Existence of Offset Polytopes

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We are given a non-convex three-dimensional polytope P whose boundary is homeomorphic to a sphere. We want to construct an *offset* polytope P_ε in which every face is translated outward by the same small distance ε. P_ε should have the same number of faces as P, and the boundary of P_ε should still be homeomorphic to a sphere. If P has a saddle-like vertex of degree 4 or larger, the result is not unique, see Figure 1. *Does such an offset polytope always exist for sufficiently small $\varepsilon > 0$?*

It is enough to solve the problem locally for each vertex v of degree $d \geq 4$. The link of v might be a convoluted spherical polygon, as in Figure 2. Such a vertex will be blown up into $d-2$ new vertices, connected by edges that form a tree. In the neighborhood of the new vertices, there should be a face of P_ε corresponding to each face incident to v, and this face should be simply connected when clipped to a neighborhood of v.

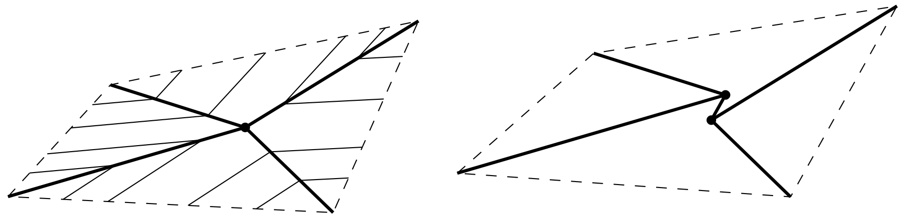

Figure 1: A saddle vertex and one of two possible offset surfaces

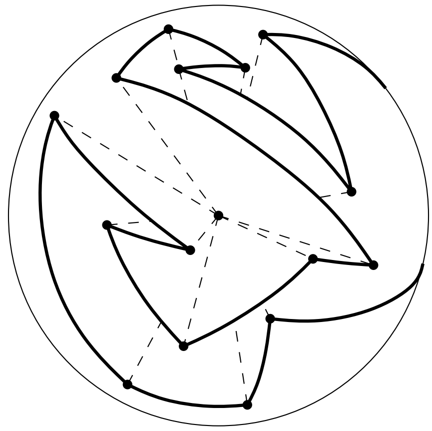

Figure 2: The link of a vertex v

References

[1] Franz Aurenhammer and Gernot Walzl. Structure and computation of straight skeletons in 3-space. In Leizhen Cai, Siu-Wing Cheng, and Tak Wah Lam, editors, *Algorithms and Computation—24th International Symposium, ISAAC 2013, Hong Kong, China, December 16–18, 2013, Proceedings*, volume 8283 of *Lecture Notes in Computer Science*, pages 44–54. Springer, 2013. doi:10.1007/978-3-642-45030-3_5.

[2] Franz Aurenhammer and Gernot Walzl. Polytope offsets and straight skeletons in 3D (video). In Siu-Wing Cheng and Olivier Devillers, editors, *30th Annual Symposium on Computational Geometry, SOCG'14, Kyoto, Japan, June 8–11, 2014*, pages 98–99. ACM, 2014. doi:10.1145/2582112.2595651, `http://www.computational-geometry.org/SoCG-videos/socg14video/`.

[3] Gill Barequet, David Eppstein, Michael T. Goodrich, and Amir Vaxman. Straight skeletons of three-dimensional polyhedra. In Dan Halperin and Kurt Mehlhorn, editors, *Algorithms—ESA 2008, 16th Annual European Symposium, Karlsruhe, Germany, September 15–17, 2008. Proceedings*, volume 5193 of *Lecture Notes in Computer Science*, pages 148–160. Springer, 2008. doi:10.1007/978-3-540-87744-8_13, arXiv:0805.0022 [cs.CG].