1. Andrey Kupavskii, kupavskii@ya.ru

“Given \( n \) slabs in \( \mathbb{R}^d \) of total divergent width, can one cover the unit ball with their translates?”

In more details: is it true that there exists \( C = C(d) \), such that for any \( n_1, \ldots, n_s \in S^{d-1}, \ d > 2, \) and any \( \varepsilon_1, \ldots, \varepsilon_s \in \mathbb{R}_+ \) with \( \sum_{i=1}^{s} \varepsilon_i > C \), there exist \( x_1, \ldots, x_s \in \mathbb{R} \) satisfying

\[
\{ x \in \mathbb{R}^d : |x| \leq 1 \} \subseteq \{ x \in \mathbb{R}^d : x_i \leq \langle x, n_i \rangle \leq x_i + \varepsilon_i \}.
\]

Asked in [Makai–Pach, 1983].


2. Dömötör Pálvölgyi, dom@cs.elte.hu

Can we 3-color any (finite) set of points such that any disk with at least 3 points is non-monochromatic? Asked originally in [Keszegh, 2012].


3. Eran Nevo, nevo@math.huji.ac.il

Fix \( d \) even, and let \( n \to \infty \):

Must \( d \)-polytopes with \( n \) vertices have only \( o(n^{d/2}) \) non-simplex facets? (The trivial upper bound is \( O(n^{d/2}) \).)

Jeff Erickson asked this in 1999, and conjectured that the answer is yes, also for \((d - 1)\)-polyhedral spheres.

For spheres the answer is no - as was proved in [Nevo–Santos–Wilson, 2016]

The case \( d = 4 \) of the above question is already very interesting. The lower bound obtained in Nevo et al. is \( \Omega(n^{3/2}) \).
Let $P_1$ and $P_2$ be two combinatorially equivalent convex polytopes in $\mathbb{R}^3$. Is it true that there exist corresponding edges $t_1$ of $P_1$ and $t_2$ of $P_2$, such that the dihedral angle of $t_1$ is not greater than the dihedral angle of $t_2$, or all the corresponded angles are equal? This problem is Conjecture 5.1 in the following preprint.


**Danzer’s problem.** A finite set of pairwise intersecting disks in the plane can be stabbed by 4 points, and there exists a configuration of 10 pairwise intersecting disks that require 4 points [Danzer, 1986].

**The problem:**
(a) Understand Danzer’s solution.
(b) Come up with a simpler solution.
(c) Make it constructive.


**Fact:** For any probability measure $\mu$ that charges no lines, there exist two order types $\omega_1(\mu)$ and $\omega_2(\mu)$ of size 6 such that if $X$ is a set of 6 points $\sim \mu$ then

$$\mathbb{P}[X \text{ realizes } \omega_1(\mu)] > 1.8 \mathbb{P}[X \text{ realizes } \omega_2(\mu)].$$

**Question:** Does there exist $c > 0$ such that $\forall \mu \exists \omega_1(\mu), \omega_2(\mu)$ with $|\omega_1(\mu)| = |\omega_2(\mu)| = n$ and

$$P[X \simeq \omega_1] > c^n \ P[X \simeq \omega_2(\mu)]?$$

7. Géza Tóth, geza@renyi.hu

Is the class of intersection graphs of lines in $\mathbb{R}^3$ (or $\mathbb{R}^d$) $\chi$-bounded? Namely, is there a function $f$ such that given $n$ lines in the $\mathbb{R}^3$, no $k$ of them pairwise crossing, the lines can be colored with $f(k)$ colors in such a way that crossing lines get different colors?


8. Imre Bárány, barany@renyi.hu

$k$-crossing curves in $\mathbb{R}^d$. A curve $\gamma$ in $\mathbb{R}^d$ is $k$-crossing if every hyperplane intersects it at most $k$ times. Thus $k \geq d$. A $d$-crossing curve is called convex.

**Theorem** (Bárány, Matoušek, Pór). *For every $d \geq 2$ there is $M(d)$ such that every $(d+1)$-crossing curve in $\mathbb{R}^d$ can be split into $M(d)$ convex curves.*

The proof gives $M(d) \leq 4^d$, $M(2) = 4$ and $M(3) \leq 22$.

**Question:** Give lower bounds for $M(d)$.


9. Pavel Valtr, valtr@kam.mff.cuni.cz

Lines, line-point incidences, and crossing families in dense sets. Let $P$ be a set of $n$ points in $\mathbb{R}^2$ such that $\min \text{dist}(P) = 1$ and $\max \text{dist}(P) = O(\sqrt{m})$. Prove or disprove:

**Conjecture 1.** $P$ contains a crossing family of size $\Omega(n)$. 
Known: $P$ contains a crossing family of size $\Omega(n^{1-\varepsilon})$.

Two lines are essentially different if either their direction differ by at least $1/n$, or their $\frac{1}{\sqrt{n}}$-neighborhoods do not intersect inside $\text{conv}(P)$.

**Conjecture 2.** $P$ determines $\Omega(n^2)$ pairwise essentially different lines.

Known: $P$ determines $\Omega(n^{2-\varepsilon})$ pairwise essentially different lines.

A point $p$ and a line $\ell$ determine a rough point-line incidence if $\text{dist}(p,\ell) \leq \frac{1}{\sqrt{n}}$.

**Conjecture 3.** Let $P$ as before and $L$ a set of $n$ pairwise essentially different lines. Then the number of rough point-line incidences is at least $\Omega(n^{4/3})$.

10. Luis Montejano, luis@matem.unam.mx

Let $X$ be a polyhedron. Let $\mathcal{F} = \{A_1, \ldots, A_m\}$ be a polyhedral cover of $X$ such that $A_i$ is not empty but not necessarily connected. Let $N$ be the nerve of $\mathcal{F}$.

**Fact:** Suppose that the following hold: (a) $H_1(X) = 0$, and (b) for every $i \neq j$, if $A_i \cap A_j \neq \emptyset$ then $A_i \cup A_j$ is connected. Then $H_1(N) = 0$.

**Question:** Suppose that the following hold: (a) $H_1(X) = 0$, (b) for every $i \neq j$, if $A_i \cap A_j \neq \emptyset$ then $A_i \cup A_j$ is connected, and (c) for every $i < j < k$, if $A_i \cap A_j \cap A_k \neq \emptyset$ then $H_1(A_i \cup A_j \cup A_k) = 0$. Is it true that $H_2(N) = 0$? The answer is yes if $m = 4$.

11. József Solymosi, solymosi@math.ubc.ca

**Question 1:** What is the minimum number of collinear triples in a subset of the integer grid $n \times n \times n$? If $|S| = n^{3-\varepsilon}$, $S \subset n \times n \times n$, then $S$ spans at least $\frac{n^6-4}{\log n}$ collinear triples. We (with Jozsi Balogh) do not think that this is sharp.

**Question 2:** Find a bipartite unit distance graph which is rigid.

12. Edgardo Roldán-Pensado, e.roldan@im.unam.mx

**Centre of $BM(2)$.** Let $\delta$ be the Banach-Mazur distance. Find the convex body $C \subset \mathbb{R}^2$ such that $\max\{\delta(C, D) : D \subset \mathbb{R}^2 \text{ a convex body}\}$ is minimized.
An update by Edgardo Roldán-Pensado: The answer to the problem is known. A solution appears in: