Notes:

Wed 16:28–16:43
4A, main hall, #62

Automatic Drawing for Tokyo Metro Map

Masahiro Onda, Masaki Moriguchi, Keiko Imai

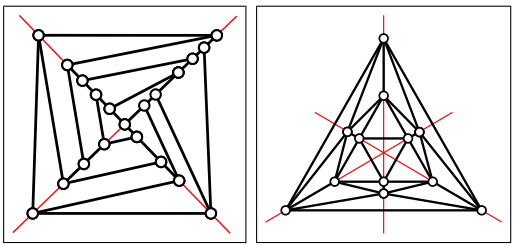


Automatic map drawing is one of the important problems in geometric information systems (GIS). Metro maps are schematic representations of geographic metro networks in GIS. It is difficult to draw complex metro networks with many stations and metro lines manually. Therefore, several methods for metro map drawings have been proposed. However, the Tokyo subway is one of the most complex networks in the world, and it is difficult to obtain its easy to read metro map using existing layout methods. In this paper, we present a new method that can generate complex metro maps like the Tokyo metro map. Our method consists of two phases. The first phase generates rough metro maps. It decomposes the metro networks into smaller subgraphs, and generates rough metro maps partially. In the second phase, we use a local search technique to improve the aesthetic quality of the rough metro maps. The experimental results including the Tokyo metro map are shown.

Wed 16:45–17:00
4A, main hall, #63

On the Weak Line Cover Numbers

Oksana Firman, Alexander Ravsky, Alexander Wolff



For a given graph, we want to find crossing-free straight-line drawings of low visual complexity. A measure for the visual complexity of a drawing that has been considered before is the minimum number of lines needed to cover all vertices. In 3D, this number, the *3D weak line cover number*, is denoted by $\pi_3^1(G)$ for a given graph G. In 2D, for any planar graph G, the *2D weak line cover number* is denoted by $\pi_2^1(G)$.

We inductively construct an infinite family of polyhedral graphs with maximum degree 6, treewidth 3, and unbounded π_2^1-value. We also determine the π_2^1- and π_3^1-values of the Platonic graphs.

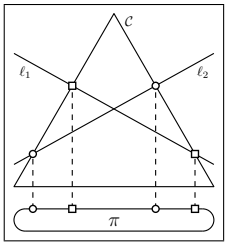
The figures here depict π_2^1-optimal drawings of (the 1-skeletons of) the dodecahedron (left) and the icosahedron (right). The two graphs are drawn such that their vertices lie on two and three lines, respectively.

Session 4B, Wed 15:20–16:50, room 005

Wed 15:20–15:35
4B, room 005, #60

Subquadratic Encodings for Point Configurations

Jean Cardinal, Timothy M. Chan, John Iacono, Stefan Langerman, Aurélien Ooms

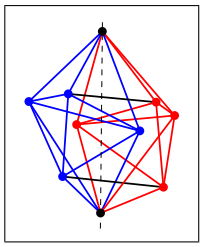


We give a compact encoding for abstract order types that allows efficient query of the orientation of any triple: the encoding uses $O(n^2)$ bits and an orientation query takes $O(\log n)$ time in the word-RAM model. We shorten the encoding to $O(n^2(\log\log n)^2/\log n)$ bits for realizable order types, giving the first subquadratic encoding for those order types with fast orientation queries.

Wed 15:37–15:52
4B, room 005, #19

Almost-Equidistant Sets

Martin Balko, Attila Pór, Manfred Scheucher, Konrad Swanepoel, Pavel Valtr



A set of points in $\mathbb{R}^d$ is called *almost-equidistant* if for any three points from the set, some two are at unit distance. Let $f(d)$ denote the largest size of an almost-equidistant set in $\mathbb{R}^d$.

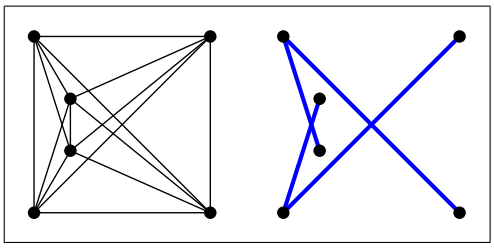
It is known that $f(2) = 7$, $f(3) = 10$, and that the extremal sets are unique.

We investigate almost-equidistant sets in higher dimensions. For every dimension $d \geq 3$, we have an example of an almost-equidistant set of $2d + 4$ points in the d-space and we prove the asymptotic upper bound $f(d) \leq O(d^{3/2})$.

Wed 15:54–16:09
4B, room 005, #21

Minimal Geometric Graph Representations of Order Types

Oswin Aichholzer, Martin Balko, Michael Hoffmann, Jan Kynčl, Wolfgang Mulzer, Irene Parada, Alexander Pilz, Manfred Scheucher, Pavel Valtr, Birgit Vogtenhuber, Emo Welzl



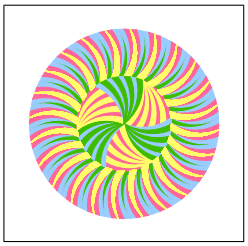
We consider the problem of characterizing small geometric graphs whose structure uniquely determines the order type of its vertex set. We describe a set of edges that prevent the order type from changing by continuous movement and identify properties of the resulting graphs.

Wed 16:11–16:26
4B, room 005, #31

A Note on Planar Monohedral Tilings

Oswin Aichholzer, Michael Kerber, István Talata, and Birgit Vogtenhuber

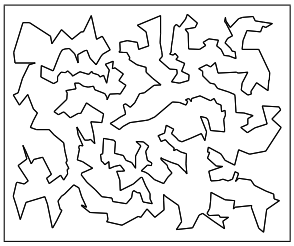
In this talk we present *planar monohedral tilings*, that is, tilings of $\mathbb{R}^2$ with congruent *tiles*. Such a tiling has the *flag property* if every triple of pairwise intersecting tiles intersects in a common point. We show that for convex tiles, there exist only three classes of tilings that are not flag, and they all consist of triangular tiles. Hence any tiling using convex tiles with $n \geq 4$ vertices is flag. We also show that an analogous statement for the case of non-convex tiles is not true by presenting a family of counterexamples.



Wed 16:28–16:43
4B, room 005, #23

Computing Crossing-Free Configurations with Minimum Bottleneck

Sándor P. Fekete, Phillip Keldenich



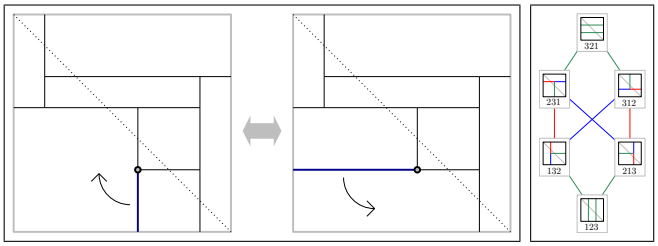
We consider problems of finding non-crossing *bottleneck* structures for a given planar point set: For a given a set of vertices V, the problem Minimum Bottleneck Polygon (MBP) is to find a simple polygon P with vertex set V whose longest edge is as short as possible; the problem Minimum Bottleneck Simple Matching (MBSM) is to find a crossing-free matching of V whose longest edge is as short as possible. Both problems are known to be NP-complete and neither admits a PTAS. We develop exact methods that can solve benchmark instances (newly generated and from the classic TSPLIB library) with up to 1,500 points for MBP and up to 20,000 points for MBSM to provable optimality. In the talk, we will highlight challenges that arise when solving these instances, including the question of how to deal with $\Omega(n^4)$ potential crossings.

Wed 16:45–17:00
4B, room 005, #46

A Note on Flips in Diagonal Rectangulations

Jean Cardinal, Vera Sacristán, Rodrigo I. Silveira

Rectangulations are partitions of a square into axis-aligned rectangles. A number of results deal with local changes involving a single edge of a rectangulation, referred to as flips, edge rotations, or edge pivoting. Such operations induce a *flip graph* on equivalence classes of rectangulations. We consider a family of flip operations on the equivalence classes of *diagonal rectangulations*, and their interpretation as transpositions in the associated *Baxter permutations*, avoiding the vincular patterns $\{3\underline{1}42, 2\underline{4}13\}$.



Notes:

Session 5. Invited Talk, Thu 9:00–10:00, main hall

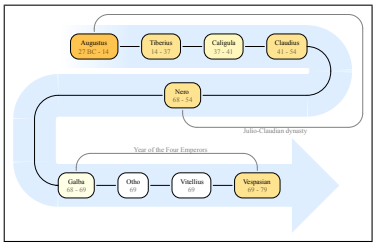
Thu 9:00–10:00 **5**, main hall, #B	**Online Competitive Routing on Delaunay Triangulations and their Variants** *Jit Bose*, Carleton University, Ottawa

A fundamental problem in computer science is that of finding a path in a graph. When the whole graph is available, standard path-finding algorithms can be applied such as Depth-First Search or Dijkstra's Algorithm. However, the problem of finding a path is more challenging in an online setting when at every step of the computation, only local information is available to the routing algorithm (such as the neighbourhood of the current vertex in the path). The difficulty is in deciding which edge to follow next in a path with only this local information. It is even more challenging to find a path with constant spanning ratio.

We will highlight different techniques for finding a short path in various types of Delaunay graphs in the online setting. We will highlight some of the difficulties involved with routing, review some of the currently best-known routing algorithms and mention a few open problems.

Session 6A, Thu 10:50–12:20, main hall

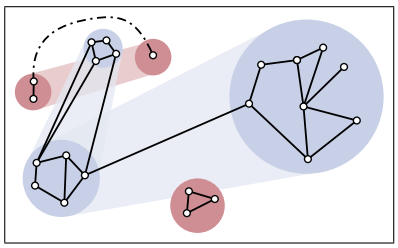
Thu 10:50–11:05 **6A**, main hall, #10	**Optimal Algorithms for Compact Linear Layouts** *Wouter Meulemans, Willem Sonke, Bettina Speckmann, Eric Verbeek, Kevin Verbeek*



Linear layouts are a simple and natural way to draw a graph: all vertices are placed on a single line and edges are drawn as arcs between the vertices. Despite its simplicity, a linear layout can be a very meaningful visualization if there is a particular order defined on the vertices. Common examples of such ordered—and often also directed—graphs are event sequences and processes: public transport systems tracking passenger check-in and check-out, banks checking online transactions, or hospitals recording the paths of patients through their system, to name a few. A main drawback of linear layouts are the usually (very) large aspect ratios of the resulting drawings, which prevent users from obtaining a good overview of the whole graph.

In this talk we present a novel and versatile algorithm to optimally fold a linear layout of a graph such that it can be drawn effectively in a specified aspect ratio, while still clearly communicating the linearity of the layout.

Thu 11:07–11:22 **6A**, main hall, #14	**Drawing Connected Planar Clustered Graphs on Disk Arrangements** *Tamara Mchedlidze, Marcel Radermacher, Ignaz Rutter and Nina Zimbel*



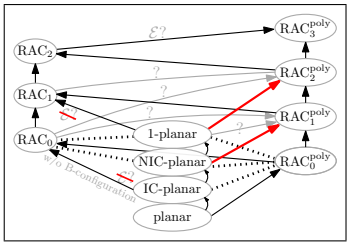
Let $G = (V, E)$ be a planar graph and let $\mathcal{V}$ be a partition of V whose *clusters*, i.e., the graphs induced by the vertex sets in $\mathcal{C}$, are connected. Let $\mathcal{D}_G$ be an arrangement of disks with a bijection between the disks and the clusters. Akitaya et al. give an algorithm to test whether $(G, \mathcal{C})$ can be embedded onto $\mathcal{D}_G$ with the additional constraint that edges are routed through an additional set of pipes between the disks. Based on such an embedding, we prove that every clustered graph with connected clusters and every disk-arrangement with non-overlapping disks has a planar straight-line drawing where every vertex is embedded in the disk corresponding to its cluster. This result can be seen as an extension of the result by Alam et al. who solely consider biconnected clusters.

Notes:

Thu 11:24–11:39 **6A**, main hall, #27	**Augmenting a Tree to a k-arbor-connected Graph with Pagenumber k** *Toru Hasunuma*

We consider an augmentation problem for a tree to increase fault-tolerance while preserving its good property on book-embeddings. A k-arbor-connected graph is a graph which has k spanning trees such that for any two vertices, the k paths between them in the k spanning trees are pairwise edge-disjoint and internally vertex-disjoint. We show that any tree with n vertices can be augmented in $O(nk)$ time to a minimum k-arbor-connected graph with pagenumber k for any k at most the radius of the tree. Besides, we extend our minimum augmentation for trees to cacti and unicyclic graphs.

Thu 11:41–11:56 **6A**, main hall, #28	**1-Bend RAC Drawings of NIC-Planar Graphs in Quadratic Area** *Steven Chaplick, Fabian Lipp, Alexander Wolff, Johannes Zink*

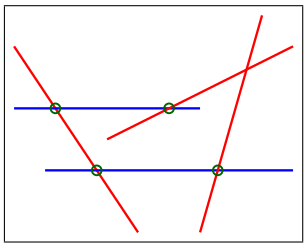


A drawing of a graph is called *1-planar* if every edge is crossed at most once. A 1-planar drawing is called *independent-crossing planar* (*IC-planar*) if no two pairs of crossing edges share a vertex. A 1-planar drawing is called *near-independent-crossing planar* (*NIC-planar*) if any two pairs of crossing edges share at most one vertex. The 1-planar, NIC-planar, and IC-planar graphs are the graphs that admit a 1-planar, NIC-planar, and IC-planar drawing, respectively. The NIC-planar graphs are a subset of the 1-planar graphs and a superset of the IC-planar graphs, which are important *beyond-planar* graph classes.

We constructively show that every n-vertex NIC-plane graph admits a NIC-planar drawing with only right-angle crossings (RAC) and at most one bend per edge on a grid of size $O(n) \times O(n)$. Given a NIC-plane graph, our construction takes linear time.

We also give an overview of the relationships between several classes of 1-planar graphs and k-bend RAC graphs (see the containment diagram above; our results are highlighted in red).

Thu 11:58–12:13 **6A**, main hall, #32	**NP-Completeness of Max-Cut for Segment Intersection Graphs** *Oswin Aichholzer, Wolfgang Mulzer, Patrick Schnider, Birgit Vogtenhuber*



We consider the problem of finding a *maximum cut* in a graph $G = (V, E)$, that is, a partition $V_1 \dot\cup V_2$ of V such that the number of edges between V_1 and V_2 is maximum. It is well known that the decision problem whether G has a cut of at least a given size is in general NP-complete. We show that this problem remains hard when restricting the input to *segment intersection graphs*. These are graphs whose vertices can be drawn as straight-line segments, where two vertices share an edge if and only if the corresponding segments intersect.

We obtain our result by a reduction from a variant of Planar Max-2-SAT that we introduce and also show to be NP-complete.

Session **6B**, Thu 10:50–12:20, room 005

Thu 10:50–11:05 **6B**, room 005, #9	**A New Lower Bound on the Maximum Number of Plane Graphs using Production Matrices** *Clemens Huemer, Alexander Pilz, Rodrigo I. Silveira*

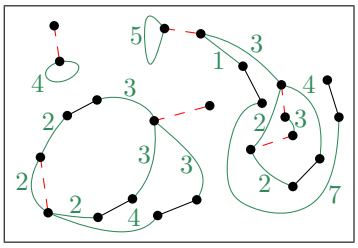
We use the concept of production matrices to show that there exist sets of n points in the plane that admit $\Omega(41.77^n)$ crossing-free geometric graphs. This improves the previously best known bound of $\Omega(41.18^n)$ by Aichholzer et al. (2007).

Notes:

Thu 11:07–11:22
6B, room 005, #51

Time-Space Trade-Offs for Euclidean Minimum Spanning Trees

Bahareh Banyassady, Luis Barba, Wolfgang Mulzer



First, I introduce the *Limited-Workspace Model* and I define the *Euclidean minimum spanning tree* of a set of points, called EMST. Then, I present an algorithm which provides a time-space trade-off to compute the EMST of a planar set of n points in the limited-workspace model. The algorithm uses $O(s)$ words of workspace and runs in $O(n^3 \log s/s^2)$ time. In the algorithm, I refer to the *relative neighborhood graph*, called RNG, and Kruskal's MST algorithm on RNG. The exciting part is defining a compact representation of planar graphs, called an *s-net*, which allows us to manipulate RNG's component structure incrementally during the execution of the algorithm and maintain its compact representation using O(s) words of workspace. The s-net structure could possibly be used by other algorithms for planar graphs in the limited-workspace model.

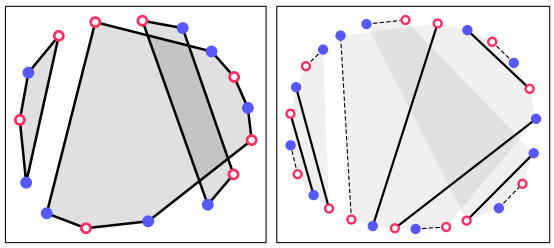
Thu 11:24–11:39
6B, room 005, #70

Bottleneck Bichromatic Non-crossing Matchings using Orbits

Marko Savić, Miloš Stojaković

Problem: BBNCM

Given a set of n red and n blue points in convex position in the plane, we are interested in matching red points with blue points by straight line segments so that the segments do not cross. We want to find such a matching that minimizes the length of the longest segment.



Tools developed: Orbits

Orbits form a partition of the point set such that two differently colored points can be matched iff they belong to the same orbit. Lots of nice properties! (E.g. orbits make a total order.)

Results: Speedy algorithms

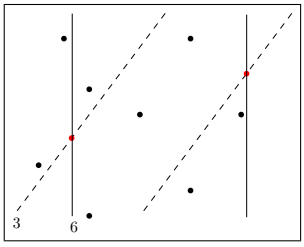
$O(n^2)$-time algorithm for points in convex position (previously best-know: $O(n^3)$).

$O(n)$-time algorithm for points on a circle (previously best-known: $O(n \log n)$).

Thu 11:41–11:56
6B, room 005, #5

A Combinatorial Measure of Closeness in Point Sets

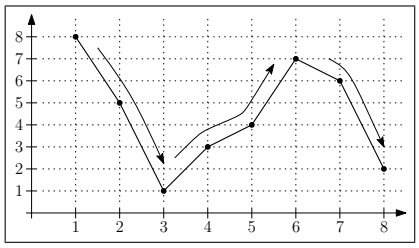
Alexander Pilz, Patrick Schnider



We introduce *stripe closeness* and *stripe remoteness*, two combinatorial measures that capture how close together or far apart a set of query points lies within another set of points. The idea behind these concepts is that we look at all possible projections of the point set to a line and count the number of points that lie between the query points. For two points in a point set, the notion of stripe closeness can be seen as a combinatorial distance measure. We give bounds on the stripe closeness of two closest points. Further, we analyze stripe remoteness for triples in point sets and show that there are always three points that have high stripe remoteness.

Notes:

Thu 11:58–12:13 **6B**, room 005, #40	**Rollercoasters: Long Sequences without Short Runs** *Therese Biedl, Ahmad Biniaz, Robert Cummings, Anna Lubiw, Florin Manea, Dirk Nowotka, Jeffrey Shallit*



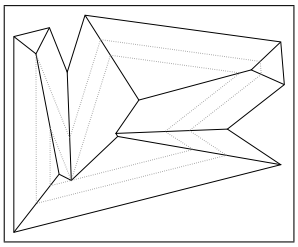
A *rollercoaster* is a sequence of real numbers for which every maximal contiguous subsequence, that is increasing or decreasing, has length at least three. By translating this sequence to a set of points in the plane, a rollercoaster can be defined as a polygonal path for which every maximal sub-path, with positive- or negative-slope edges, has at least three points. Given a sequence of distinct real numbers, the rollercoaster problem asks for a maximum-length (not necessarily contiguous) subsequence that is a rollercoaster. It was conjectured that every sequence of n distinct real numbers contains a rollercoaster of length at least $\lceil n/2 \rceil$ for $n > 7$, while the best known lower bound is $\Omega(n/\log n)$. In this paper we prove this conjecture. Our proof is constructive and implies a linear-time algorithm for computing a rollercoaster of this length. We also show how to compute a maximum-length rollercoaster in an arbitrary array with n elements in $O(n \log n)$-time; a maximum-length rollercoaster in a permutation of $\{1, \ldots, n\}$ can be computed in $O(n \log \log n)$ time.

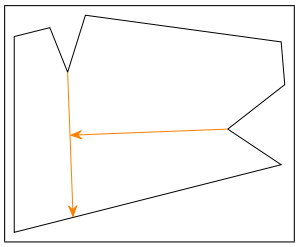
Session 7A, Thu 13:20–14:50, main hall

Thu 13:20–13:35 **7A**, main hall, #17	**The Topology of Skeletons and Offsets** *Stefan Huber*

Two skeleton structures (Voronoi diagram, straight skeleton) of polygons with holes and their topology are considered. We start with their homotopy equivalence to the underlying shape. Then we continue to the evolution of offset curves each skeletons induces and investigate the (persistent) topology of these processes. The duality of skeletons and the evolution of offsets leads to a simple and efficient persistence algorithm.

Thu 13:37–13:52 **7A**, main hall, #42	**On Merging Straight Skeletons** *Franz Aurenhammer, Michael Steinkogler*

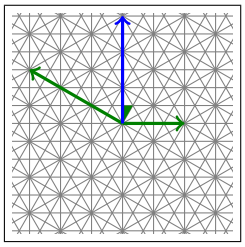




Since the introduction of the straight skeleton of simple polygons about two decades ago, the search for efficient algorithms to compute the straight skeleton has produced a variety of algorithms.

We present a new approach that applies the divide-and-conquer paradigm based on the motorcycle graph (a structure that encodes information about the behaviour of the polygon's reflex vertices during the shrinking process that defines the straight skeleton). The resulting algorithm uses simple building blocks and has a running time of $\mathcal{O}(d\, n \log n)$, where d is the decomposition depth of the motorcycle graph.

Thu 13:54–14:09 **7A**, main hall, #4	**Coxeter Triangulations have Good Quality** *Aruni Choudhary, Siargey Kachanovich, Mathijs Wintraecken*



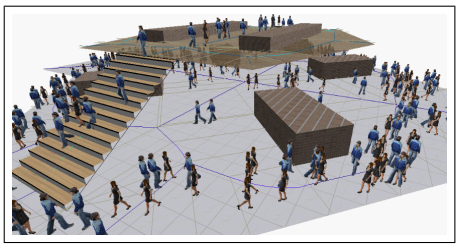
Coxeter triangulations are triangulations of Euclidean space based on a single simplex. By this we mean that given an individual simplex we can recover the entire triangulation of Euclidean space $\mathbb{R}^d$ by inductively reflecting through the faces of the simplex. In this paper we establish that the quality of the simplices in all Coxeter triangulations is $O(1/\sqrt{d})$ of the quality of regular simplex. We further investigate the Delaunay property for these triangulations. Moreover, we consider an extension of the Delaunay property, namely protection, which is a measure of non-degeneracy of a Delaunay triangulation. In particular, one family of Coxeter triangulations achieves the protection $O(1/d^2)$. We conjecture that both bounds are optimal for triangulations in Euclidean space.

Thu 14:11–14:26 **7A**, main hall, #36	**Integer and Mixed Integer Tverberg Numbers** *Jesús A. De Loera, Thomas Hogan, Frédéric Meunier, Nabil Mustafa*

Tverberg's theorem says that sufficiently many points in Euclidean space can always be partitioned with intersecting convex hulls. We consider a variation where the coordinates of the points are required to be integer. We will show that for integer m at least three, any $4m-3$ lattice points in the plane can be partitioned into m subsets so that the convex hulls of each subset intersect at a lattice point.

We will also mention some results for this problem in higher dimensions, as well as a "mixed-integer" version.

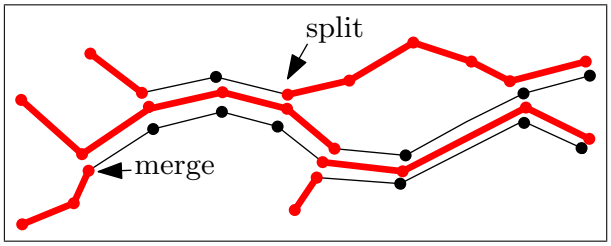
Thu 14:28–14:43 **7A**, main hall, #66	**On the Topology of Walkable Environments** *Benjamin Burton, Arne Hillebrand, Maarten Löffler, Saul Schleimer, Dylan Thurston, Stephan Tillmann, Wouter van Toll*



Motivated by motion planning applications, we study 2-dimensional surfaces embedded in 3-dimensional space with the property that their vertical projection is an immersion. We provide bounds on the complexity of a triangulation of such a surface, given that the projection of the boundary is a polygon with m segments. We then show how these bounds lead to efficient algorithm to compute such a triangulation. Finally, we relate our result to concrete motion planning setting and review related open questions.

Session **7B**, Thu 13:20–14:50, room 005

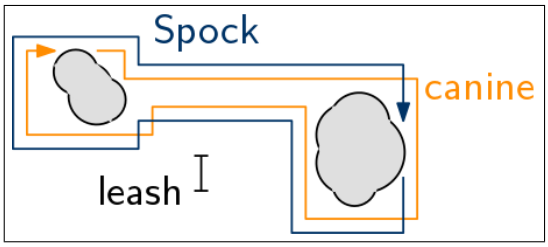
Thu 13:20–13:35 **7B**, room 005, #7	**Group Diagrams for Representing Trajectories** *Maike Buchin, Bernhard Kilgus*



The amount of available movement data such as GPS data collected by mobile devices has increased massively during the last years. One of the challenges associated with the large amount of available data is to compactly represent it.

We propose the group diagram as a representation for multiple trajectories representing one or several moving groups. Given a distance threshold, a similarity measure and a minimality criterion a minimal group diagram is a minimal representation of the groups maintaining the spatio-temporal structure of the groups' movement. We give hardness results and approximation algorithms for computing several variants of the group diagram with Fréchet distance and equal-time distance as similarity measure.

Thu 13:37–13:52 **7B**, room 005, #43	**The k-Fréchet Distance of Polygonal Curves** *Maike Buchin, Leonie Ryvkin*



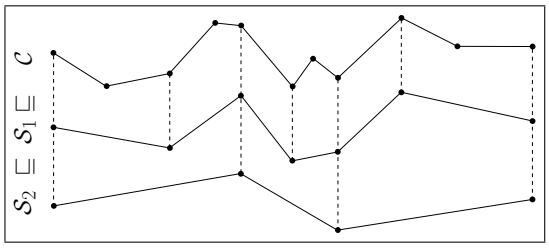
We introduce a new distance measure for comparing polygonal chains: the k-Fréchet distance. It is closely related to the Fréchet distance but allows to compare polygonal chains piecewise, i.e., we cut the input curves a number of times and match the resulting subcurves using the weak Fréchet distance.

To give an intuition we use the well-known man and dog analogy: this time Mr. Spock has to walk the Alfa 177 canine. Their leash is too short, but asking Scotty to beam either him or the canine a number of times will enable them to traverse their respective paths. Unfortunately, Scotty's task is hard: Reducing from Minimum Common String Partition we prove NP-Completeness of deciding the k-Fréchet-distance as well as APX-Completeness for the optimization variant. On the positive side, we present algorithmic approaches, in particular a 2-approximation algorithm.

Thu 13:54–14:09
7B, room 005, #13

Progressive Simplification of Polygonal Curves

Kevin Buchin, Maximilian Konzack, Wim Reddingius

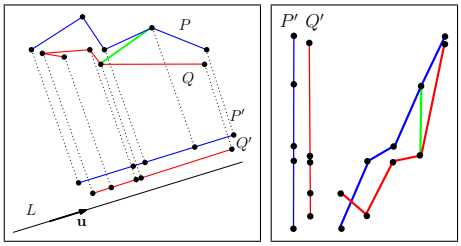


Simplifying polygonal curves at different levels of detail is an important problem with many applications. Existing geometric optimization algorithms are only capable of minimizing the complexity of a simplified curve for a single level of detail. We present an $O(n^3m)$-time algorithm that takes a polygonal curve of n vertices and produces a set of consistent simplifications for m scales while minimizing the cumulative simplification complexity. This algorithm is compatible with distance measures such as Hausdorff, Fréchet and area-based distances, and enables simplification for continuous scaling in $O(n^5)$ time.

Thu 14:11–14:26
7B, room 005, #26

Probabilistic Embeddings of the Fréchet Distance

Anne Driemel, Amer Krivošija

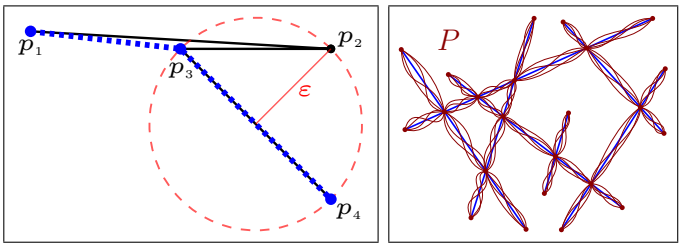


The Fréchet distance is a popular distance measure for curves which naturally lends itself to fundamental computational tasks, such as clustering, nearest-neighbor searching, and spherical range searching in the corresponding metric space. However, its inherent complexity poses considerable computational challenges in practice. To address this problem we study distortion of the probabilistic embedding that results from projecting the curves to a randomly chosen line. Such an embedding could be used in combination with, e.g. locality-sensitive hashing. We show that in the worst case and under reasonable assumptions, the discrete Fréchet distance between two polygonal curves in $\Re^2$ or $\Re^3$ of complexity t degrades by a factor linear in t with constant probability. We show upper and lower bounds on the distortion.

Thu 14:28–14:43
7B, room 005, #25

On Optimal Polyline Simplification using the Hausdorff and Fréchet Distance

Marc van Kreveld, Maarten Löffler, Lionov Wiratma



We consider the polyline simplification problem using the pure form of the Hausdorff and Fréchet distance: for a given $\varepsilon > 0$, choose a minimum size subsequence of the vertices of the input such that the Hausdorff or Fréchet distance between the input and output polylines is at most ε.

We present examples of input polylines such that their output simplifications from the well-known Douglas-Peucker and Imai-Iri simplification algorithms are not optimal. Even if both algorithms use a considerably larger ε, they may still need many more vertices than an optimal simplification.

We prove that computing an optimal simplification under the undirected Hausdorff distance is NP-hard. Then, we give a polynomial time algorithm to compute an optimal simplification under the Fréchet distance.

Nachweis Bernd Schönberger

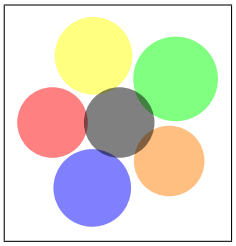
Session 8. Invited Talk, Fri 9:00–10:00, main hall

Fri 9:00–10:00 **8**, main hall, #C	**Geometric Issues in Self-Driving Cars** *Raúl Rojas*, Freie Universität Berlin

I will give an overview of a new iteration of the architecture of the autonomous cars which have been developed at the Dahlem Center for Machine Learning and Robotics, Freie Universität Berlin. I will explain how we mix reactive with deliberative control. I will explain how we have experimented with geometry-based localization and the ideas we have for localization and driving under tough weather conditions. In one project we are investigating swarm behavior in traffic. At the end, I will present some ideas about the evolution of the commercial introduction of autonomous vehicles in the near future.

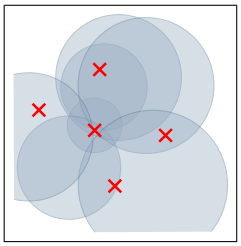
Session 9A, Fri 10:50–12:20, main hall

Fri 10:50–11:05 **9A**, main hall, #57	**$L(2,1)$-labeling of disk intersection graphs** *Konstanty Junosza-Szaniawski, Joanna Sokół*



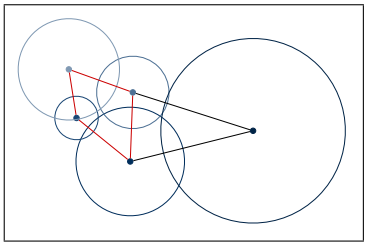
We consider the problem of $L(2,1)$-labeling of intersection graphs of disks. An $L(2,1)$-labeling is a mapping $f\colon V(G) \to \{0,1,2,\ldots\}$ in which labels assigned to adjacent vertices differ by at least 2, and labels assigned to vertices at distance 2 are different. The span of f is the difference between the maximum and the minimum label used, and the span $\lambda(G)$ of a graph G is the minimum span of an $L(2,1)$-labeling of G. We show that if G is an intersection graph of disks, then $\lambda(G) \leq \frac{4}{5}\Delta(G)^2 + 25\Delta(G) + 22$, where $\Delta(G)$ denotes the maximum degree in graph G. Notice that the bound does not depend on the sizes of the disks.

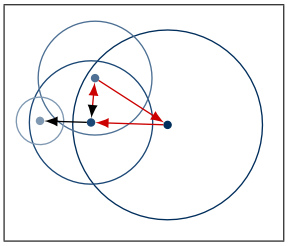
Fri 11:07–11:22 **9A**, main hall, #29	**Stabbing Pairwise Intersecting Disks by Five Points** *Sariel Har-Peled, Haim Kaplan, Wolfgang Mulzer, Liam Roditty, Paul Seiferth, Micha Sharir, Max Willert*



Once upon a time (in 1959) in a place far far away (Oberwolfach) there was a man who proved that every set of n *pairwise intersecting disks* can be stabbed by at most 4 points, i.e., each disk contains at least one of the 4 *stabbing points*. The name of this man was ...
His proof is fairly involve**d**, and there seems to be no obvious w**a**y to turn it into an efficient algorithm. We prese**n**t fast algorithms to find a stabbing point set of si**z**e 5. Moreover, we give a simple construction with 13 pairwise inters**e**cting disks that cannot be stabbed by th**r**ee points.

Fri 11:24–11:39 **9A**, main hall, #68	**Finding the Girth in Disk Graphs and a Directed Triangle in Transmission Graphs** *Haim Kaplan, Katharina Klost, Wolfgang Mulzer, Liam Roditty*

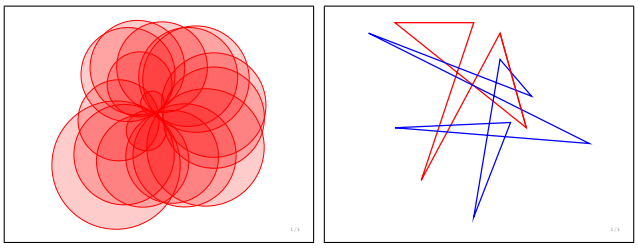




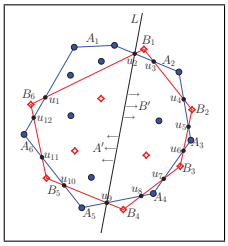
In this talk we consider the complexity of problems related to cycles in disk intersection and transmission graphs. Given a set $S \subset \mathbb{R}^2$ of n sites, each with an associated radius. The disk intersection graph of S has a vertex for each site and an edge between sites s, t with $|st| \leq r_s + r_t$. The transmission graph of S has a directed edge $s \to t$, if $|st| \leq r_s$.
We show that the *weighted girth* of a disk graph can be computed in $O(n \log n)$ expected time. Additionally we show that in $O(n \log^2 n)$ time, we can decide whether a transmission graph given by the sites contains a *directed triangle*.

