Spanners and Reachability Oracles for Directed Transmission Graphs

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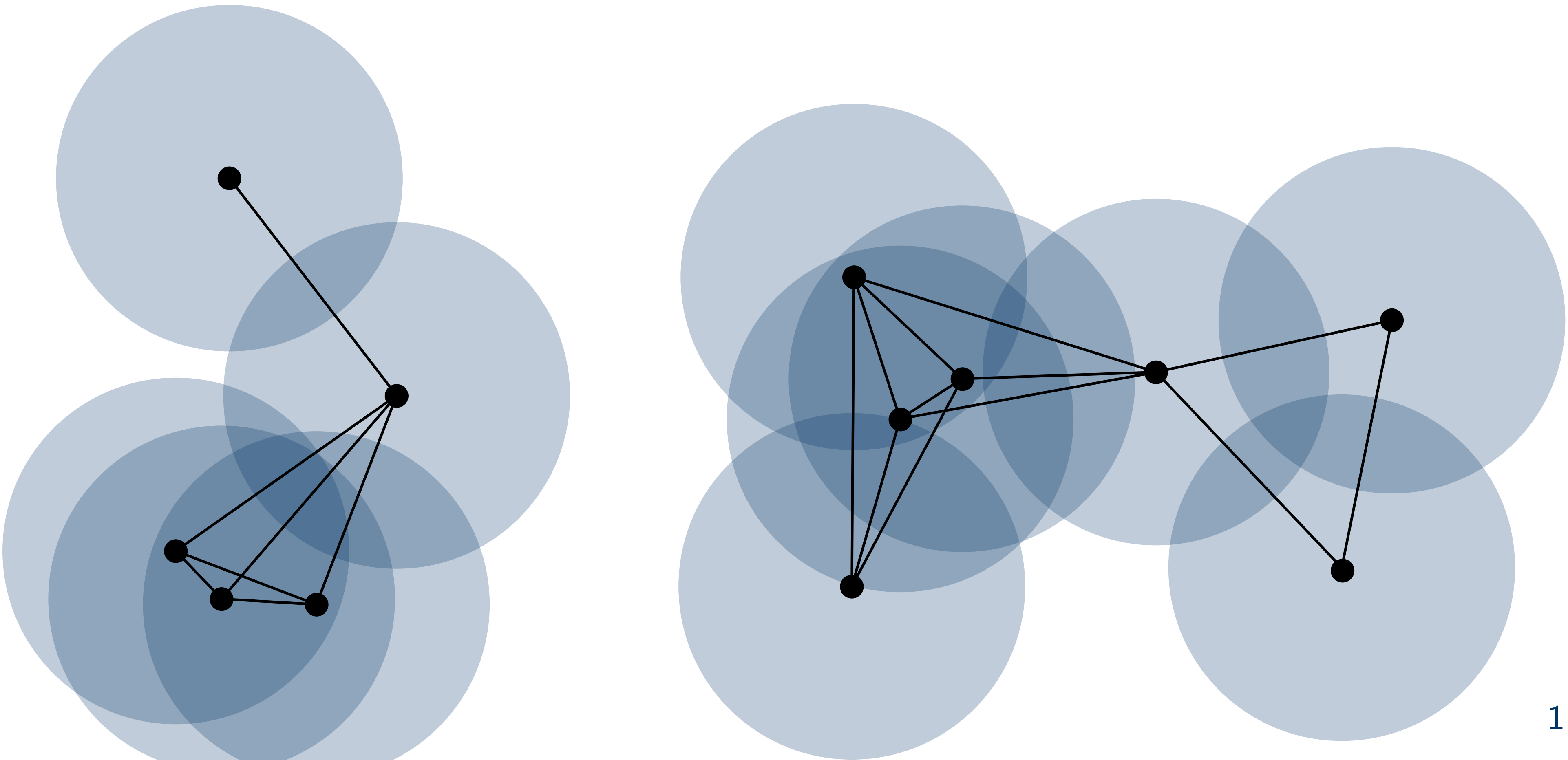
Transmission Graphs

- Model for sensor networks: n unit-disks with centers $P \subset \mathbb{R}^2$



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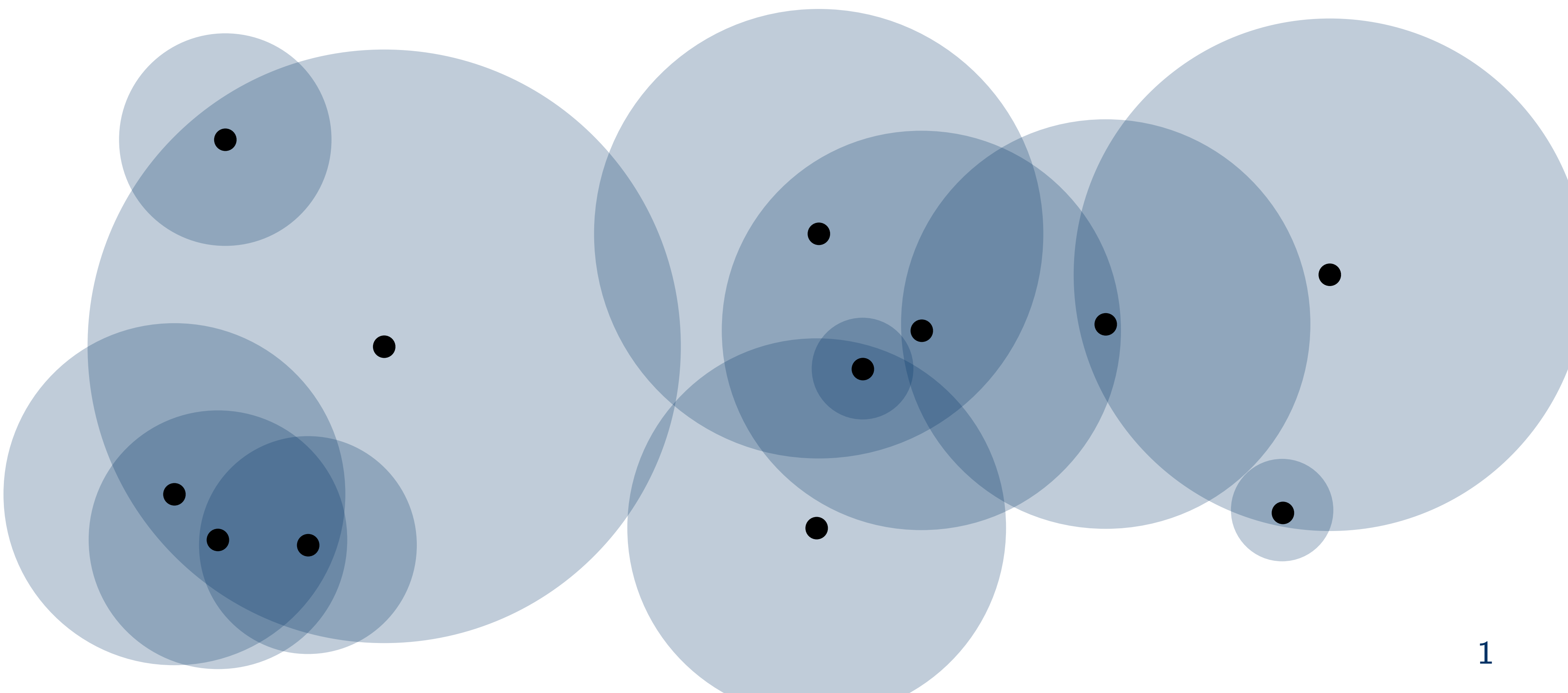

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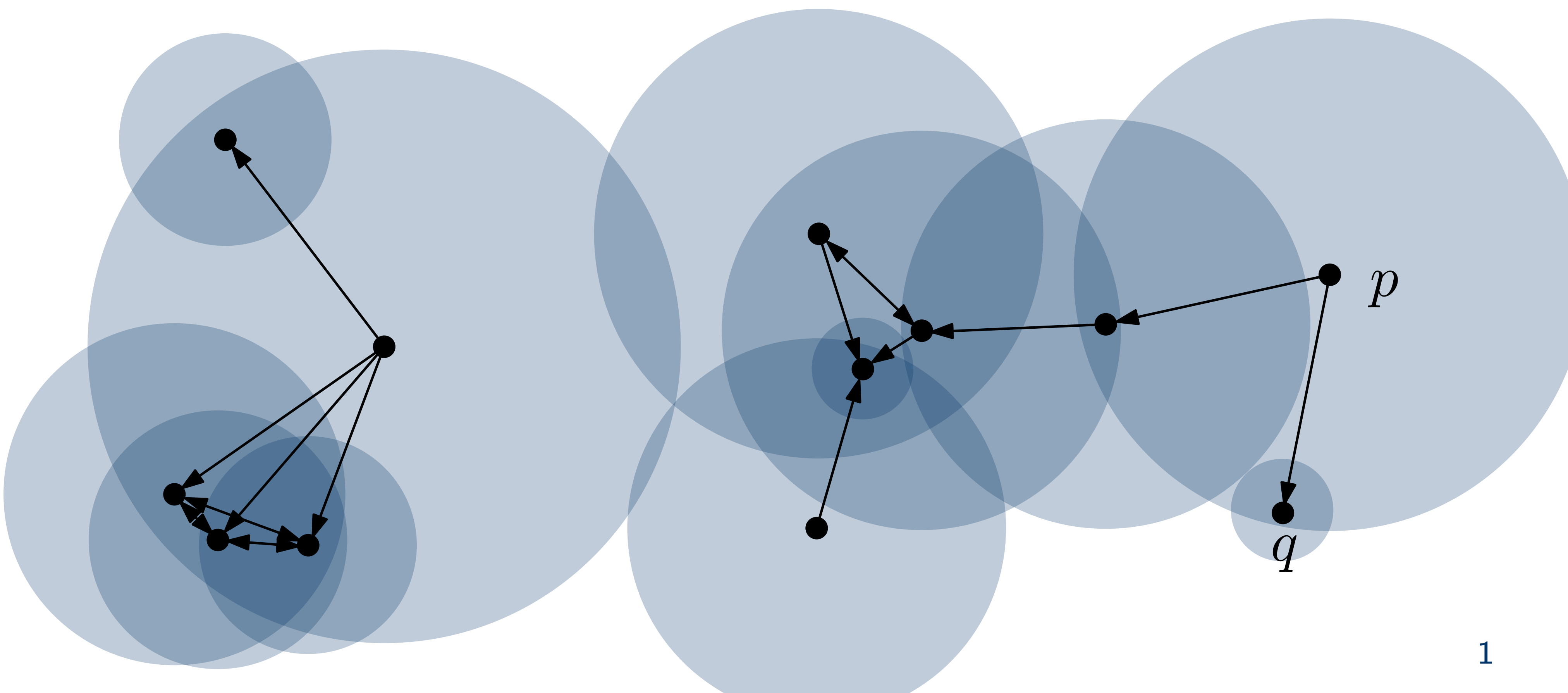
- Transmission graph G on P: directed edge $pq \Leftrightarrow d(p,q) \leq r_p$



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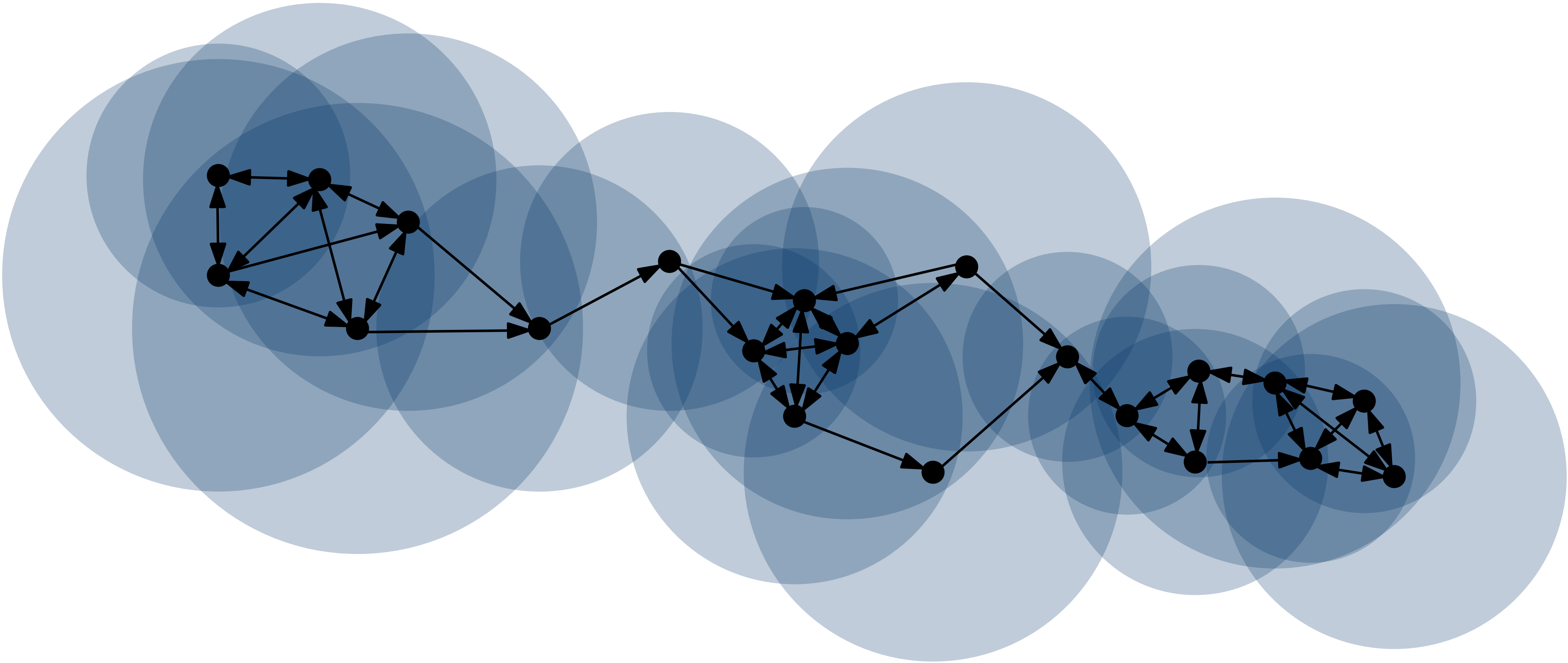
- linear representation ($P \subset \mathbb{R}^2$, radii r_p) VS. $\Theta(n^2)$ edges in G



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- parameter $t > 1$

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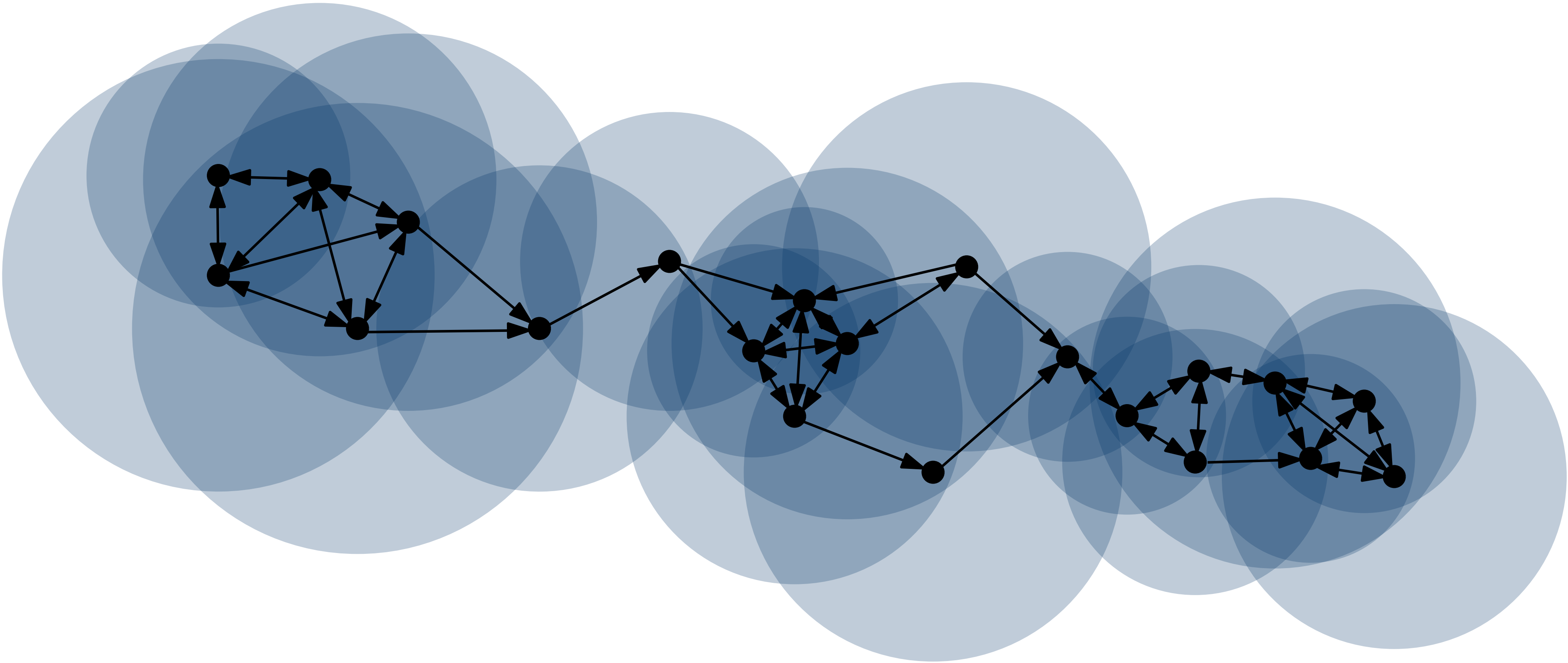

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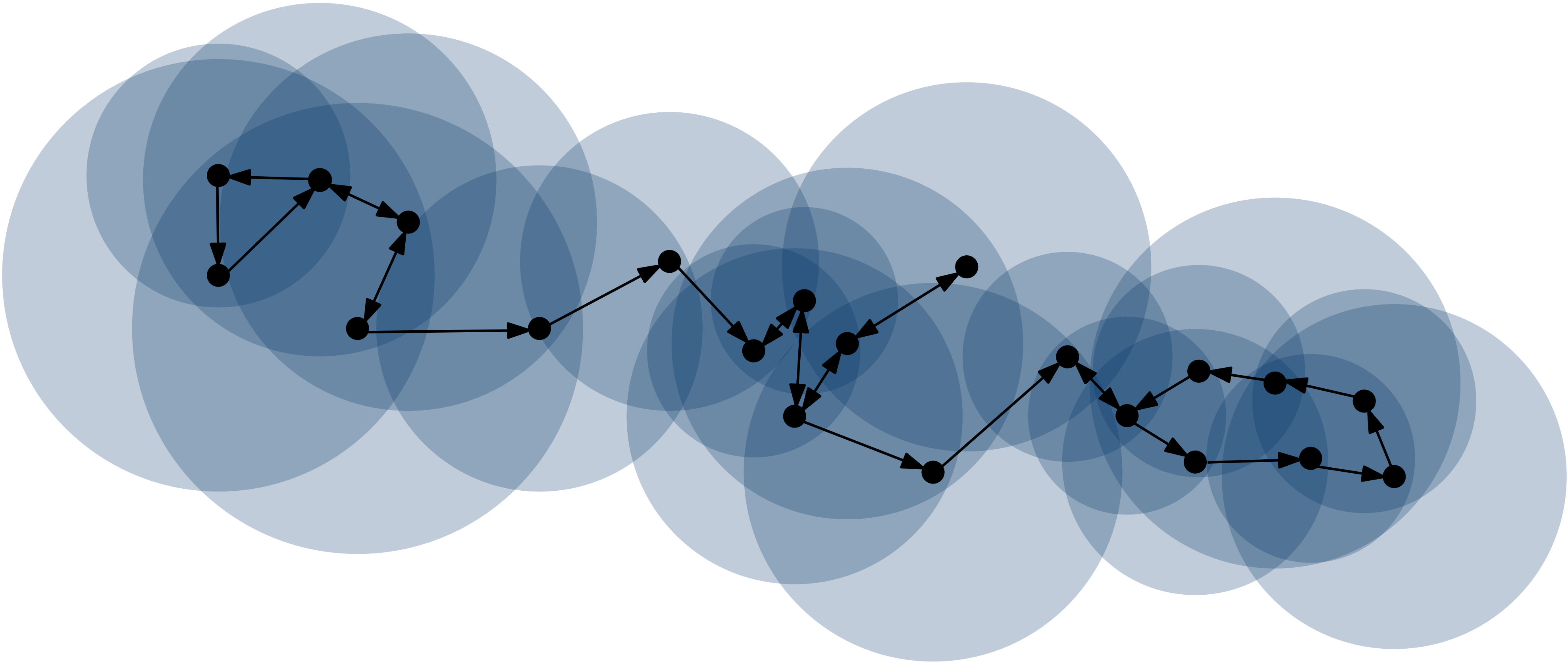

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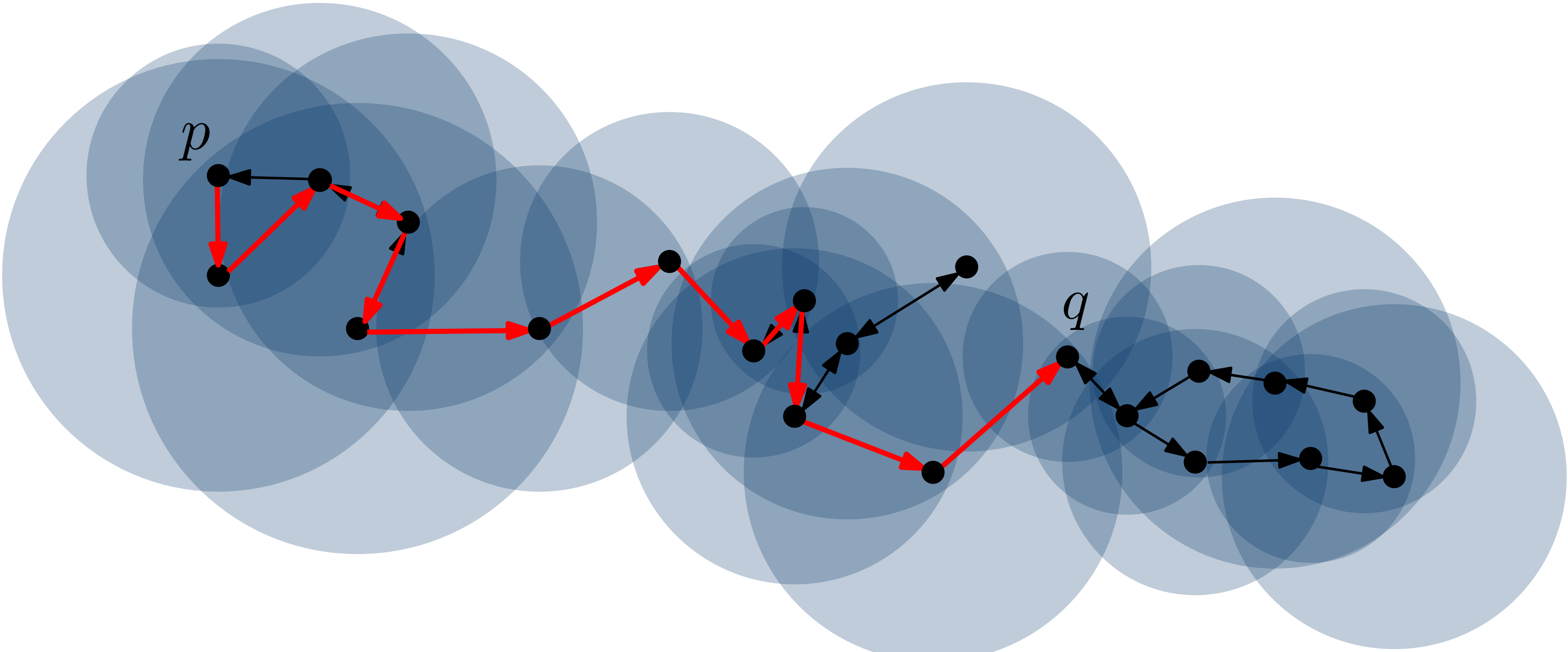

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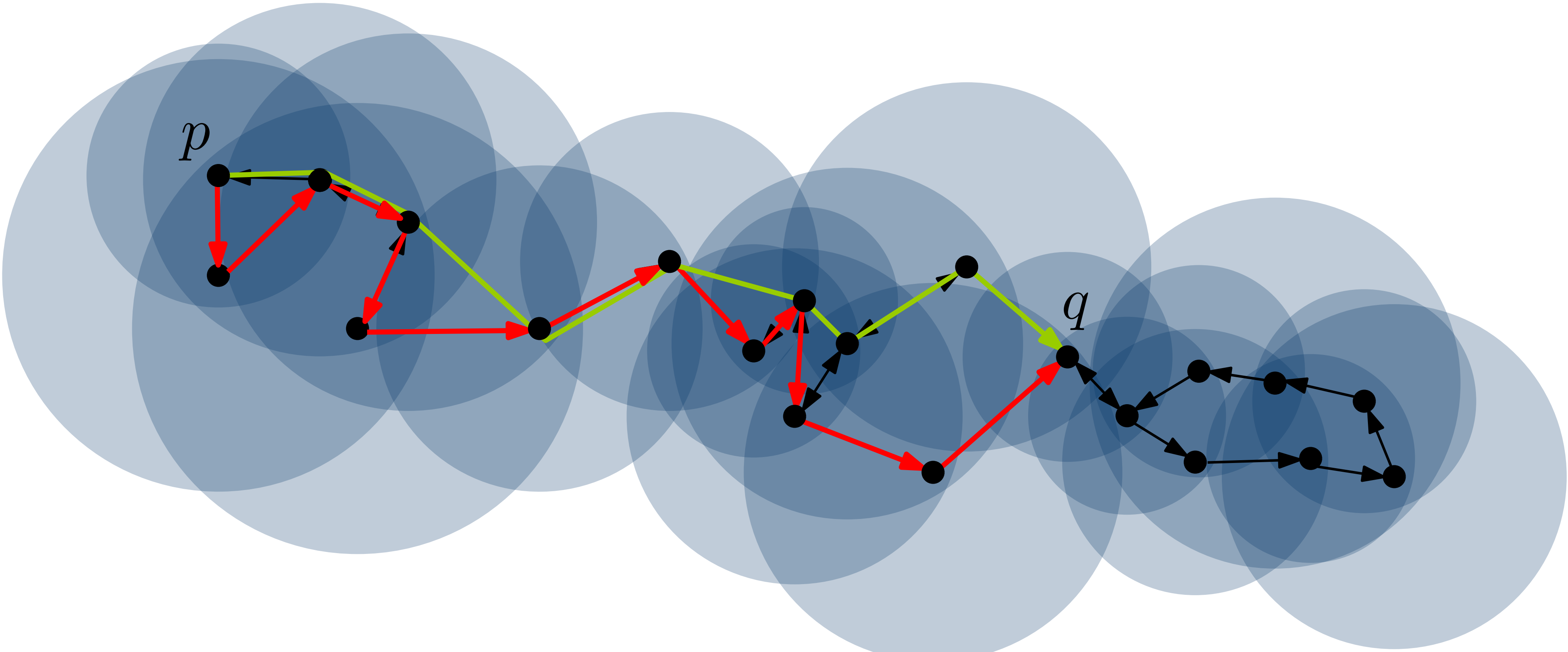
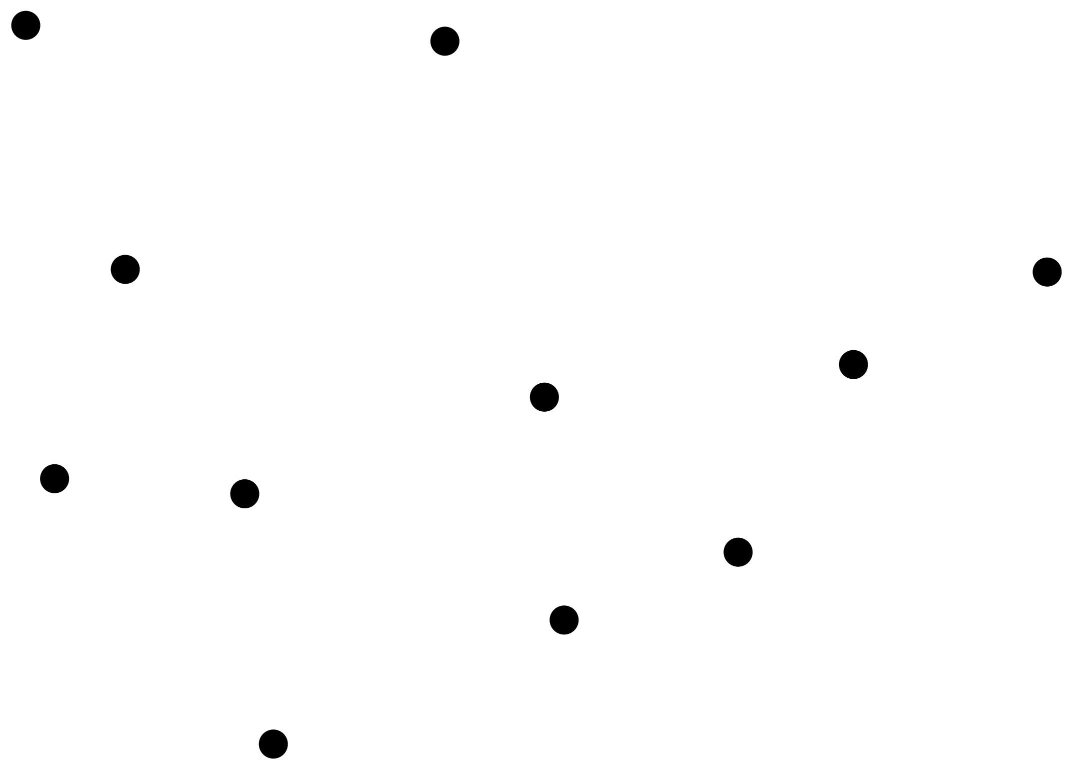


Modified Yao Graph

- Given: point set $P \subset \mathbb{R}^2$ with radii, parameter $k(t)$



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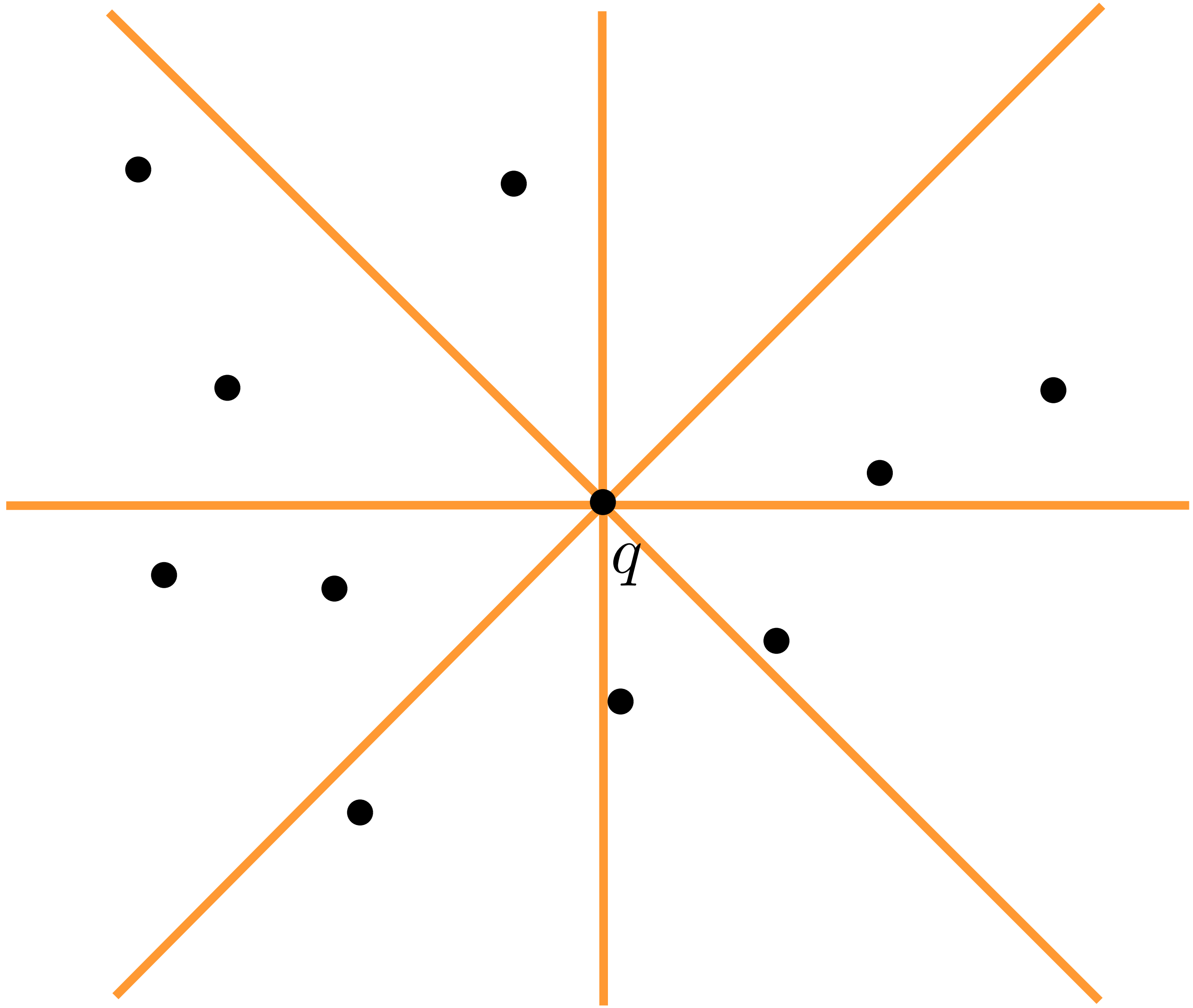

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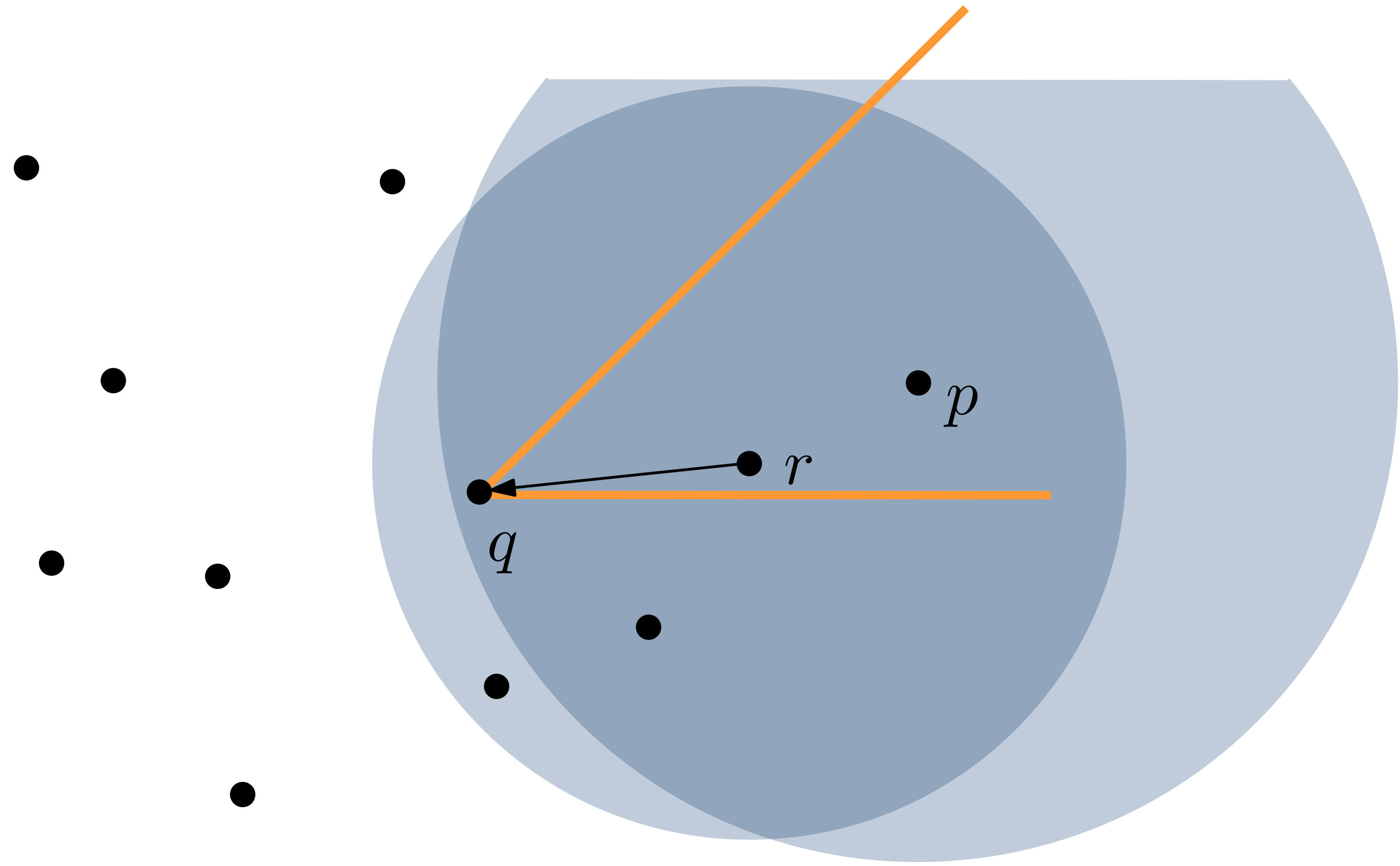

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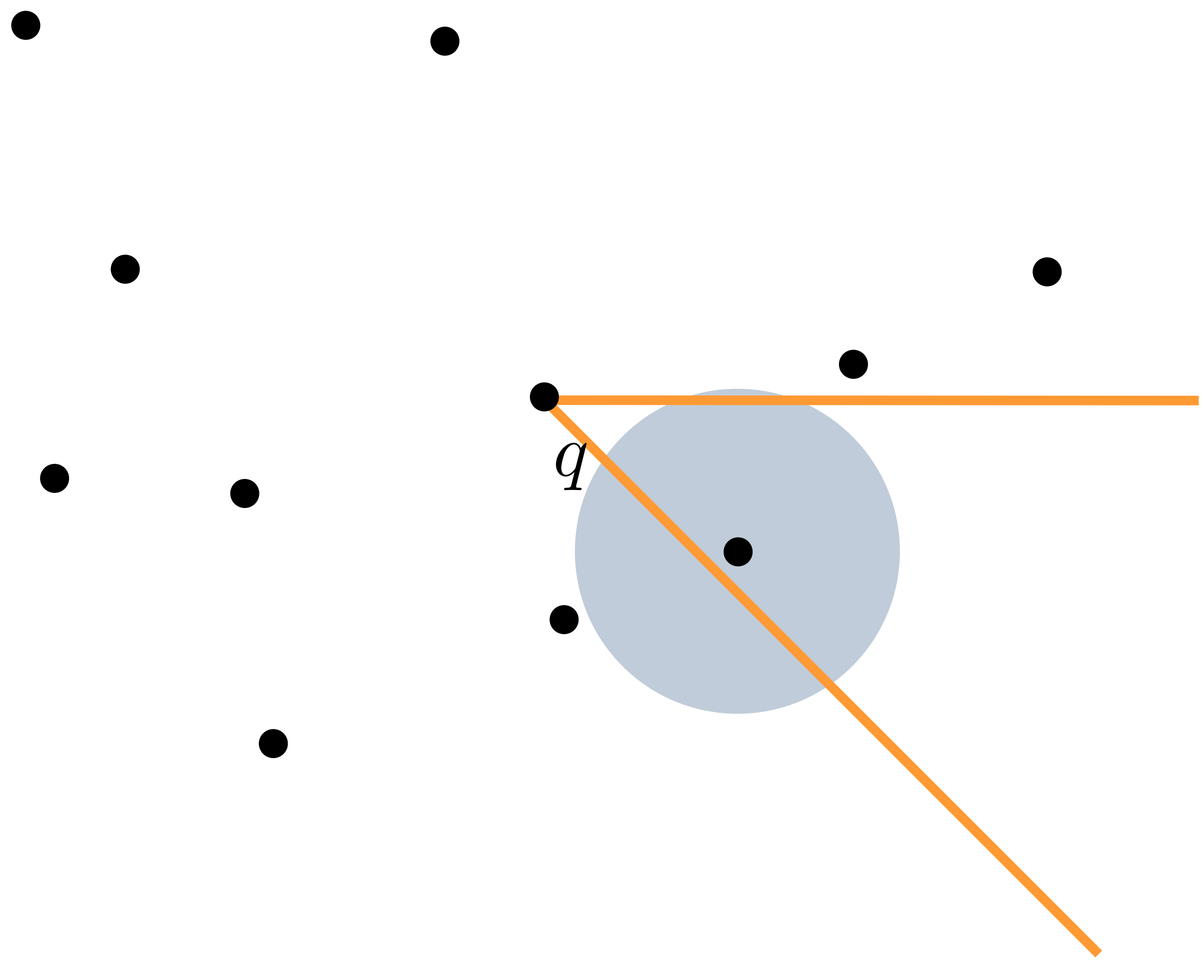

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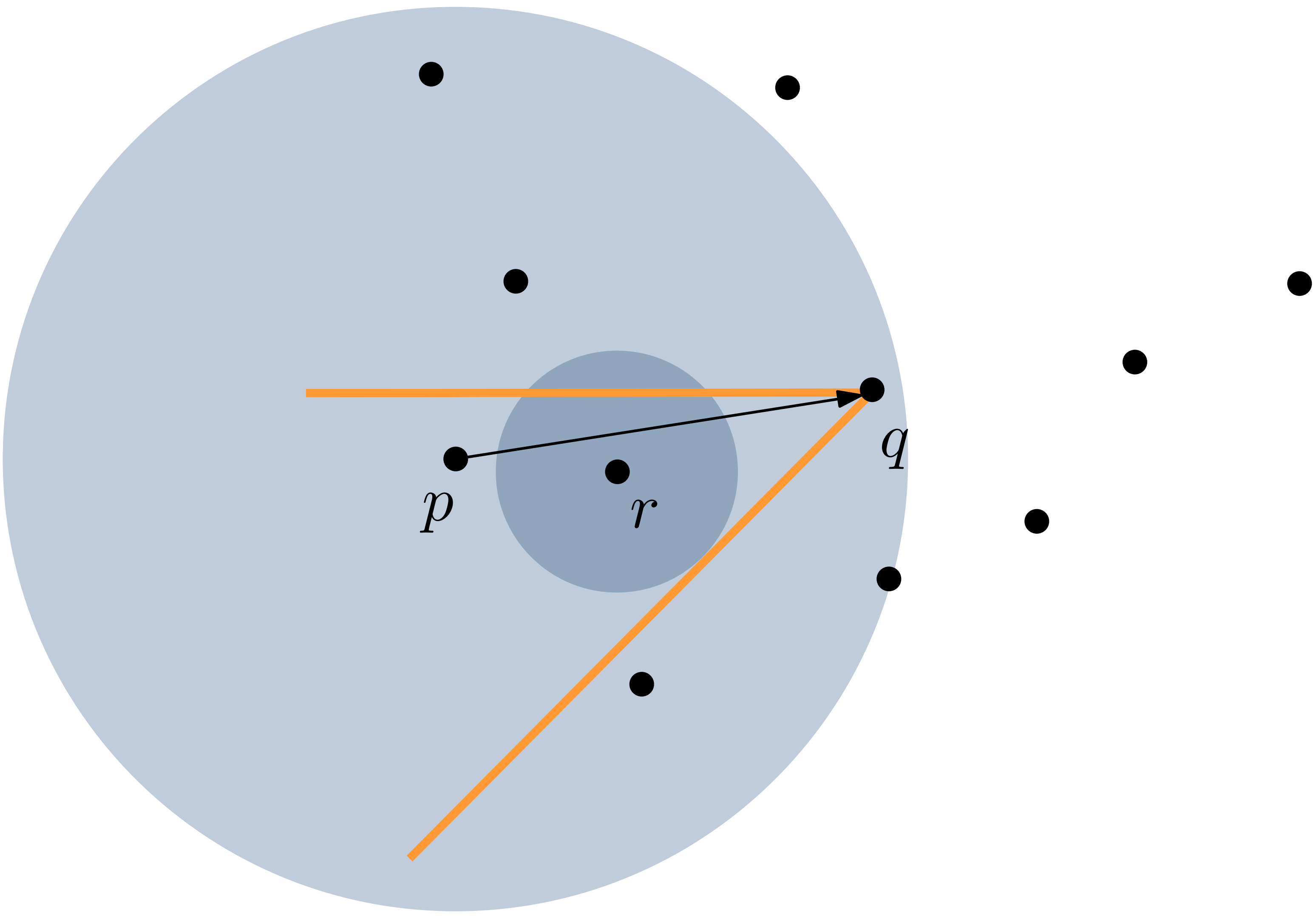

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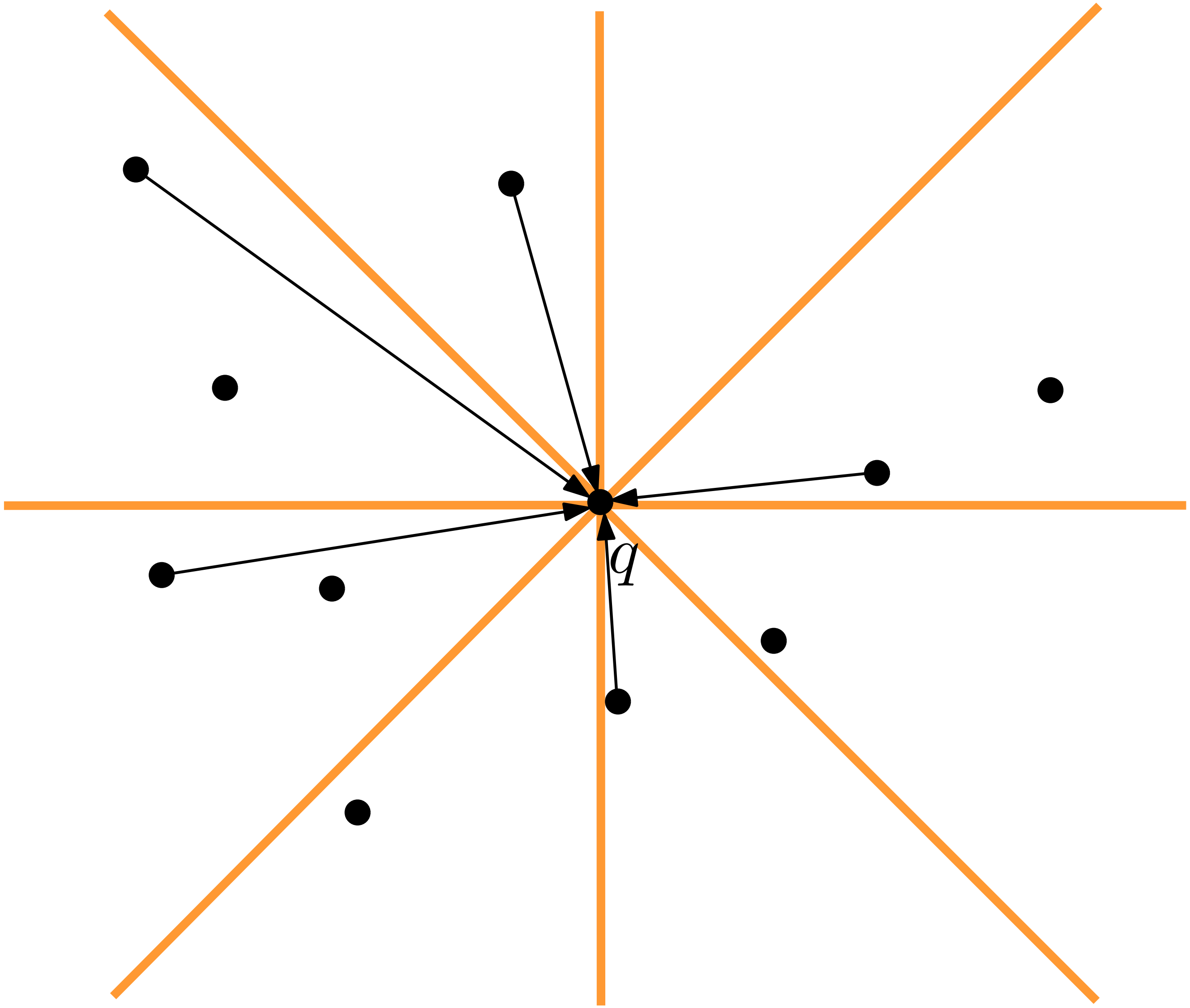

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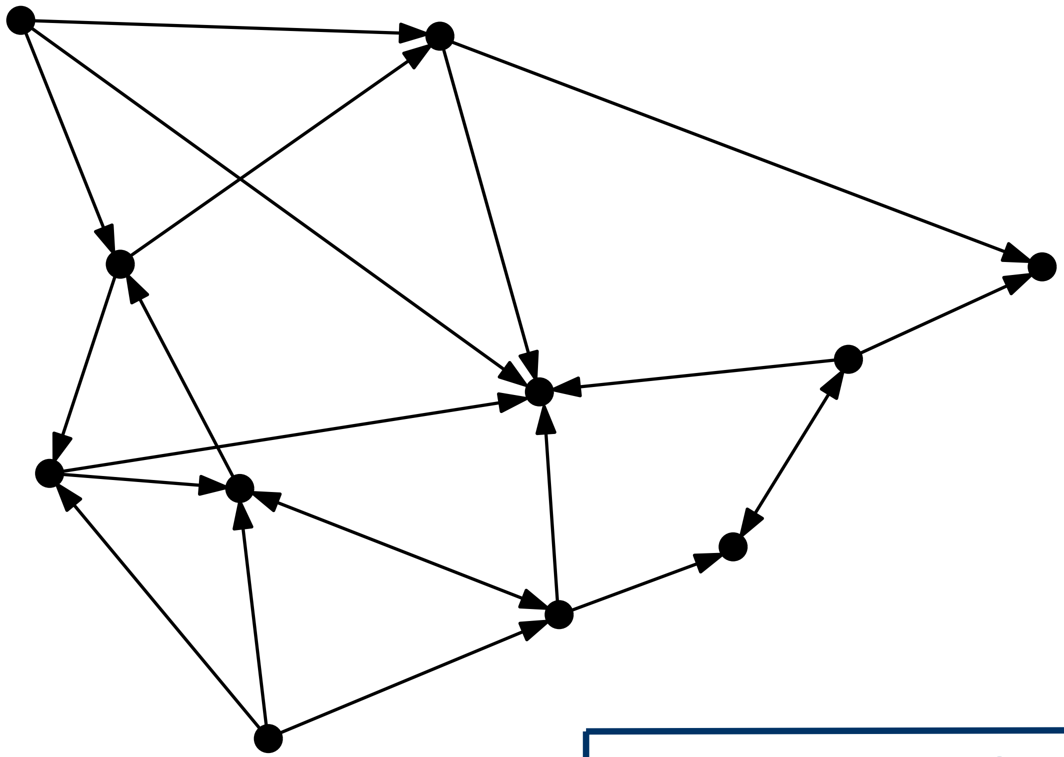


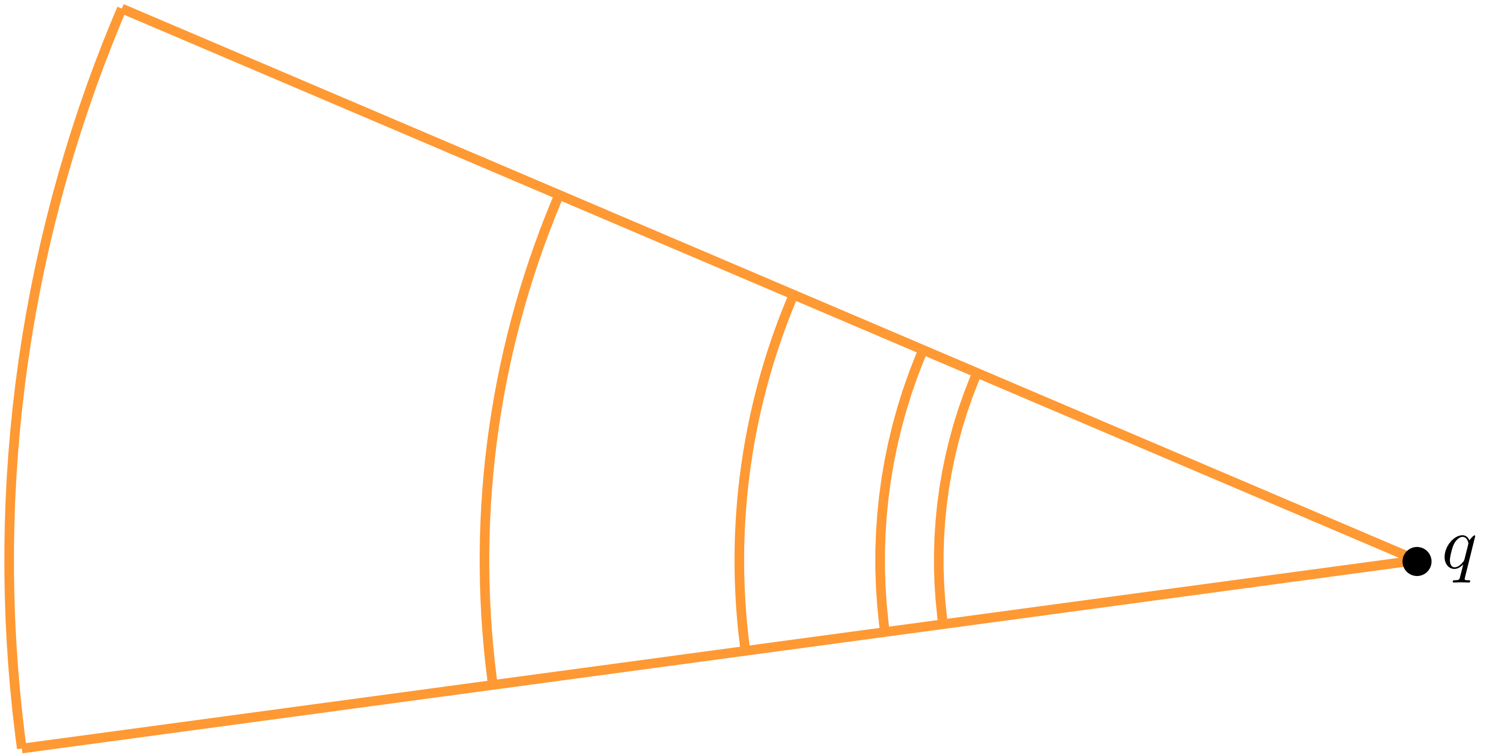
- Trival $O(n^2)$ algorithm

Theorem: Can be done in time $O(n(\log n + \log \Phi))$.

spread of P

The Idea

- partition each cone in $O(\log \Phi)$ intervals



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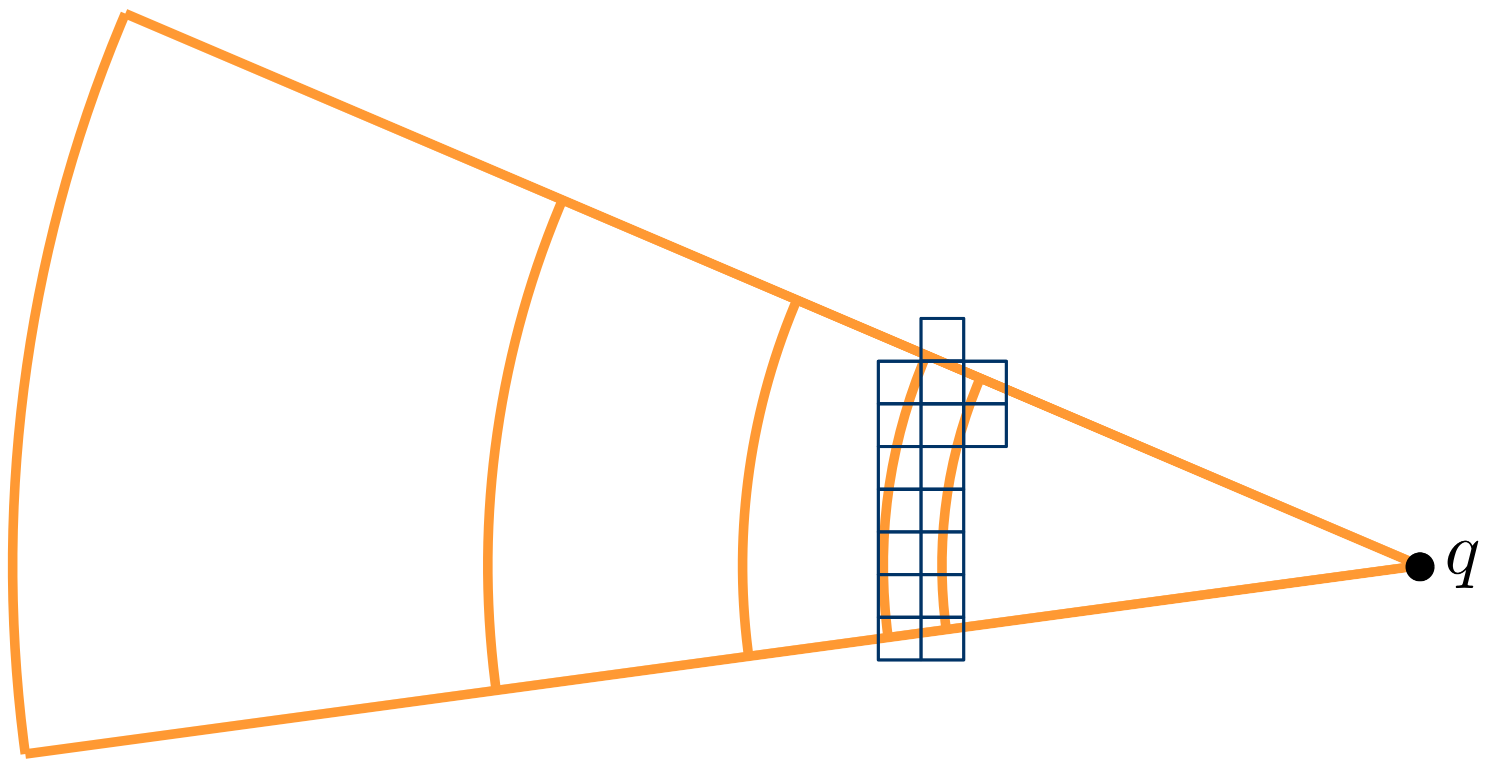

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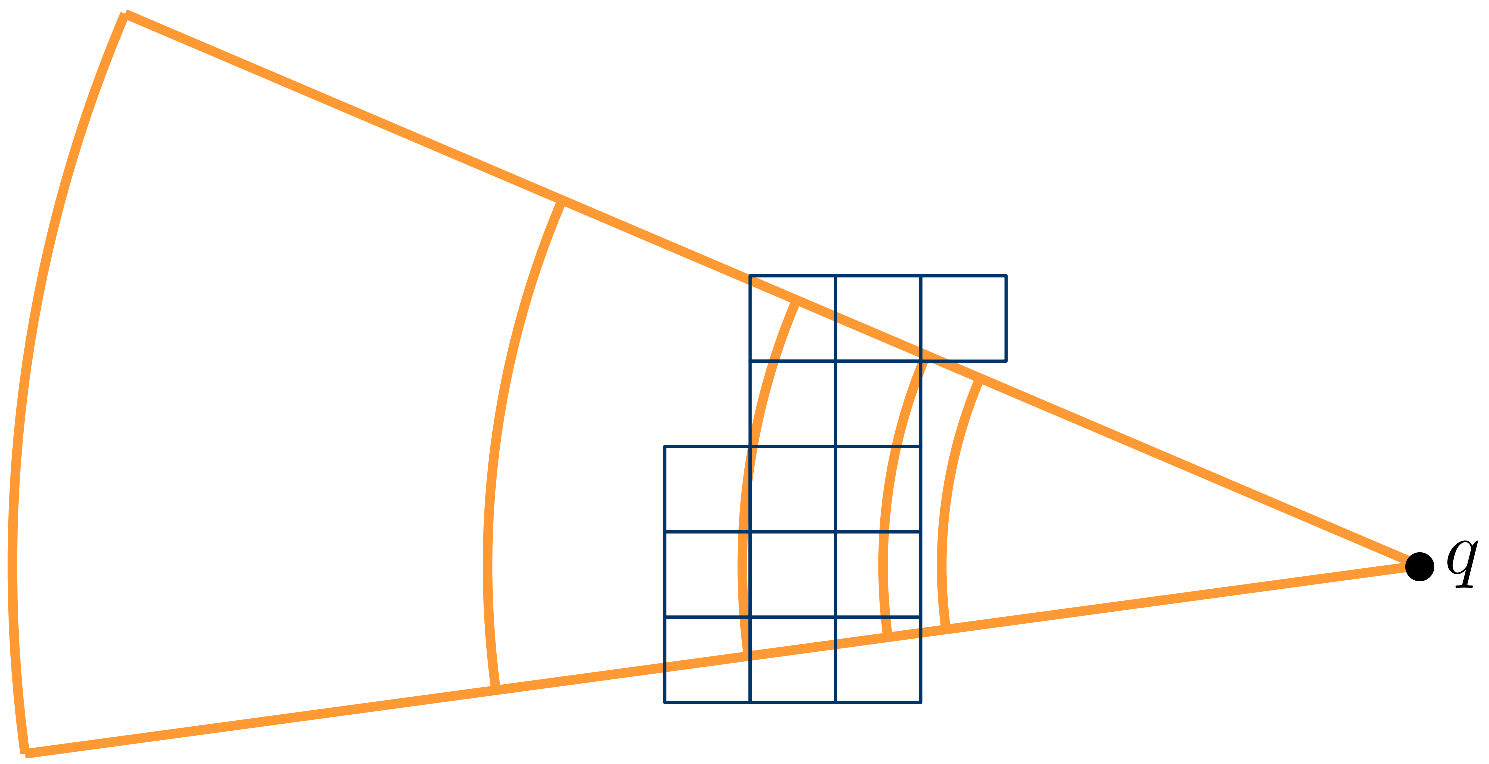

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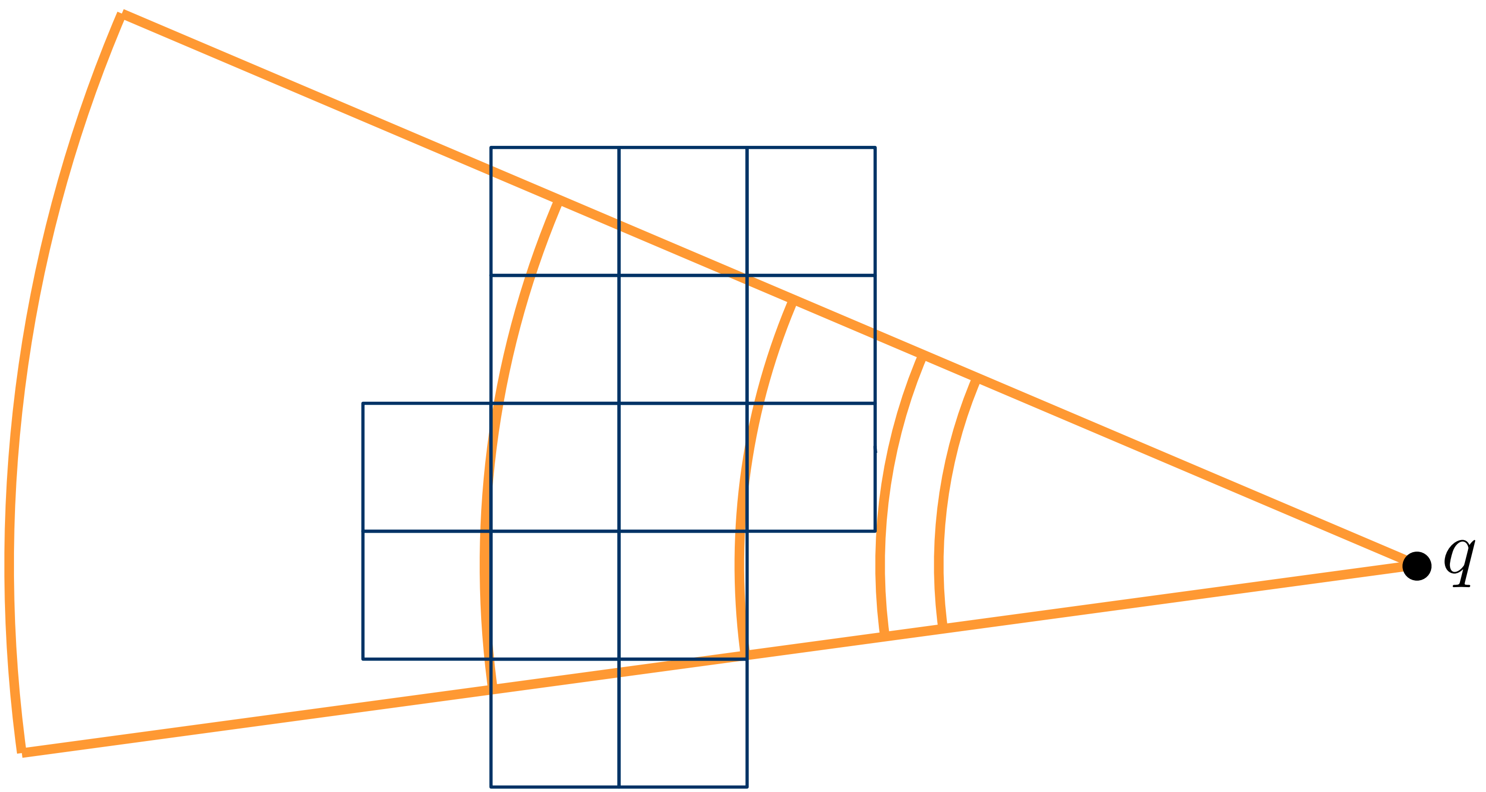

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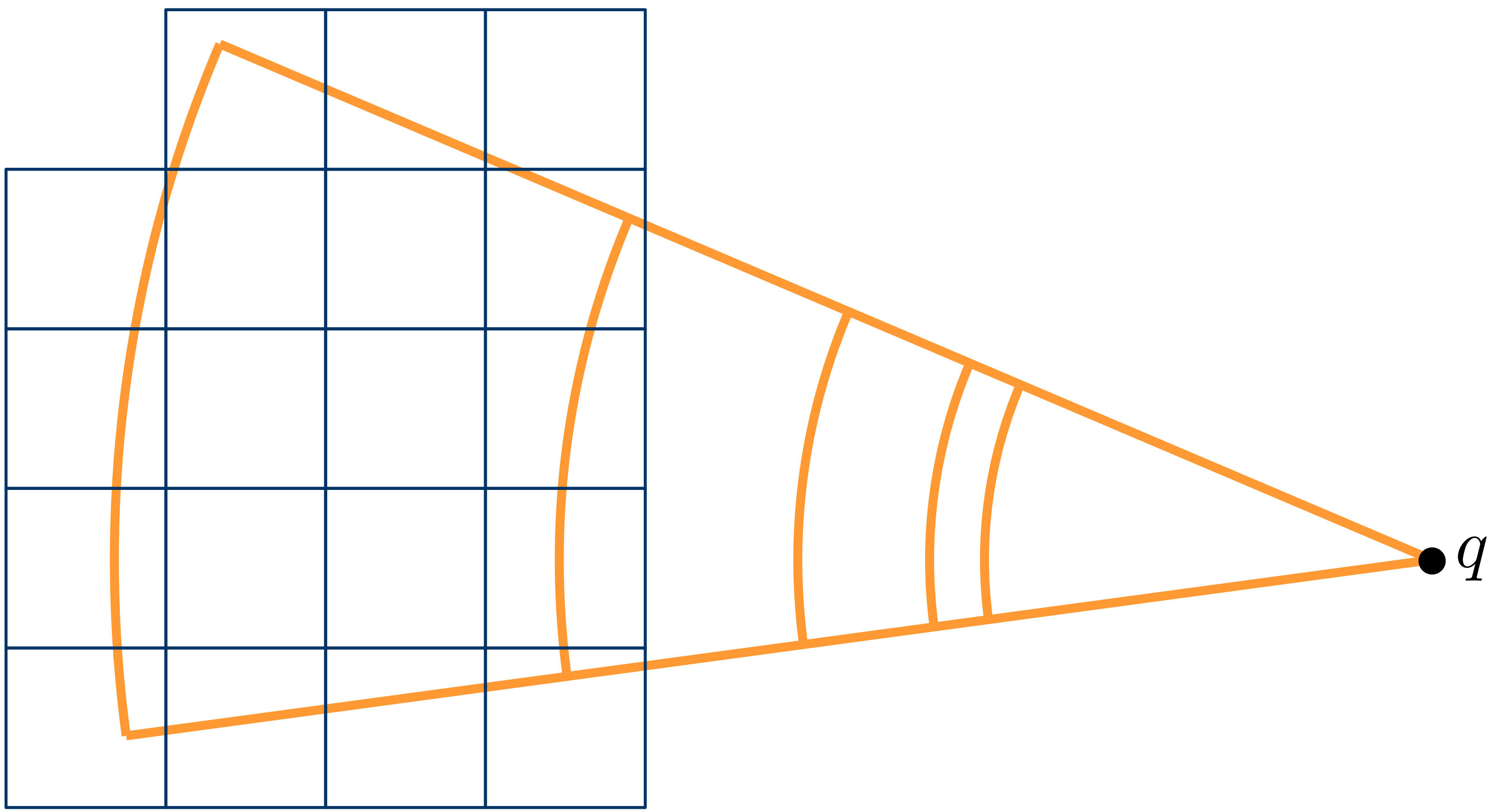

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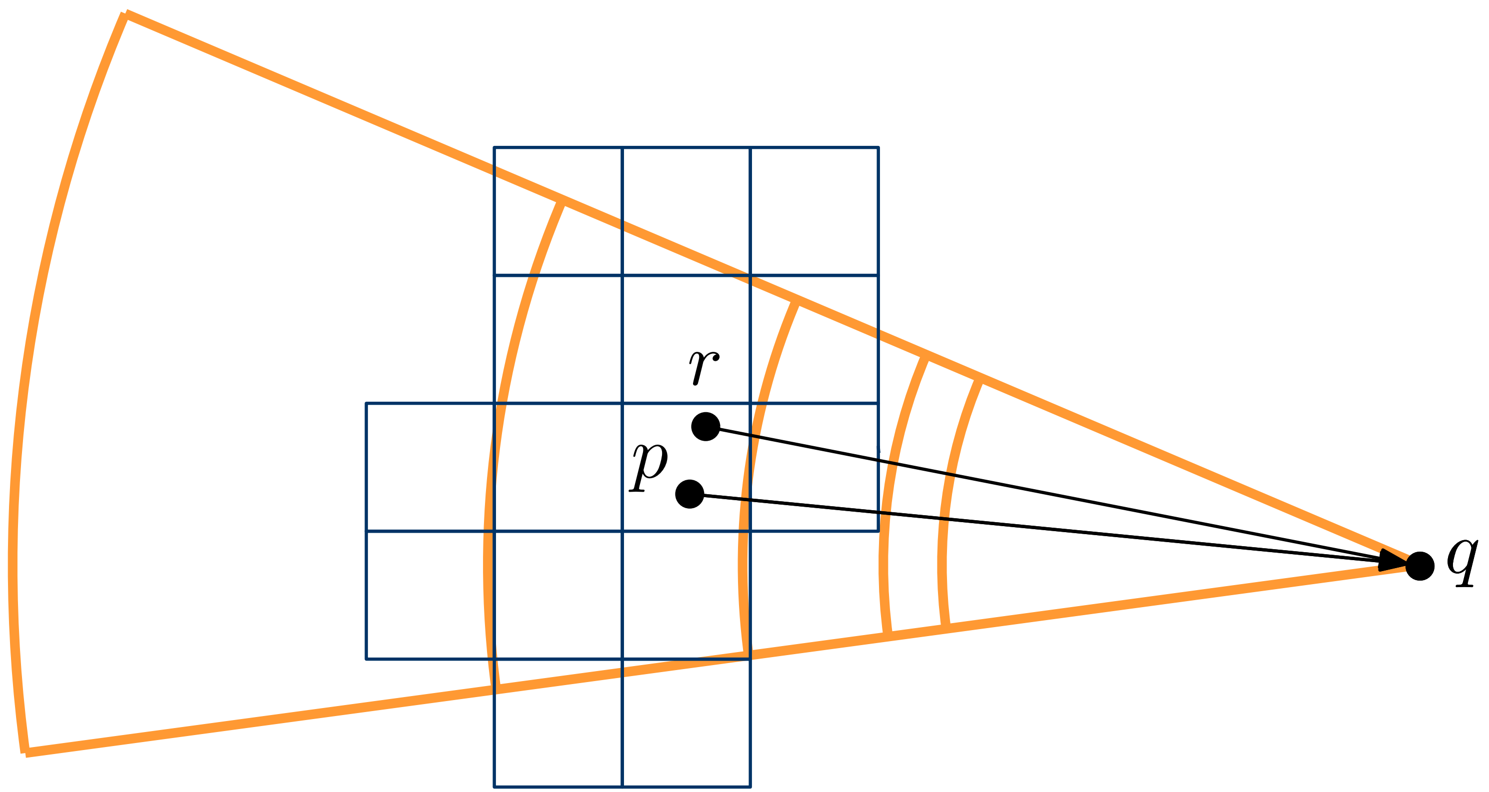
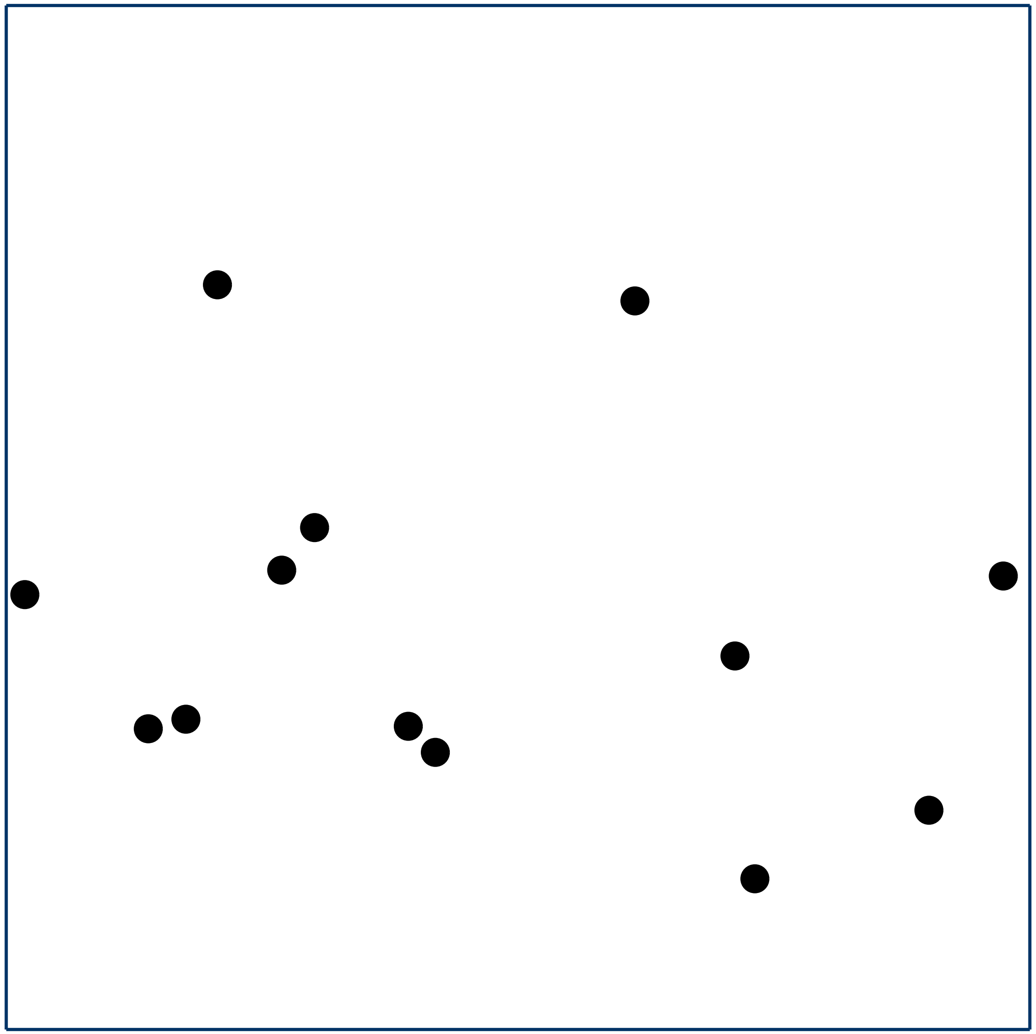


The Spanner Construction

- Compute quadtree with depth $O(\log \Phi)$



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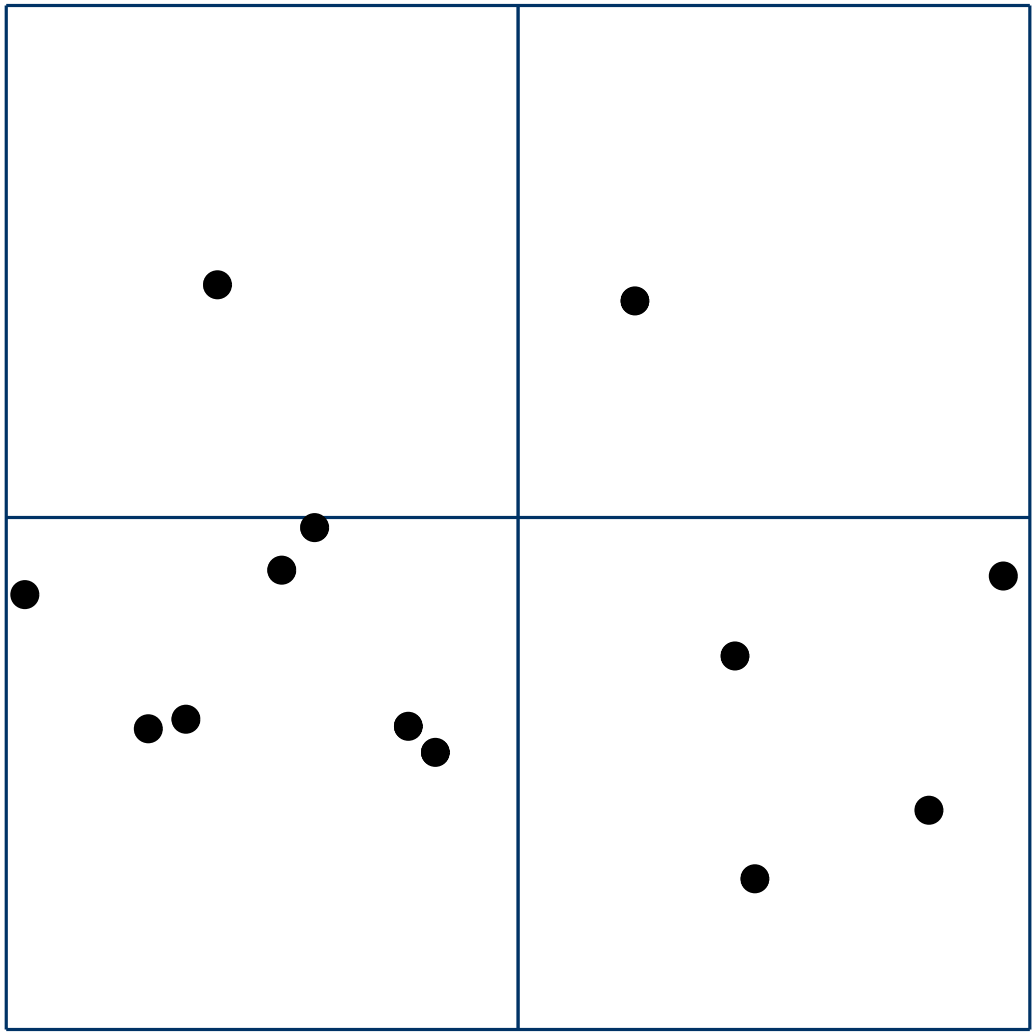

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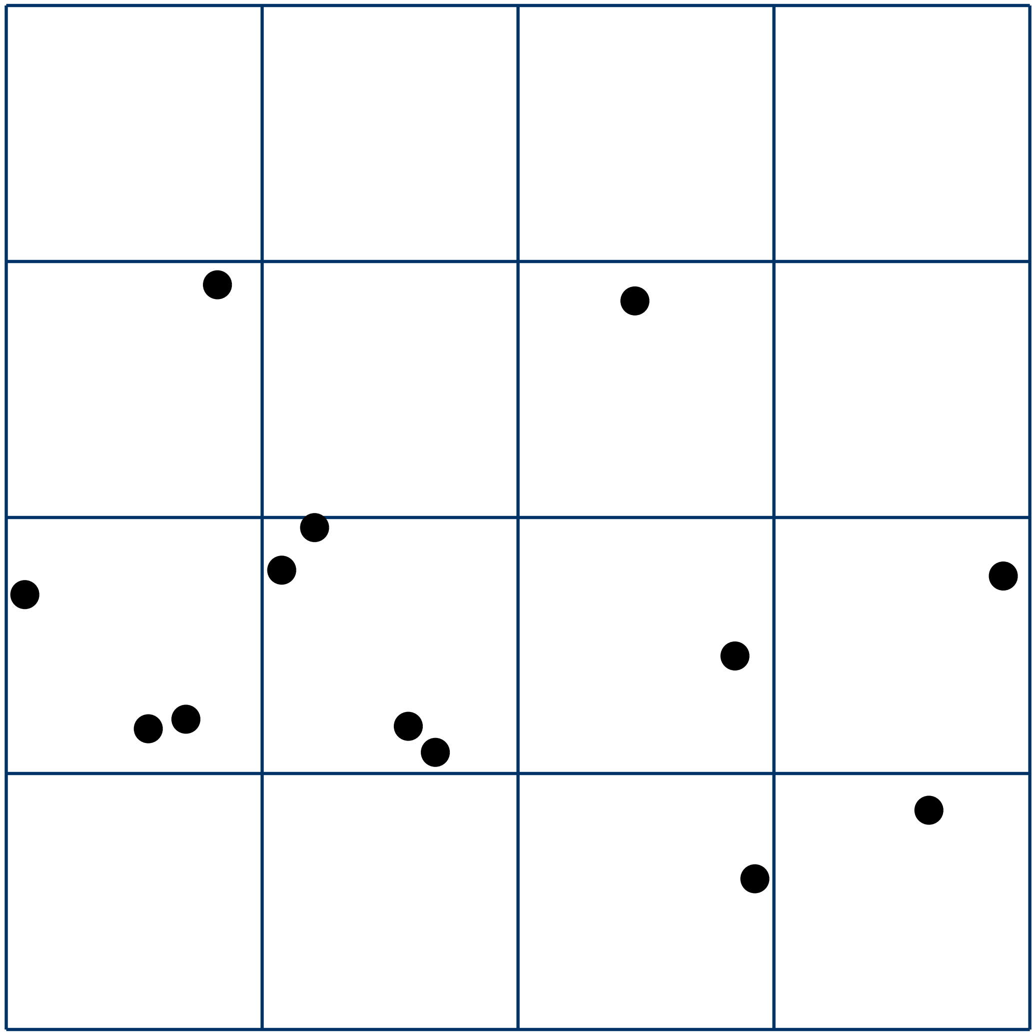

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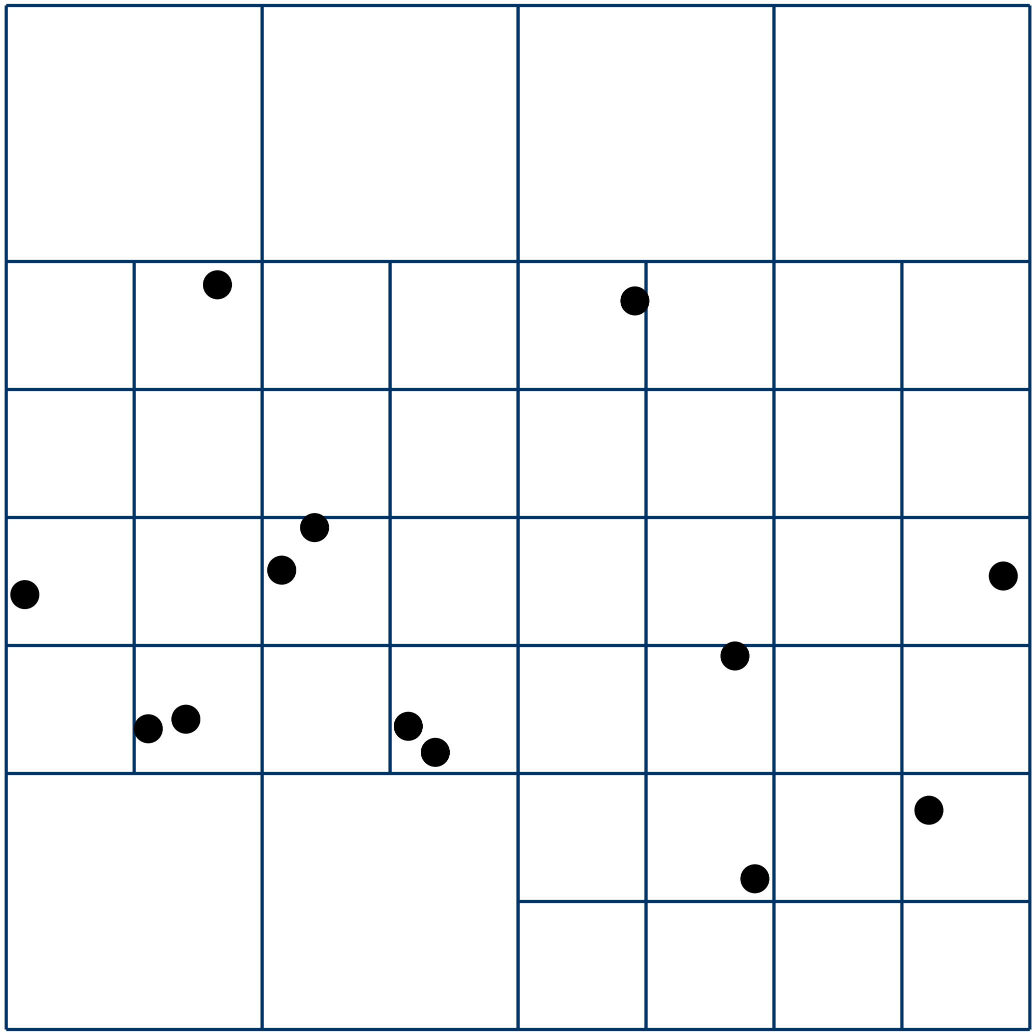

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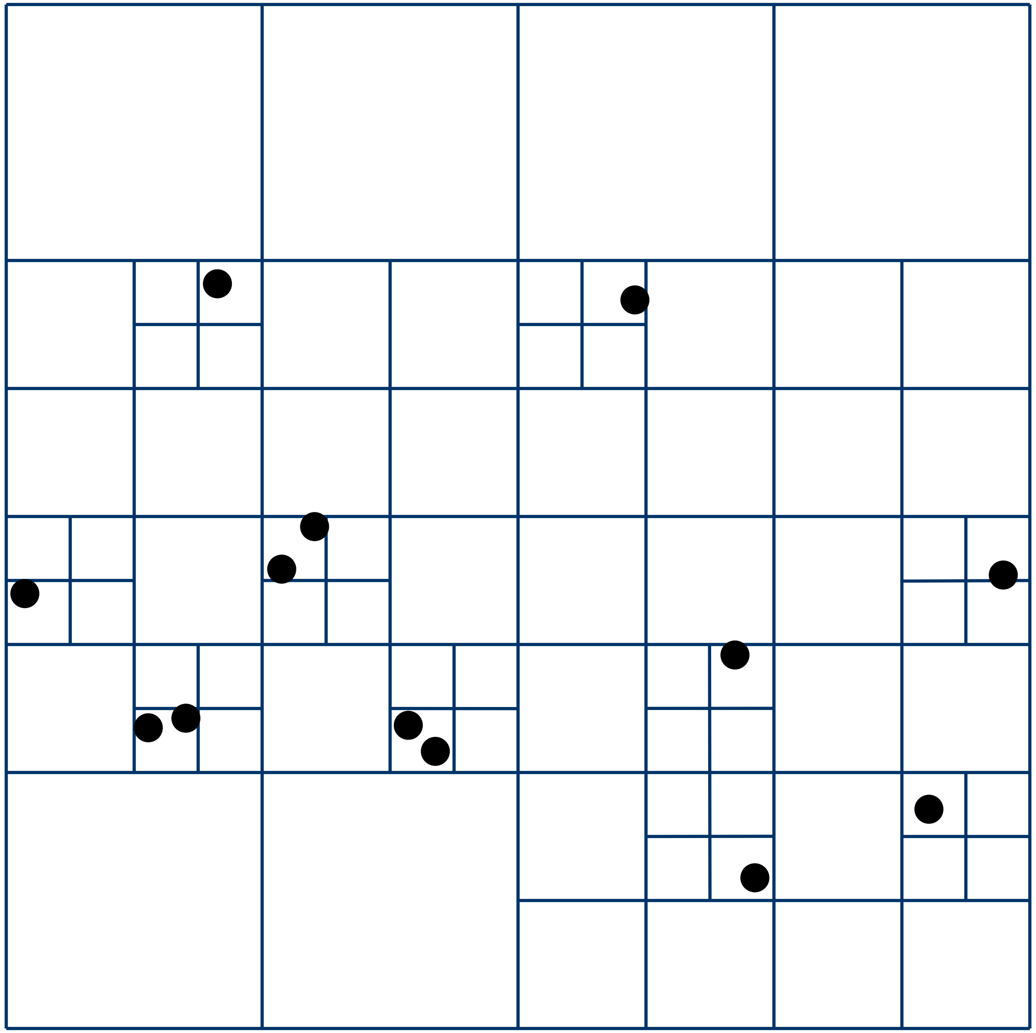

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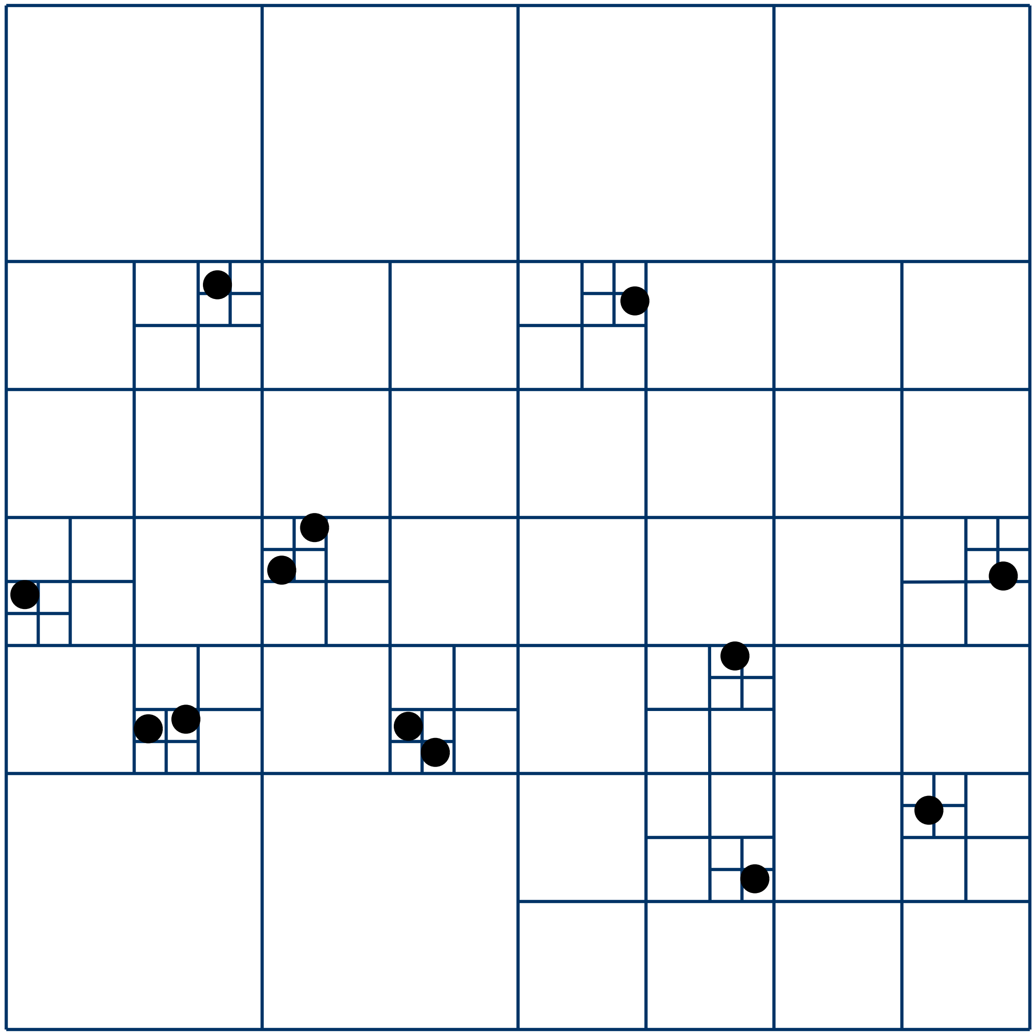

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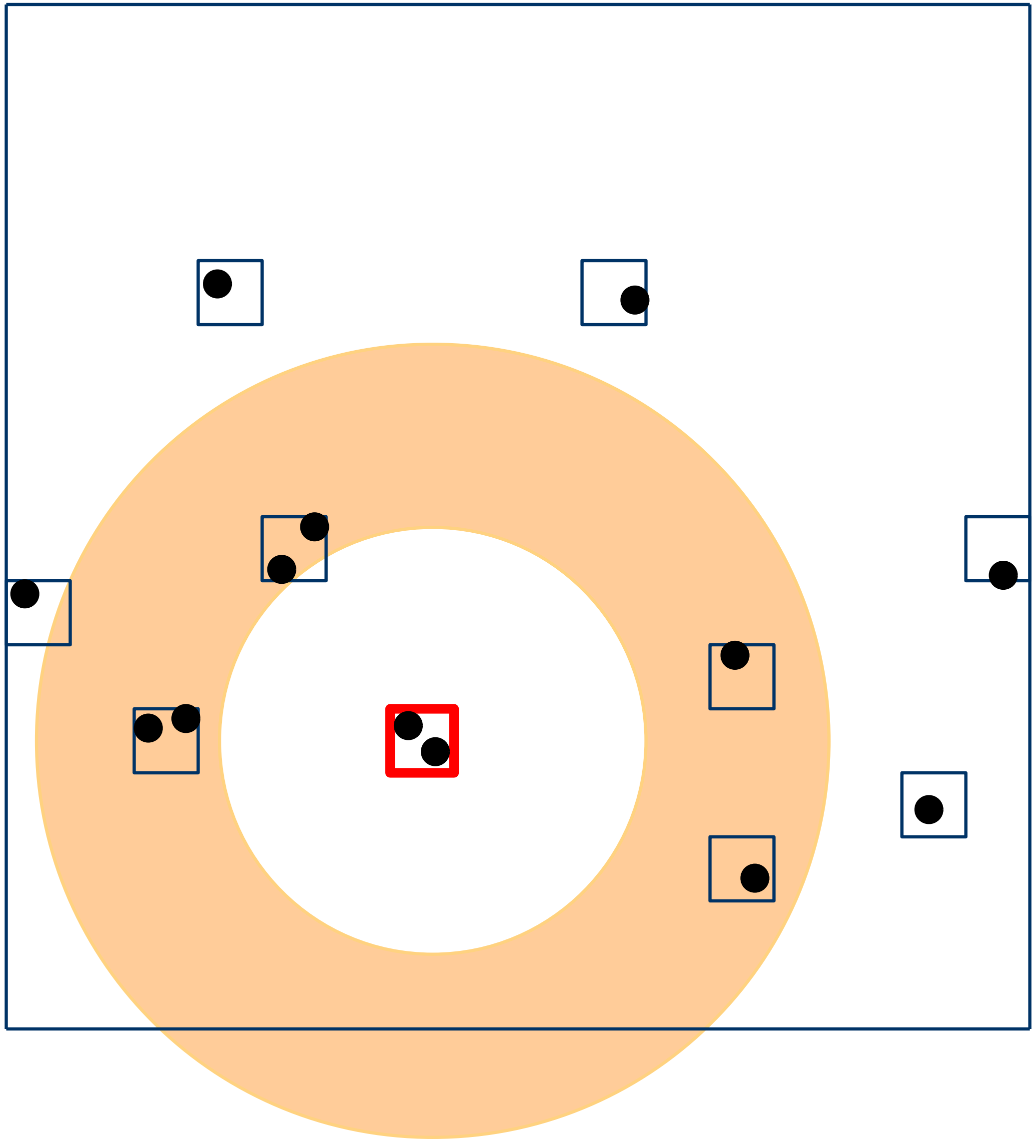

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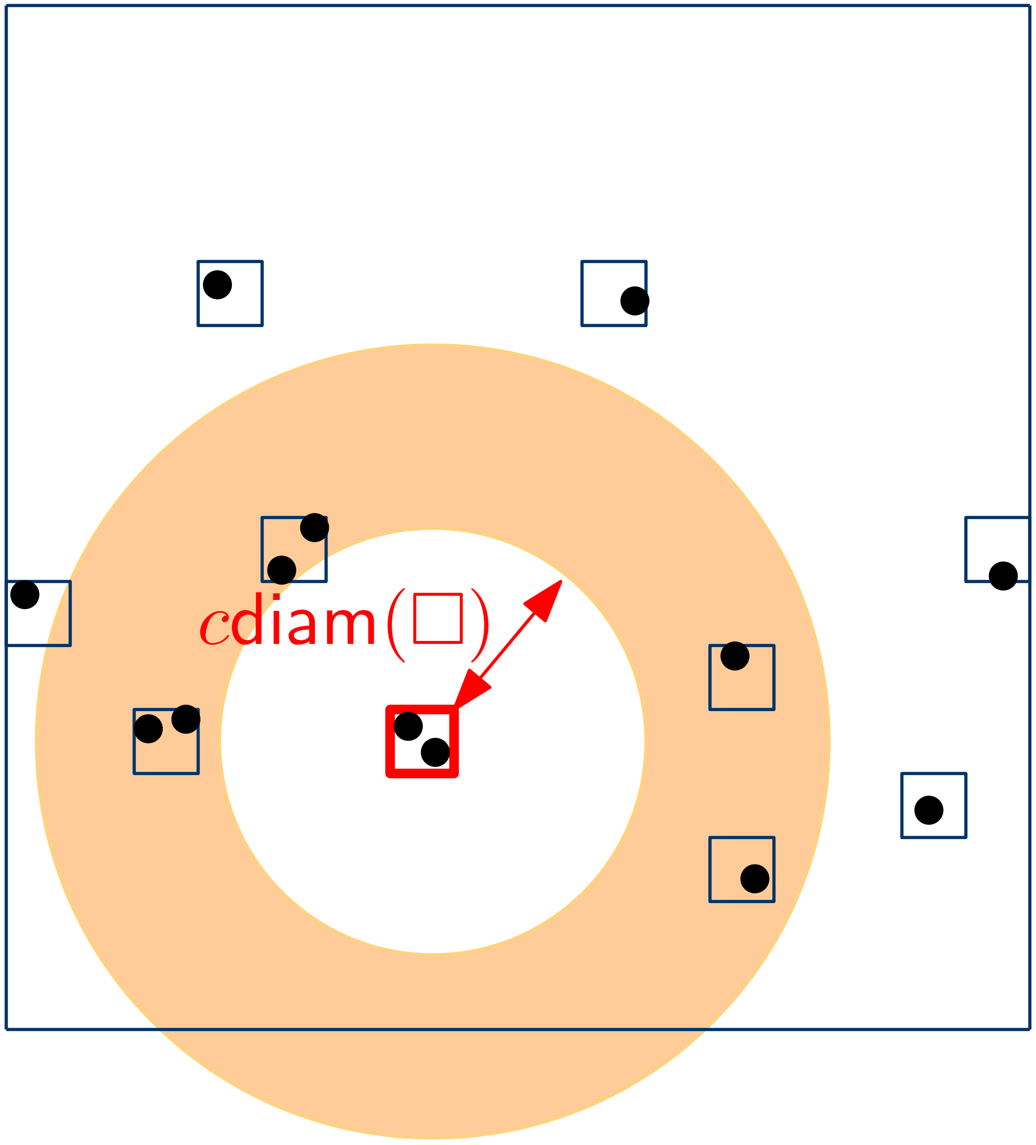

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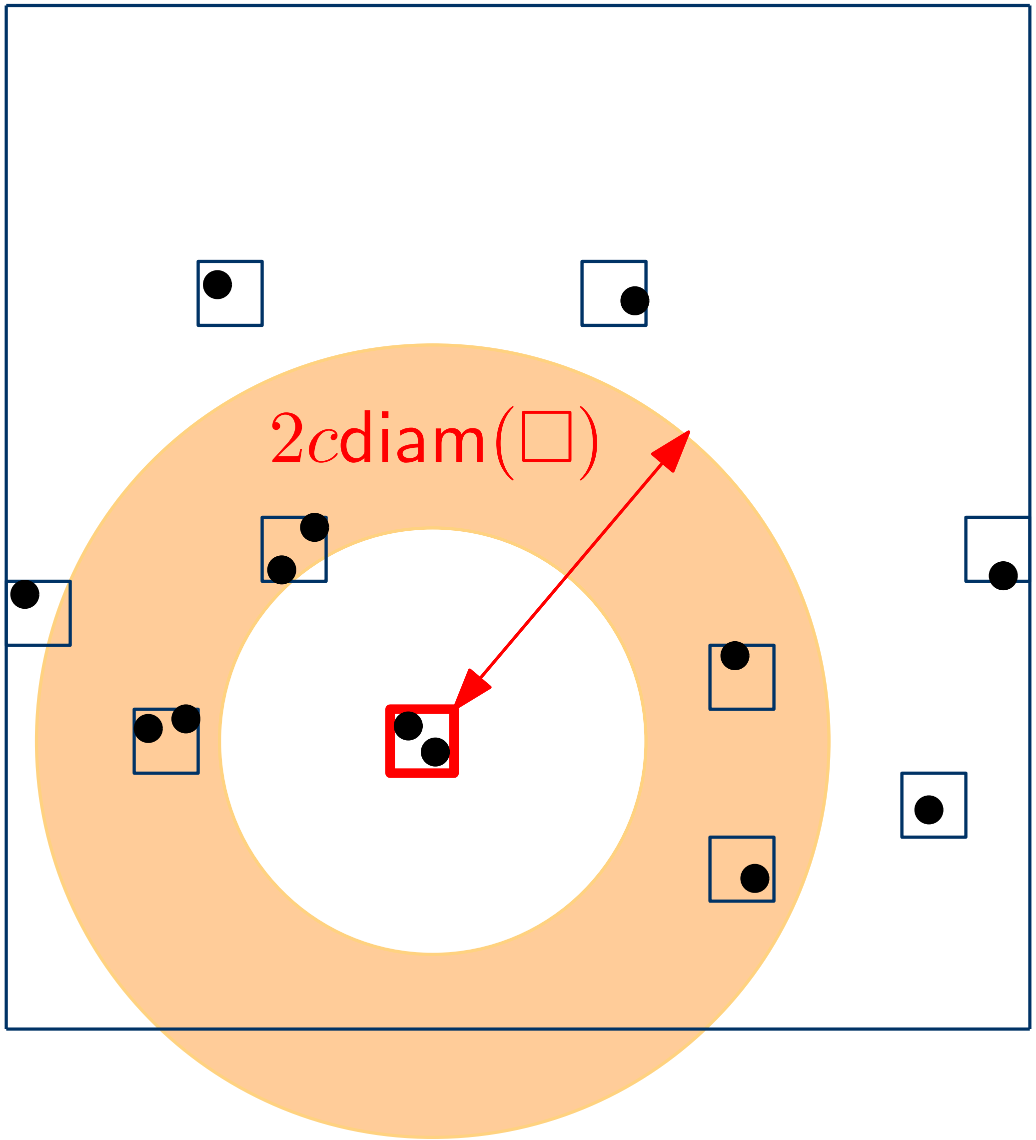

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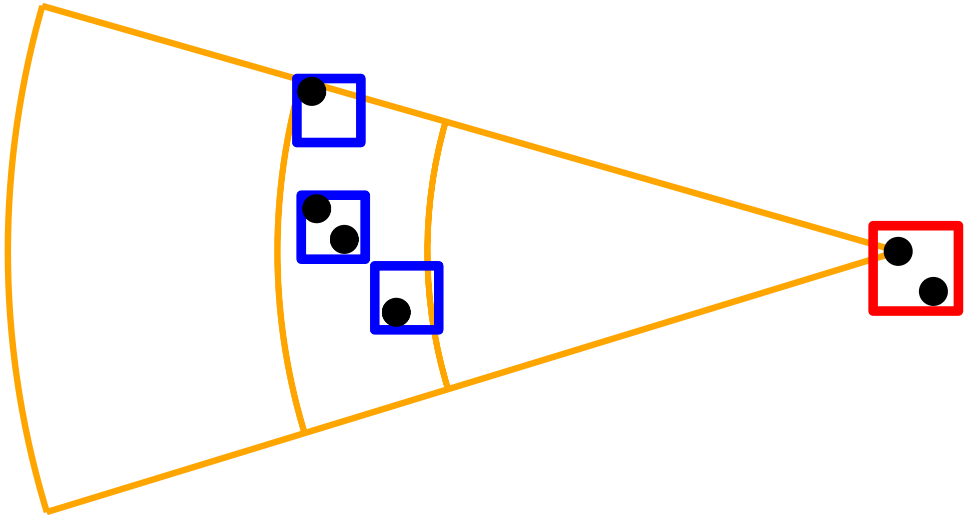

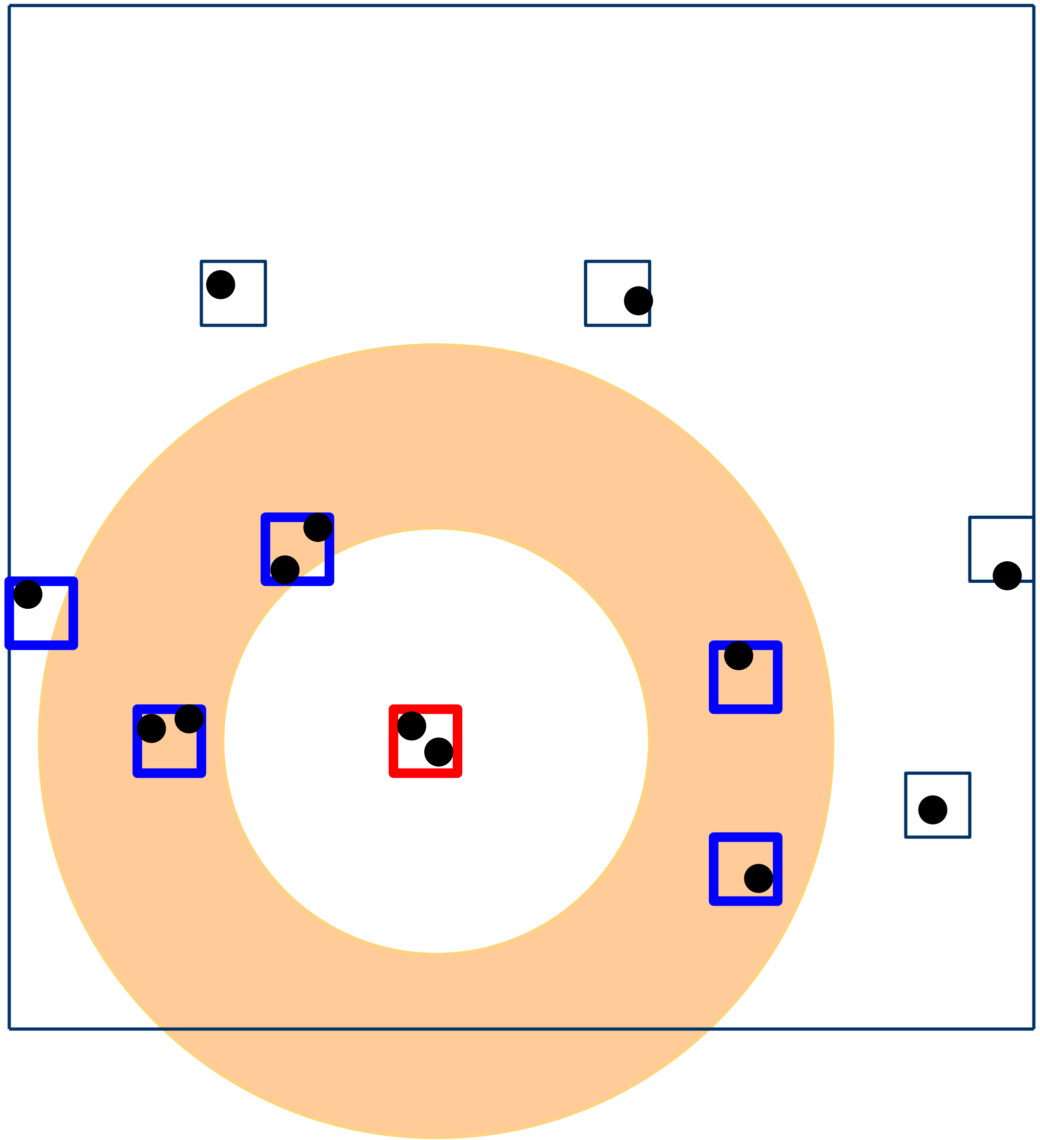

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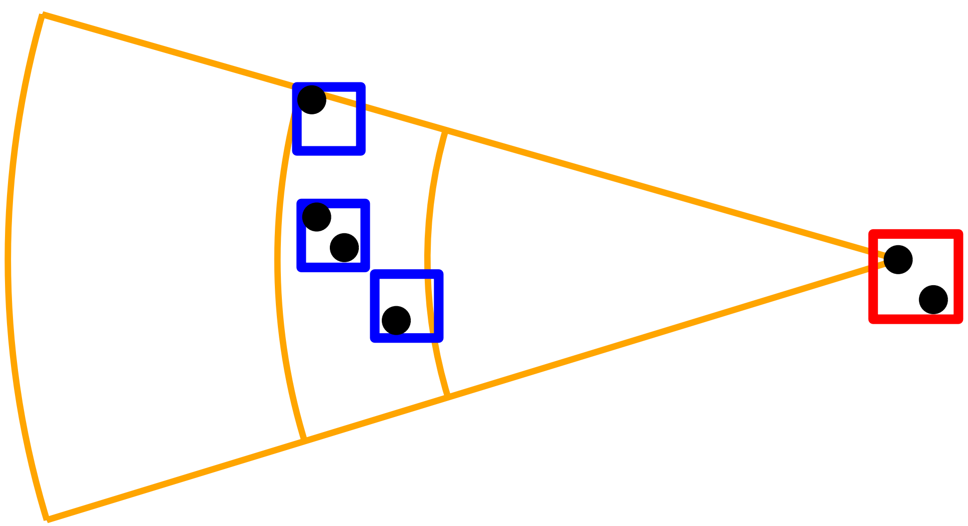

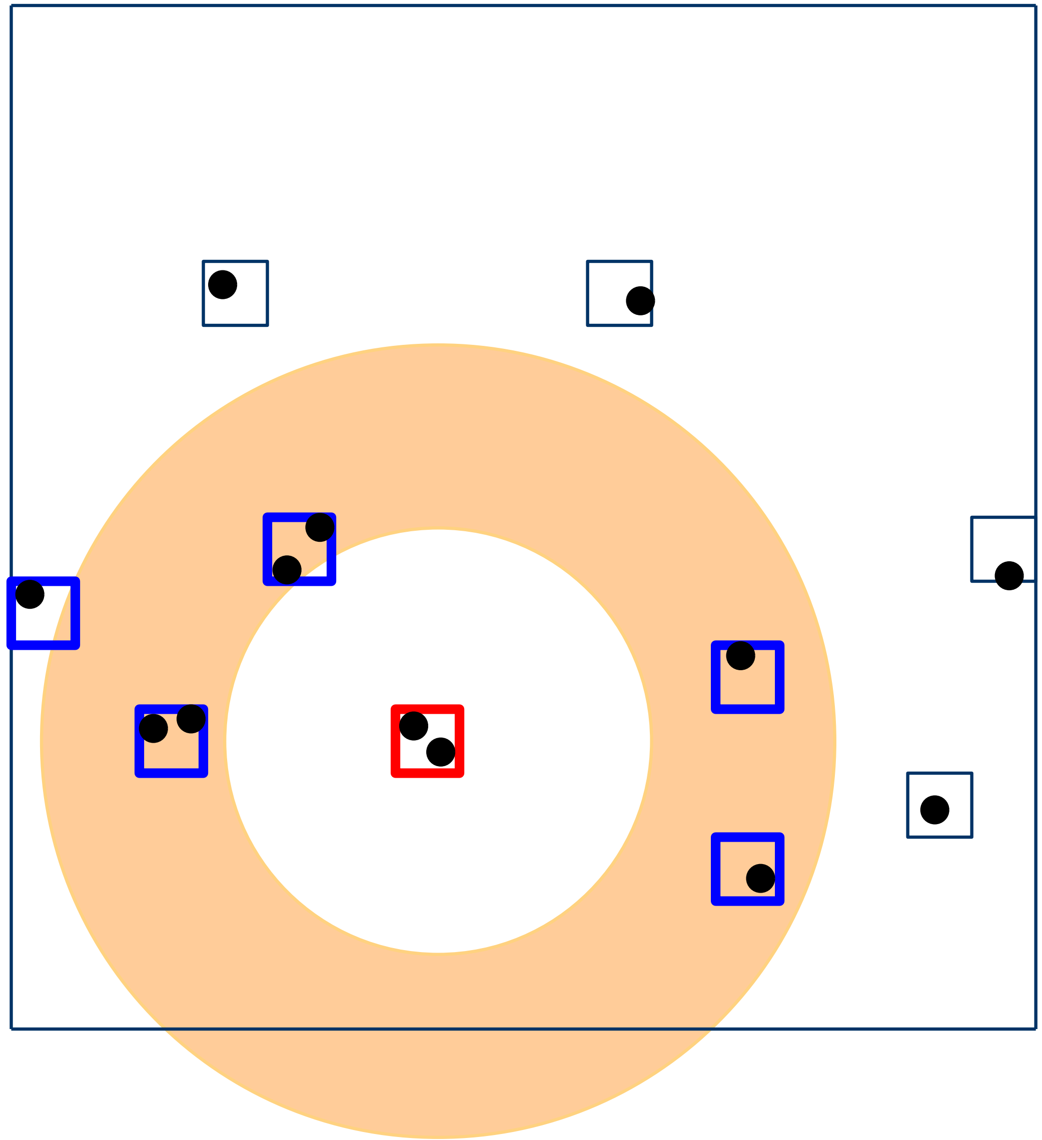

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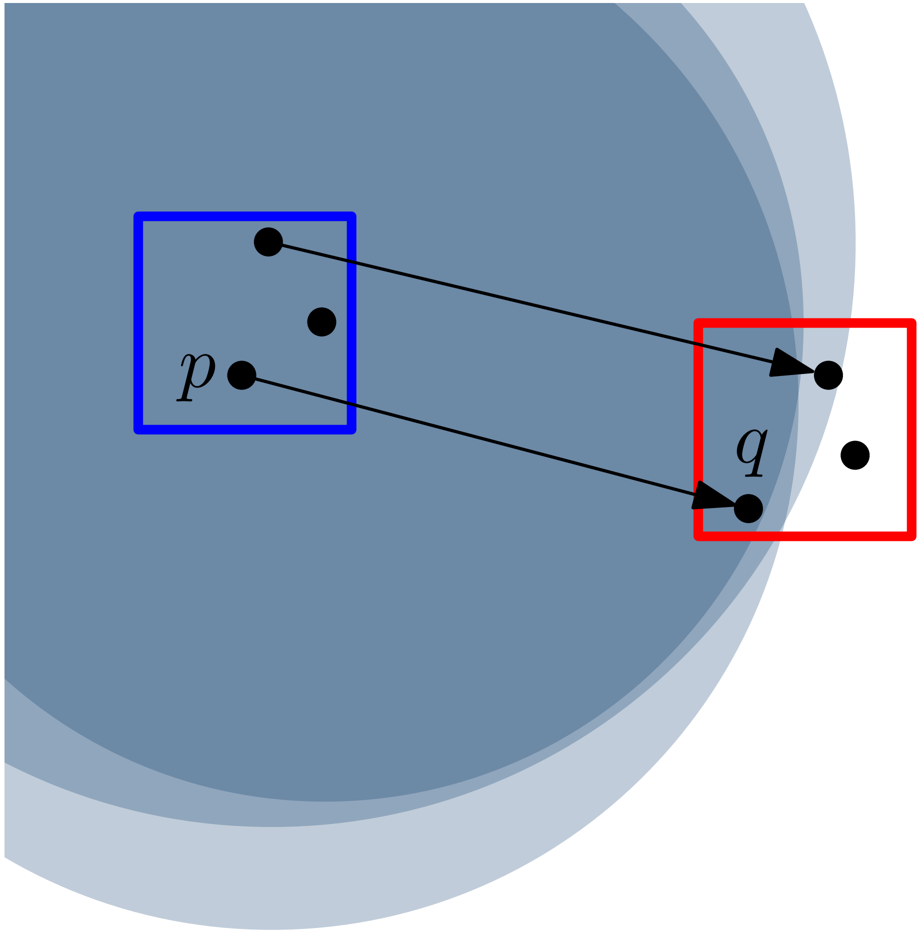

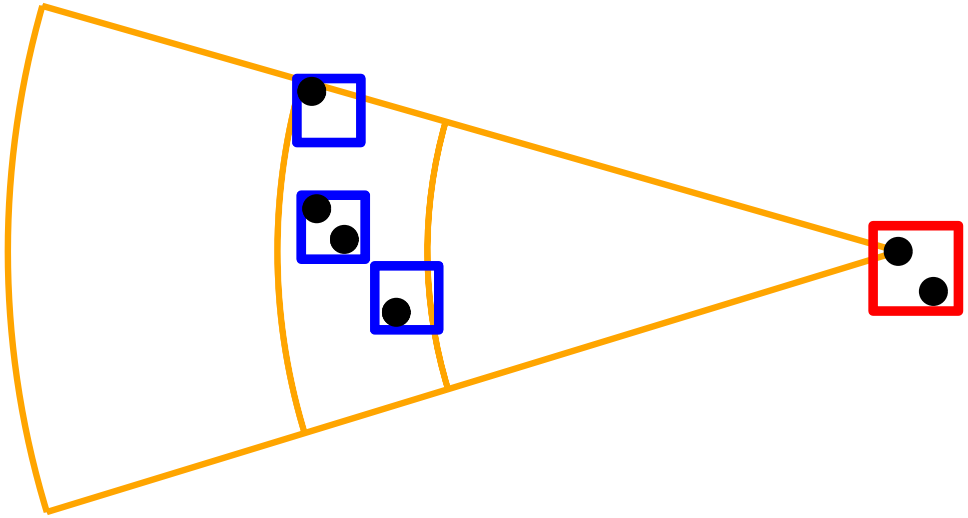

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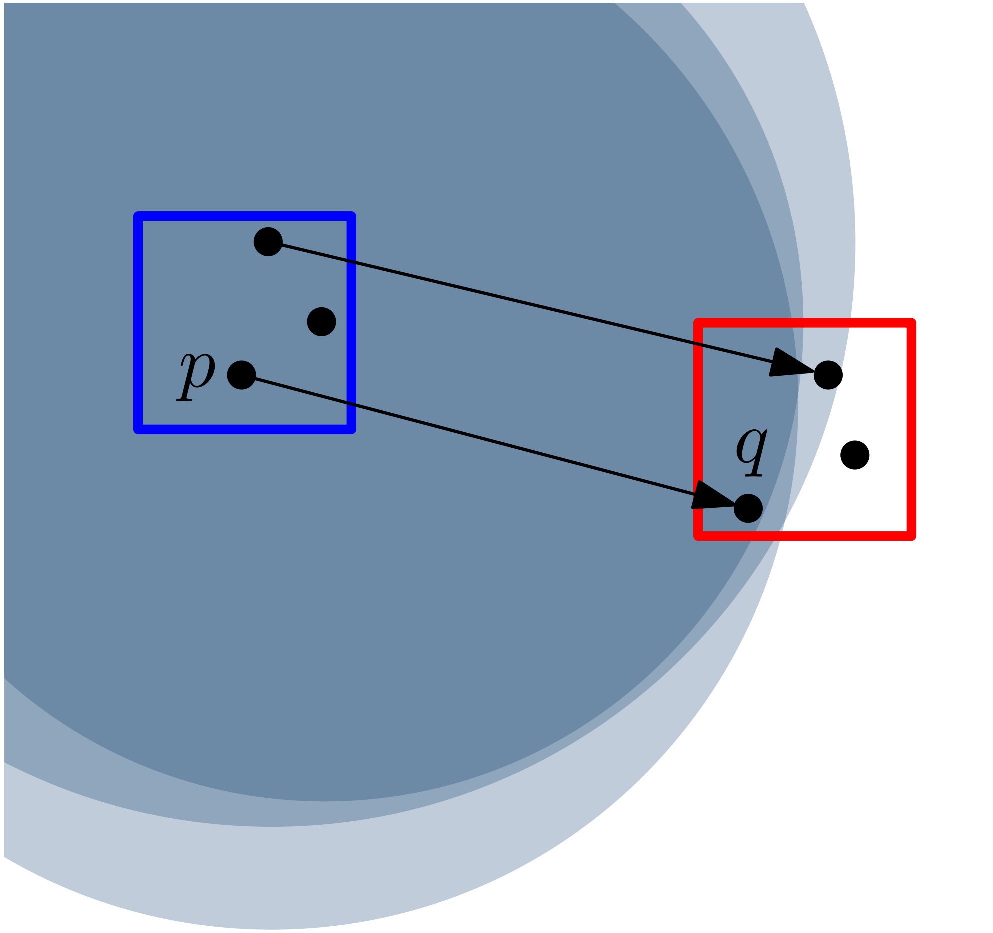

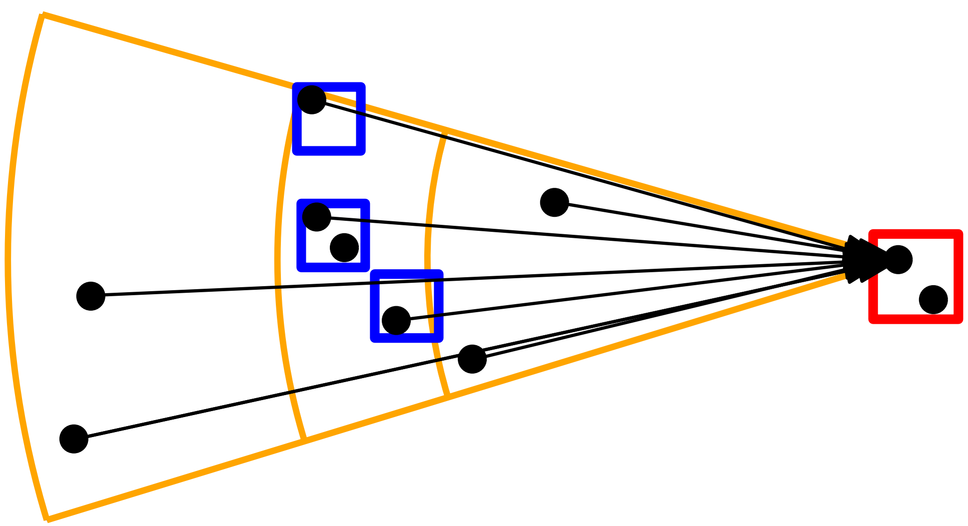

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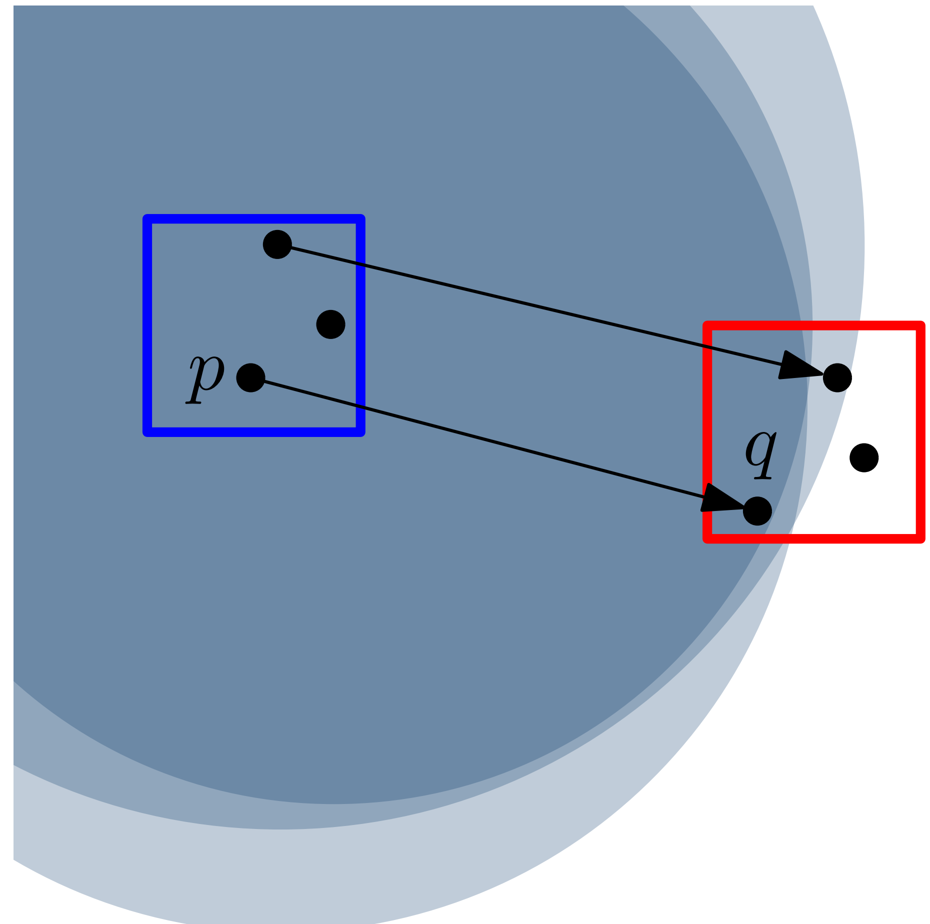

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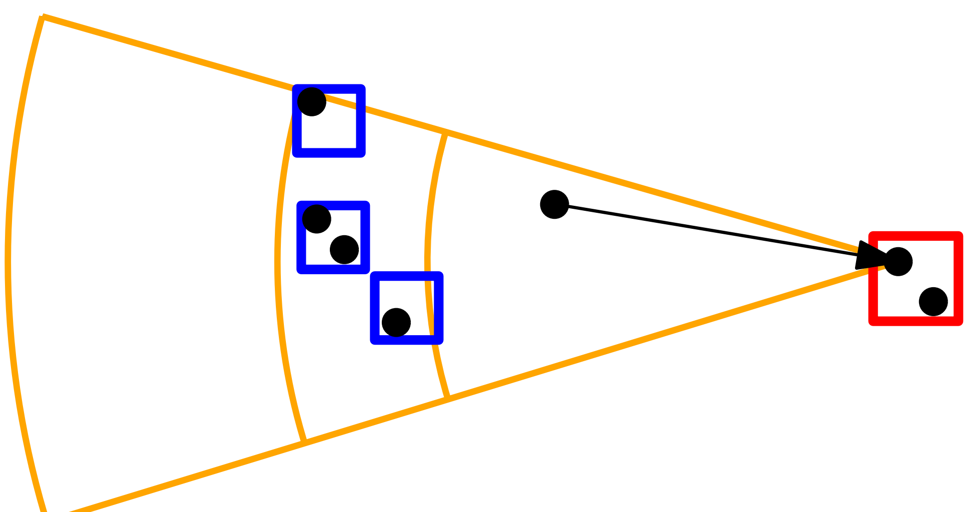

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Theorem 3: For any $t > 1$, we can compute a t-spanner in expected time $O(n \log^6 n)$.

Chan's dynamic NN datastructure

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Chan and Tsakalidis, SoCG '15

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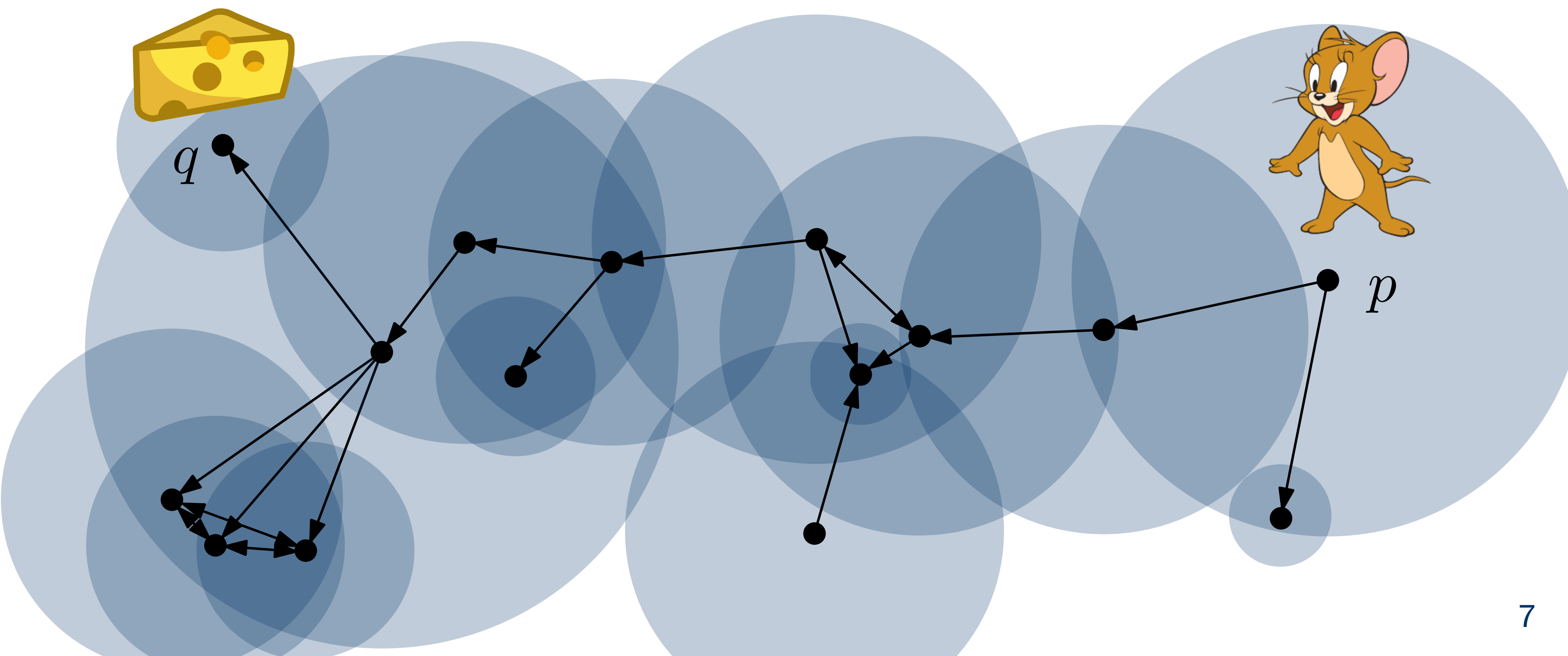

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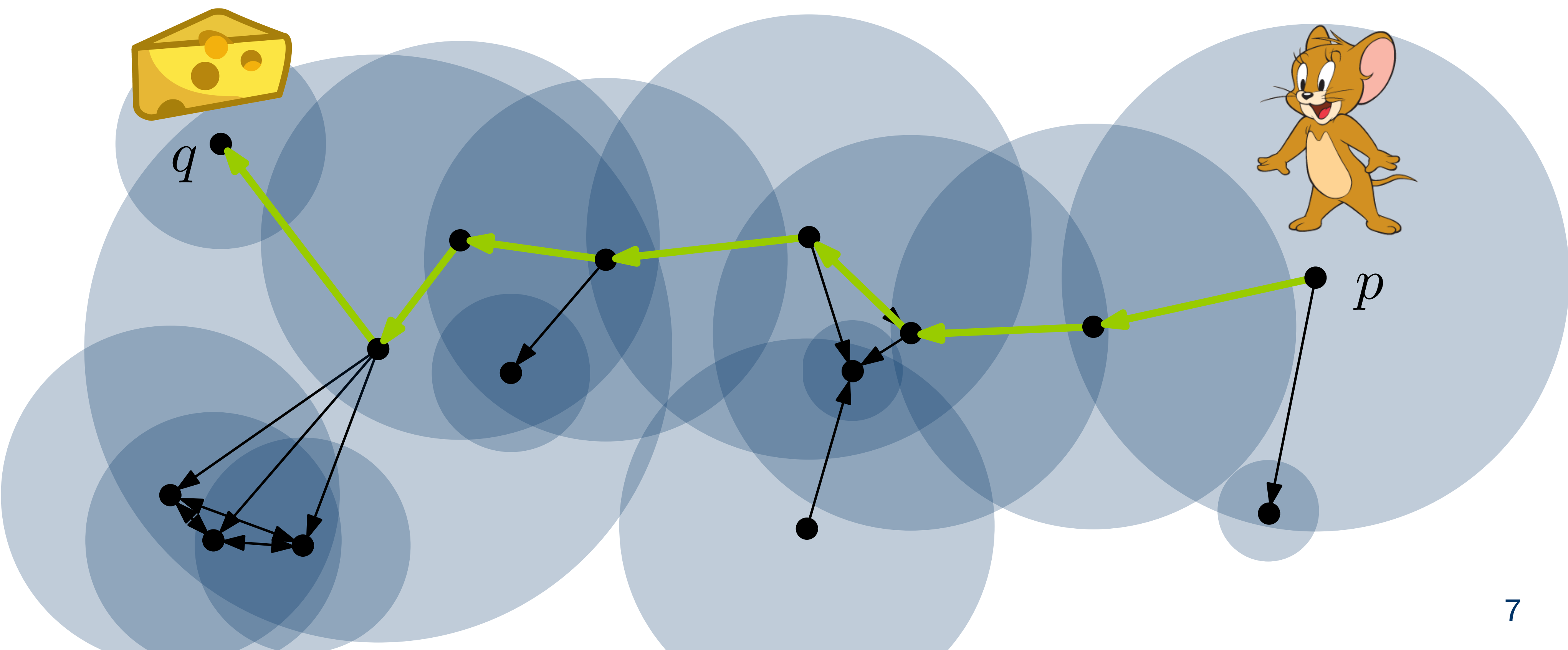