Fri 11:41–11:56 **9A**, main hall, #61	**QPTAS and Sub-exponential Algorithm for Maximum Clique on Disk Graphs** *Édouard Bonnet, Panos Giannopoulos, Eun Jung Kim, Paweł Rzążewski, Florian Sikora*

A disk graph is the intersection graph of closed disks in the plane. We show the structural result that a disjoint union of cycles is the complement of a disk graph if and only if at most one of those cycles is of odd length. From that, we derive the first QPTAS and sub-exponential algorithm running in time $2^{\tilde{O}(n^{2/3})}$ for Maximum Clique on disk graphs. In contrast, the problem is unlikely to have such algorithms on intersection graphs of filled ellipses or filled triangles.



Fri 11:58–12:13 **9A**, main hall, #3	**Geometric Clustering in Normed Planes** *Pedro Martín, Diego Yáñez*



Given two finite sets of points A and B in a normed plane, we prove that there are two linearly separable sets A' and B' such that $\mathrm{diam}(A') \leq \mathrm{diam}(A)$, $\mathrm{diam}(B') \leq \mathrm{diam}(B)$, and $A' \cup B' = A \cup B$. This extends a result for the Euclidean distance to symmetric convex distance functions. As a consequence, some Euclidean k-clustering algorithms are adapted to normed planes, for instance, those that minimize the maximum, the sum, or the sum of squares of the k cluster diameters. The result is useful in order to solve the 2-clustering problem when two different bounds are imposed to the diameters. The well-known Euclidean approach to this problem does not work for every normed plane. We point out that the Hershberger-Suri's data structure for managing ball hulls can be used in this context.

Session 9B, Fri 10:50–12:20, room 005

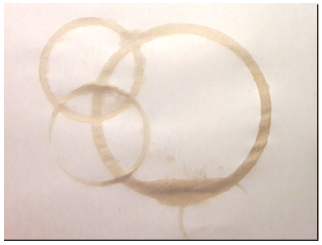
Fri 10:50–11:05 **9B**, room 005, #33	**Non-Monochromatic and Conflict-Free Colorings in Tree Spaces** *Boris Aronov, Mark de Berg, Aleksandar Markovic, Gerhard Woeginger*

A non-monochromatic coloring (NM-coloring) of a set S of intervals in $\mathbb{R}^1$ is a coloring such that for any point $p \in \mathbb{R}^1$, if the set S_p of intervals containing p contains at least two intervals, then it contains two intervals of different color. A conflict-free coloring (CF-coloring) of S is defined similarly, except that S_p should now contain an interval with a unique color.

It is well known that any set of n intervals in $\mathbb{R}^1$ admits a NM-coloring with two colors and a CF-coloring with three colors. We investigate generalizations of this result to colorings of objects in more complex 1-dimensional spaces, namely so-called tree spaces.

Fri 11:07–11:22 **9B**, room 005, #15	**Arrangements of Pseudocircles: On Circularizability** *Stefan Felsner, Manfred Scheucher*

An arrangement of pseudocircles is a collection of simple closed curves on the sphere or in the plane such that every pair is either disjoint or intersects in exactly two crossing points. An arrangement is circularizable if there is a combinatorially equivalent arrangement of circles.



Kang and Müller showed that every arrangement of at most 4 pseudocircles is circularizable. Linhart and Ortner found an arrangement of 5 pseudocircles which is not circularizable.

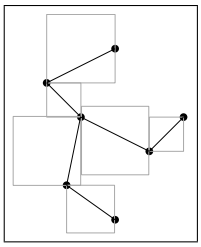
We show that there are exactly four non-circularizable arrangements of 5 pseudocircles and exactly three non-circularizable digon-free arrangements of 6 pairwise intersecting pseudocircles.

Notes:

Fri 11:24–11:39
9B, room 005, #49

Sequences of Spanning Trees for L_∞-Delaunay Triangulations

Prosenjit Bose, Pilar Cano, Rodrigo I. Silveira



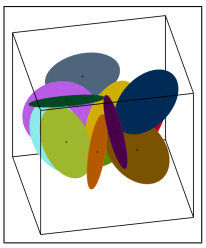
In the L_∞-Delaunay triangulation of a point set S in the plane, two points are adjacent if there exists an empty axis-aligned square that contains them on its boundary. Now, consider an arbitrary crossing-free spanning tree T_0 of S. Using a *constrained* L_∞-Delaunay triangulation, we define a sequence of crossing-free spanning trees $T_0, T_i, \ldots$ of S such that the tree T_{i+1} is a minimum spanning tree in the *constrained* L_∞-Delaunay triangulation of T_i, for $i \geq 0$.

We show that this sequence converges to a minimum spanning tree of S in the L_∞ metric. This extends a result for the L_2-Delaunay Triangulation to the L_∞ metric using a different approach.

Fri 11:41–11:56
9B, room 005, #47

3D-Disk-Packing

Helmut Alt, Otfried Cheong, Ji-won Park, Nadja Scharf



In our talk, we consider the problem of finding a minimum volume axis-parallel cuboid container into which a given set of three dimensional unit size disks can be packed under translations. The problem is neither known to be NP-hard nor to be in NP. The audience will learn about an approximation algorithm and the idea why this algorithm computes a constant factor approximation. On the way, one will see how this problem is related to finding the shortest Hamiltonian path in a graph and as a byproduct why it is impossible to pack all unit disks simultaneously into a finite size convex container under translation. This is exciting since in two dimensions, it is possible to pack all line segments into a finite size convex container, i.e. a circle. To the best of the authors' knowledge, the given approximation algorithm is the first algorithm with proven performance bound to pack three-dimensional objects under translations other than axis-parallel boxes.

Fri 11:58–12:13
9B, room 005, #69

Lower Bounds for Coloring of the Plane

Konstanty Junosza-Szaniawski, Krzysztof Węsek

One of the most challenging questions in discrete geometry is the Hadwiger-Nelson problem: how many colors are needed to color the Euclidean plane so that points at distance 1 receive distinct colors? In other words, what is the chromatic number of the unit distance graph of the plane? The question is widely open for over 60 years as the answer is only known to be between 4 and 7. However, it appears that if we consider a modified problem with a wider distance condition, it is possible to make further progress or even give the exact answer.

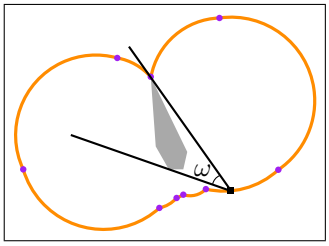
For $b > 1$, let $G_{[1,b]}$ be the graph with the set of vertices $\mathbb{R}^2$ and adjacency between points at distance in the interval $[1, b]$. We obtain new lower bounds on $\chi(G_{[1,b]})$ for certain values of b. Combined with known upper bounds, this result gives two intervals of values of b for which we exactly determine $\chi(G_{[1,b]})$ to be 7 and 9, respectively. The first interval contains and substantially enlarges the only known set of values of b with determined $\chi(G_{[1,b]})$, coming from the work of Exoo (2004).

Session 10A, Fri 13:20–14:50, main hall

Fri 13:20–13:35
10A, main hall, #44

Reconstructing a Convex Polygon from its ω-Cloud

Prosenjit Bose, Jean-Lou De Carufel, Elena Khramtcova, Sander Verdonschot



Consider probing a convex polygon P with rotating calipers, and try to reconstruct P from the resulting diameter function. You will not always be able to do so!

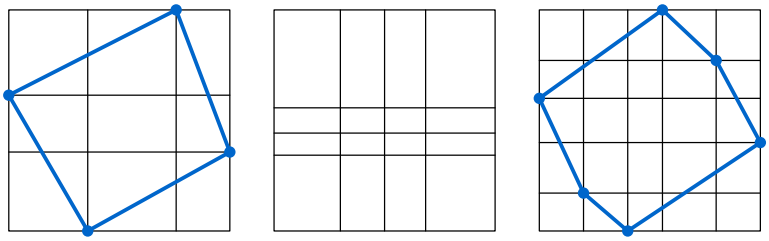
Imagine now using another probing tool, an *ω-wedge*, that consists of two rays emanating from a single point (the apex) and separated by an angle $\omega < \pi$. Instead of the diameter function you will get the *ω-cloud* of P, which is the curve traced by the apex of the ω-wedge as it rotates around P while maintaining tangency to P in both rays.

We investigate reconstructing a polygon P from its ω-cloud. We show that if ω is known, the ω-cloud alone uniquely determines P, and we give a linear-time reconstruction algorithm. Furthermore, even if we only know that $\omega < \pi/2$, we can still reconstruct P, albeit in cubic time in the number of vertices. This reduces to quadratic time if in addition we are given the location of one of the vertices of P.

Fri 13:37–13:52
10A, main hall, #39

On Convex Polygons in Cartesian Products

Jean-Lou De Carufel, Adrian Dumitrescu, Wouter Meulemans, Tim Ophelders, Claire Pennarun, Csaba D. Tóth, Sander Verdonschot

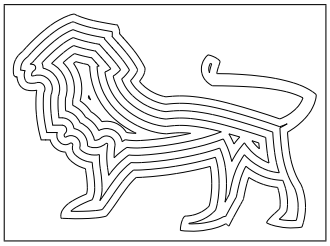


We study several problems concerning convex polygons whose vertices lie in a Cartesian product (for short, *grid*) of two sets of n real numbers. First, we prove that all such grids contain a convex polygon with $\Omega(\log n)$ vertices and that this bound is tight up to constant factors. We also present polynomial-time algorithms for computing the largest convex chain and cap that contains no two points of the same x- or y-coordinate. These algorithms give a constant approximation ratio for a largest convex polygon under this restriction. Try to find such a largest polygon for the middle grid.

Fri 13:54–14:09
10A, main hall, #52

Mitered Offsets and Skeleton for Circular Arc Polygons

Bastian Weiß, Bert Jüttler, Franz Aurenhammer



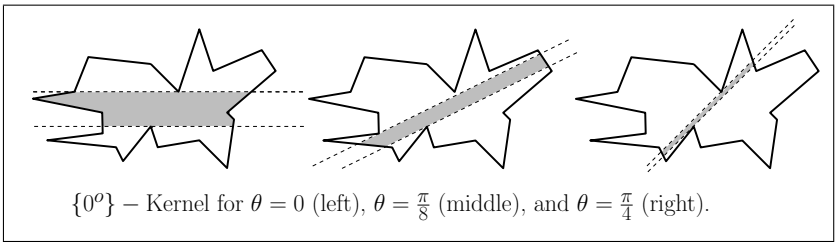
We generalize the offsetting process that defines straight skeletons of polygons to circular arc polygons. The offsets and the associated skeleton are obtained by applying an evolution process to the boundary and tracing the paths of vertices. These paths define the associated patch decomposition. While the skeleton is a forest, the patches of the decomposition possess a radial monotonicity property. Analyzing the events that occur during the evolution process is non-trivial. This leads us to an event-driven algorithm for offset and skeleton computation. Several examples (both manually created ones and approximations of planar free-form shapes by arc polygons) are presented and used to analyze the performance of our algorithm.

Notes:

Fri 14:11–14:26
10A, main hall, #74

Generalized Kernels of Polygons under Rotation

David Orden, Leonidas Palios, Carlos Seara, Paweł Żyliński



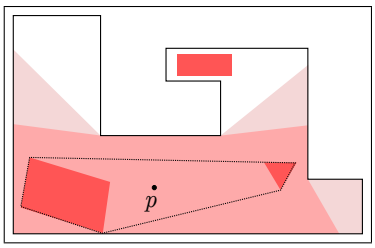
$\{0^o\}$ – Kernel for $\theta = 0$ (left), $\theta = \frac{\pi}{8}$ (middle), and $\theta = \frac{\pi}{4}$ (right).

Given a set $\mathcal{O}$ of k orientations in the plane, two points inside a simple polygon P *$\mathcal{O}$-see each other* if there is an $\mathcal{O}$-staircase contained in P that connects them. The $\mathcal{O}$-kernel of P is the subset of points which $\mathcal{O}$-see all the other points in P. This work initiates the study of the computation and maintenance of the $\mathcal{O}$-Kernel of a polygon P as we rotate the set $\mathcal{O}$ by an angle θ, denoted $\mathcal{O}$-Kernel$_\theta(P)$. In particular, we design efficient algorithms for (i) computing and maintaining $\{0^o\}$-Kernel$_\theta(P)$ while θ varies in $[-\frac{\pi}{2}, \frac{\pi}{2})$, obtaining the angular intervals where the $\{0^o\}$-Kernel$_\theta(P)$ is not empty and (ii) for orthogonal polygons P, computing the orientation $\theta \in [0, \frac{\pi}{2})$ such that the area and/or the perimeter of the $\{0^o, 90^o\}$-Kernel$_\theta(P)$ are maximum or minimum.

Fri 14:28–14:43
10A, main hall, #75

Generalized Visibility Kernel

Eyüp Serdar Ayaz, Alper Üngör



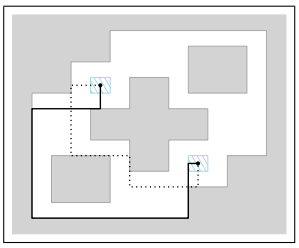
We propose a generalization of the concept visibility kernel which finds use in art gallery problems. For a point p in a simple polygon P and a subset X of P, we refer the set of points visible by every point in X that is seen by p as the generalized visibility kernel of p with respect to X. We present an $O(n + m \log m)$ time algorithm for computing the generalized visibility kernel, where n, m are the complexities of P and X respectively. As essential components of our approach, we also propose two efficient algorithms for computing the relative convex hull and the complete visibility polygon of a set of points. Our algorithm consists of four steps:

1. We use the linear-time algorithm by Joe and Simpson to compute the visibility polygon, $\mathcal{V}(p)$.
2. We present a angular plane sweep algorithm to find $\mathcal{V}(p) \cap X$.
3. We calculate the relative convex hull of $\mathcal{V}(p) \cap X$.
4. We compute the complete visibility polygon of $\mathcal{V}(p) \cap X$ relying on the relative convex hull computed in Step 3.

Fri 14:45–15:00
10A, main hall, #2

Rectilinear Link Diameter and Radius in a Rectilinear Polygonal Domain

Man-Kwun Chiu, Elena Khramtcova, Aleksandar Markovic, Yoshio Okamoto, Aurélien Ooms, André van Renssen, Marcel Roeloffzen



We study the computation of the diameter and radius under the rectilinear link distance within a rectilinear polygonal domain of n vertices and h holes. We use a *graph of oriented distances* to encode the distance between pairs of points of the domain. This helps us transform the problem so that we can search through the candidates more efficiently. Our algorithm computes both the diameter and the radius in $O(\min(n^\omega, n^2 + nh \log h + \chi^2))$ time, where $\omega < 2.373$ denotes the matrix multiplication exponent and $\chi \in \Omega(n) \cap O(n^2)$ is the number of edges of the graph of oriented distances. We also provide a faster algorithm for computing the diameter that runs in $O(n^2 \log n)$ time.

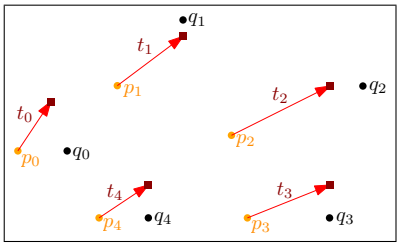
Notes:

Session 10B, Fri 13:20–14:50, room 005

Fri 13:20–13:35
10B, room 005, #6

An FPTAS for an Elastic Shape Matching Problem with Cyclic Neighborhoods

Christian Knauer, Luise Sommer, Fabian Stehn



The elastic geometric shape matching (EGSM) problem class is a generalisation of the well-known geometric shape matching problem class: Given two geometric shapes, the 'pattern' P and the 'model' Q, find a *single* transformation from a given transformation class that, if applied to the pattern, minimizes the distance between the transformed pattern and the model with respect to a suitable distance measure.

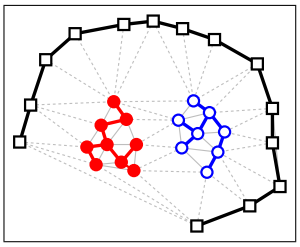
In EGSM, the pattern is divided into subshapes that are transformed by a 'transformation ensemble' T, i.e., a set of transformations. The goal is to minimize the distance between the union of the transformed subpatterns and the model in object space as well as the distance between specific transformations of the ensemble. The 'neighborhood graph' encodes which translations should be similar.

We present an FPTAS for EGSM instances for point sequences $P = (p_0, \ldots, p_{n-1})$ under translations $T = (t_0, \ldots, t_{n-1})$ with fixed correspondence where the neighborhood graph G is a simple cycle.

Fri 13:37–13:52
10B, room 005, #58

A Polynomial Algorithm for Balanced Clustering via Graph Partitioning

Luis Evaristo Caraballo, José-Miguel Díaz-Báñez, Nadine Kroher

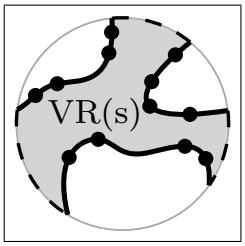


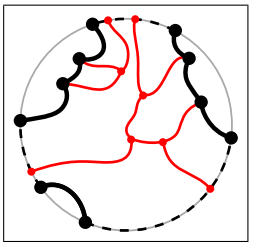
We consider a graph partitioning problem using a novel and natural quality measure for clusters: given a cluster C of a graph G (C is a connected component of G) the quality measure of C is the ratio between the heaviest outgoing edge from C and the minimum edge in the maximum spanning tree of C. And, we consider the measure of a clustering Π as the measure of the worst cluster in Π. In this talk we present some interesting properties of the optimal solution that allow us to build a quadratic algorithm on the number of nodes of G.

Fri 13:54–14:09
10B, room 005, #37

Deletion in Abstract Voronoi Diagrams in Expected Linear Time

Kolja Junginger, Evanthia Papadopoulou



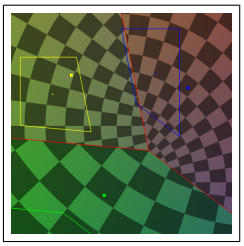


Updating an abstract Voronoi diagram in linear time, after deletion of one site, has been an open problem for a long time. Similarly for concrete Voronoi diagrams of generalized sites, other than points. In this abstract we present a simple, expected linear-time algorithm for this task. We introduce the concept of a *Voronoi-like diagram*, a relaxed version of a Voronoi construct, that has a structure similar to an abstract Voronoi diagram without however being one. Voronoi-like diagrams serve as intermediate structures, which are considerably simpler to compute, thus, making an expected linear-time construction possible.

Fri 14:11–14:26
10B, room 005, #34

Fair Voronoi Split-Screen for N-Player Games

Tobias Lenz

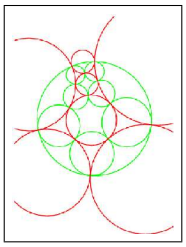


We introduce a special kind of subdivision called the Voronoi Split-Screen which evolves naturally from multiplayer games. At first glance the problem is closely related to Voronoi diagrams but much harder since all cells must have equal area. The talk is more lighthearted than the paper, omits most formulas, and illustrates different approaches with nice animations. Many possible and impossible variations and open problems might give you something to chew on during lunch.

Fri 14:28–14:43 **10B**, room 005, #56	**A New Algorithm for Finding Polygonal Voids in Delaunay Triangulations and its Parallelization** *Nancy Hitschfeld, José Ojeda, Rodrigo Alonso*

A known geometrical problem is to find low density zones (voids) in bidimensional planar point sets and to represent them as simple polygons. In this paper we formally characterize the properties of the subvoid candidates over a triangulation, present a linear algorithm to find subvoids taking as input a Delaunay triangulation, and show that this new solution can be naturally parallelized using gpu computing. We also show preliminary experimental results.

Fri 14:45–15:00 **10B**, room 005, #72	**On Primal-Dual Circle Representations** *Stefan Felsner, Günter Rote*



The Koebe-Andreev-Thurston Circle Packing Theorem tells us that every triangulated planar graph has a circle-contact representation. The theorem has been generalized in various ways. The arguably most prominent generalization assures the existence of a primal-dual circle representation for every 3-connected planar graph.

The most commonly known proof is based on an iterative algorithm. In this talk we explain the algorithm and emphasize some details in the analysis which lead to a very compact proof.

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Nachweis Ingo Joseph

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