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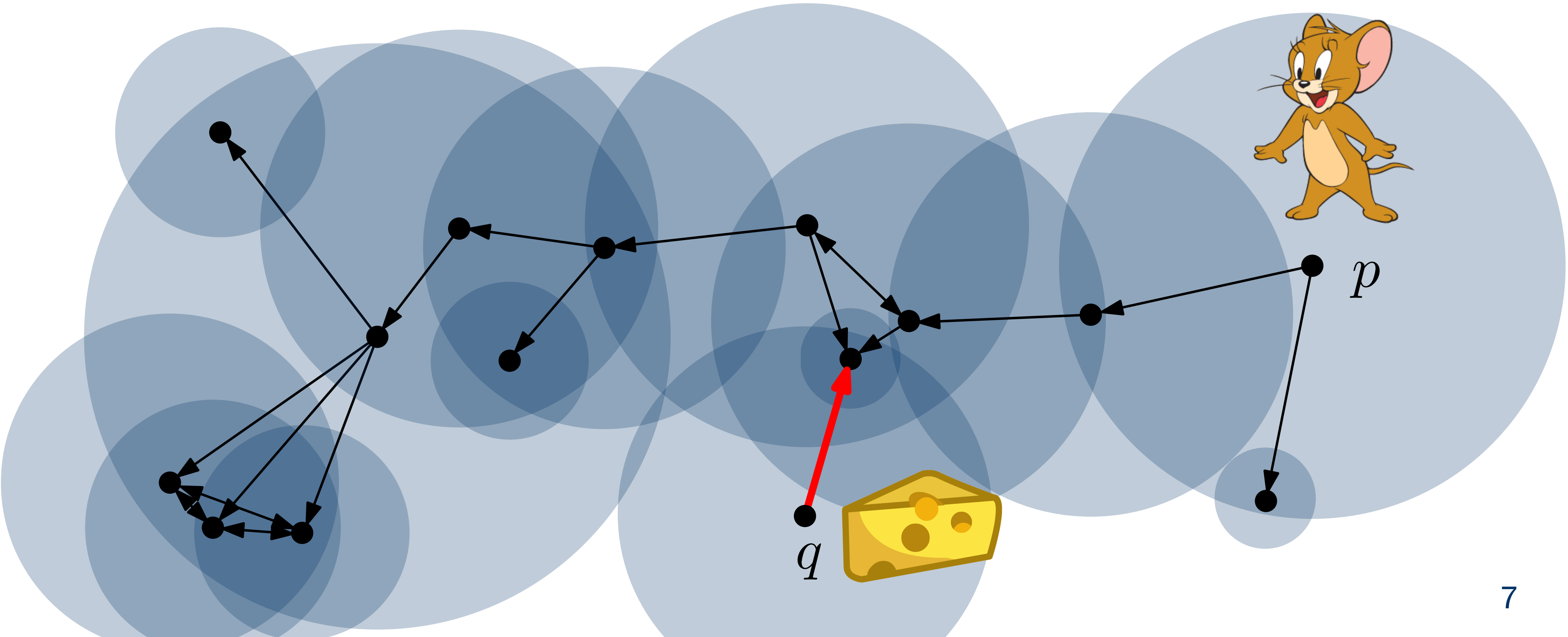

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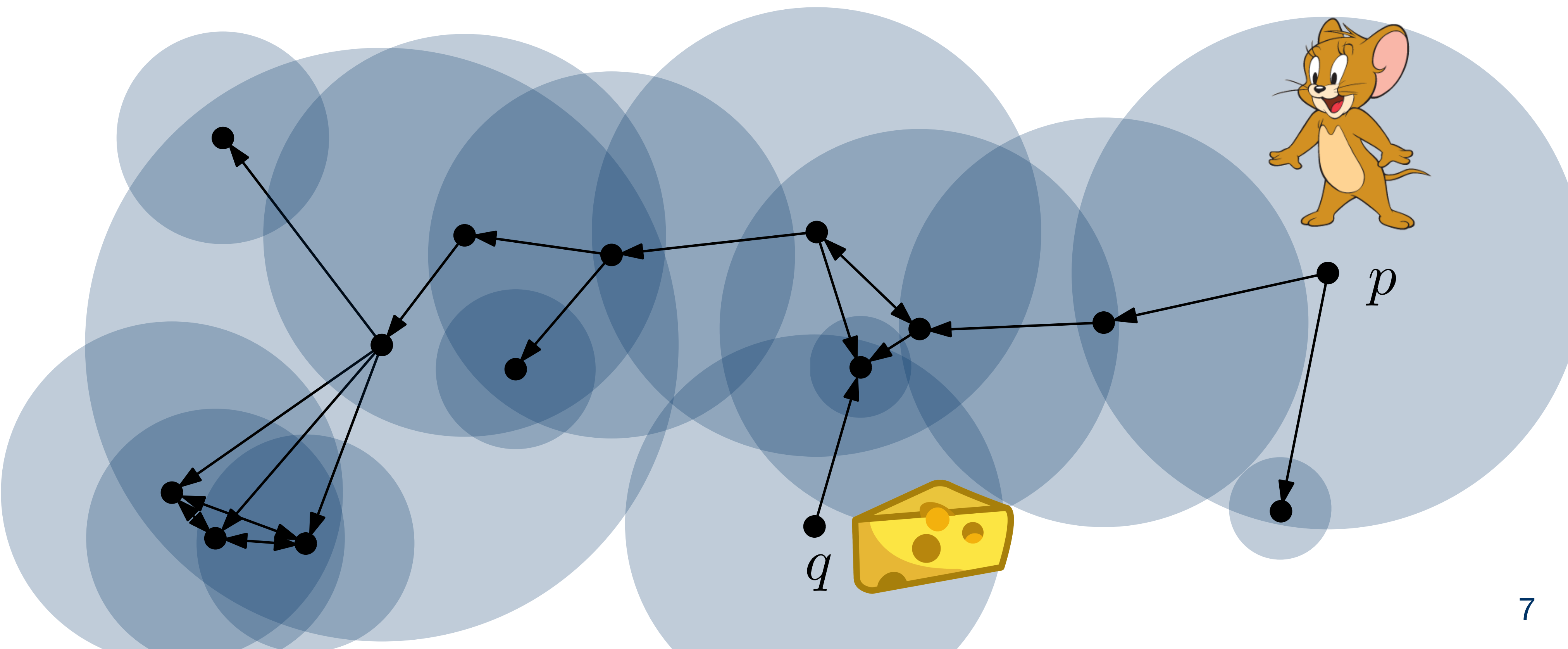
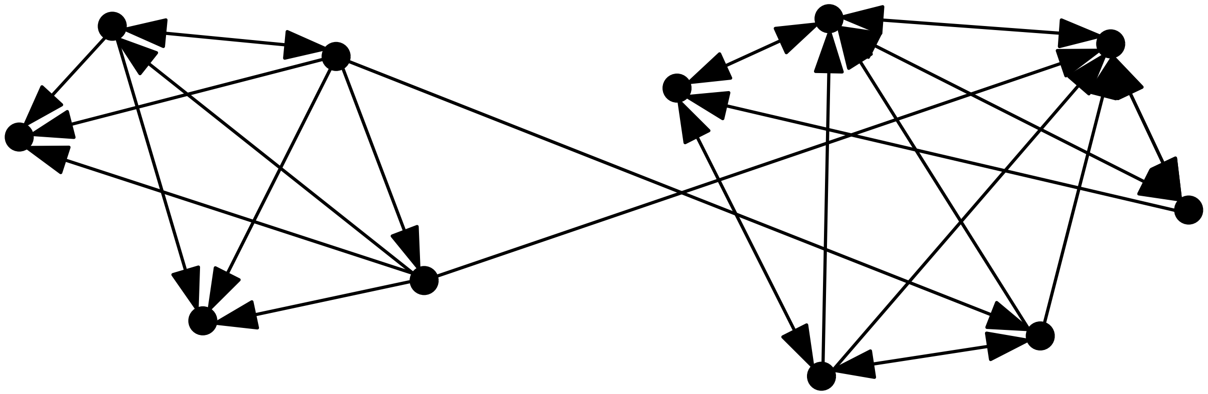
$S(n)$	$\Psi < \sqrt{3}$	$O(n)$	$O(\Psi^5 n^{3/2})$	$\Psi = O(n^c)$	$O(n^{5/3} \log n)$
$Q(n)$		$O(1)$	$O(\Psi^3 \sqrt{n})$		$O(n^{2/3} \log n)$



Reachability Oracles: $\Psi < \sqrt{3}$

- G transmission graph



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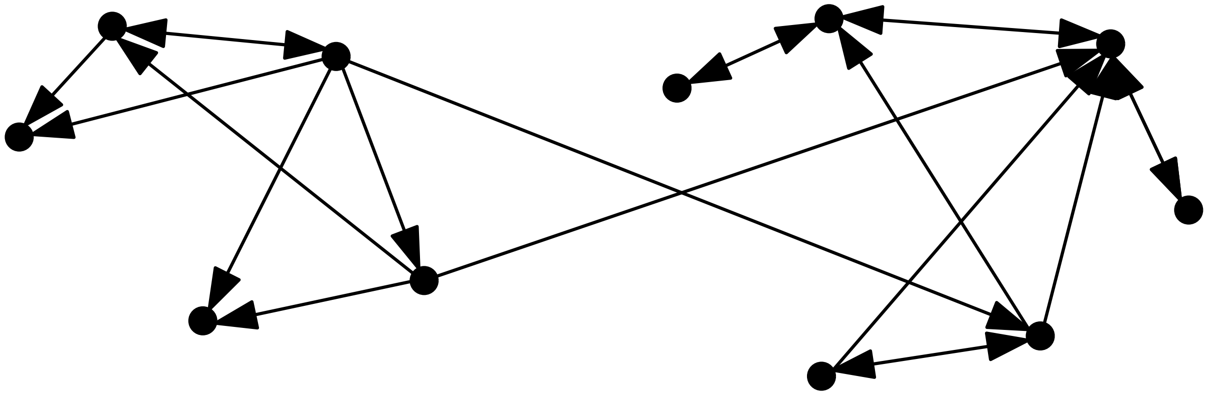
- $H \subseteq G$ with $O(n)$ edges and $O(n)$ crossings



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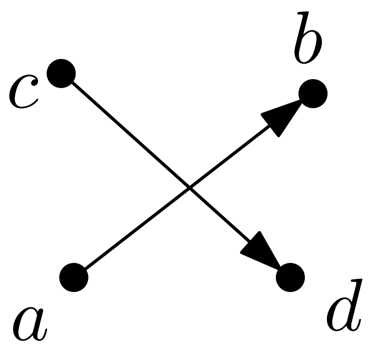

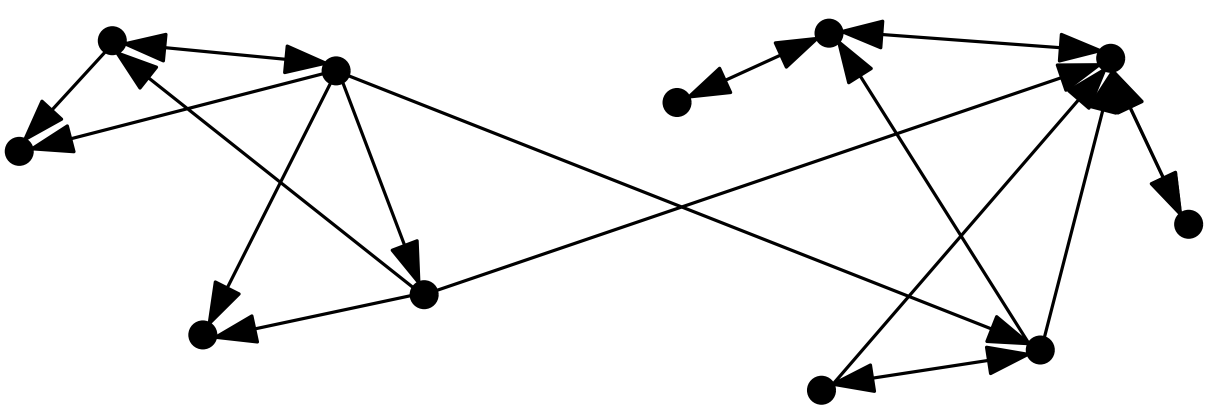

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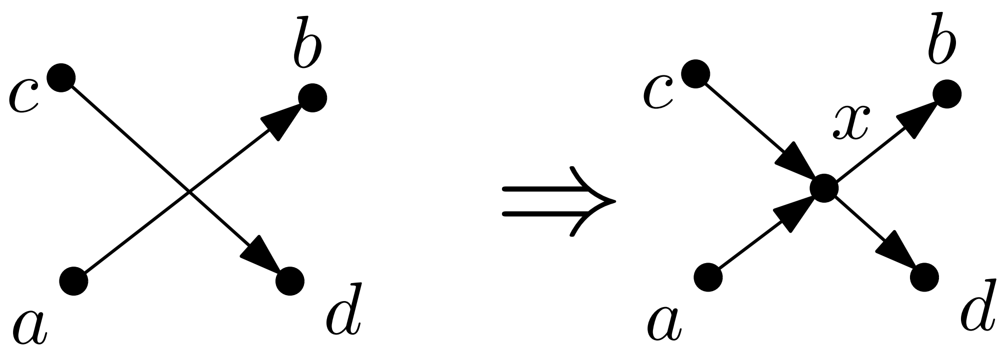

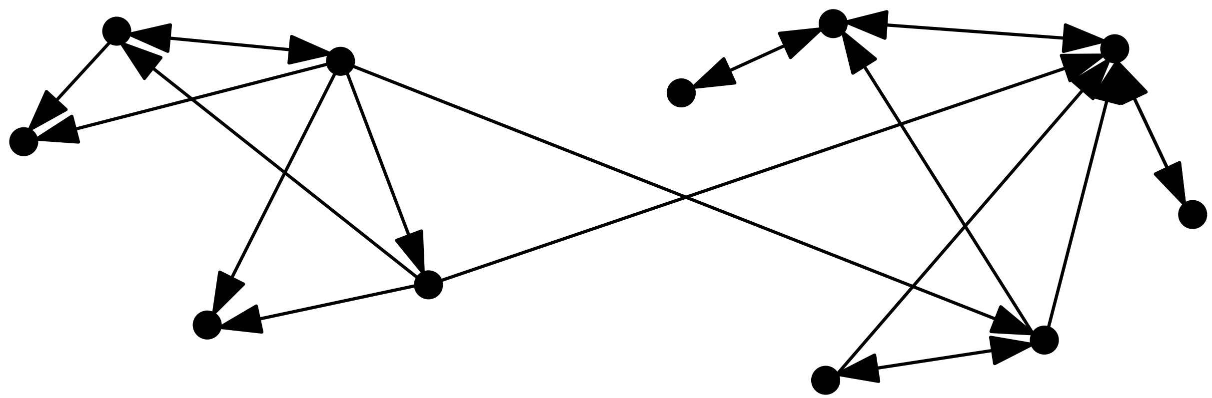

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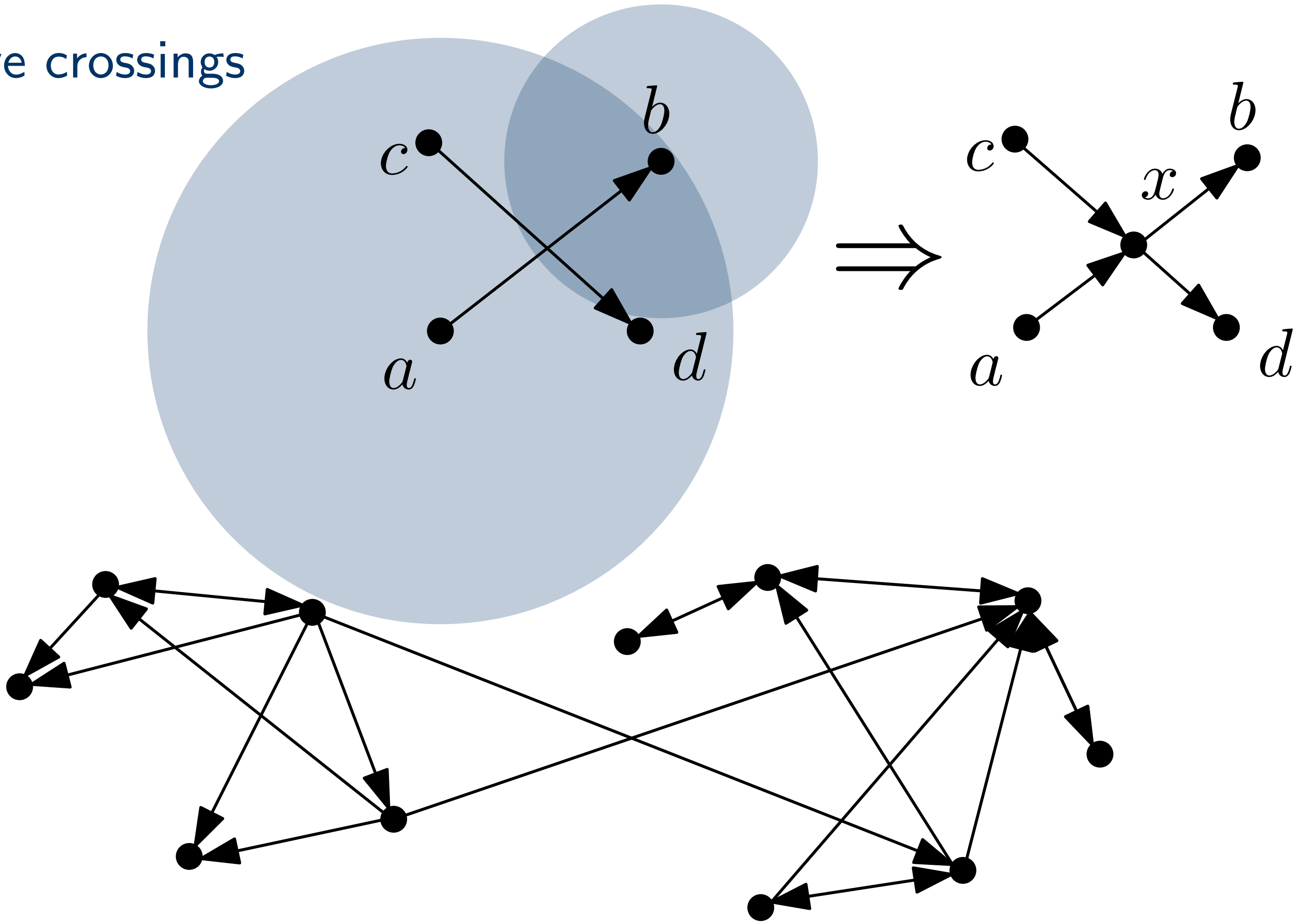

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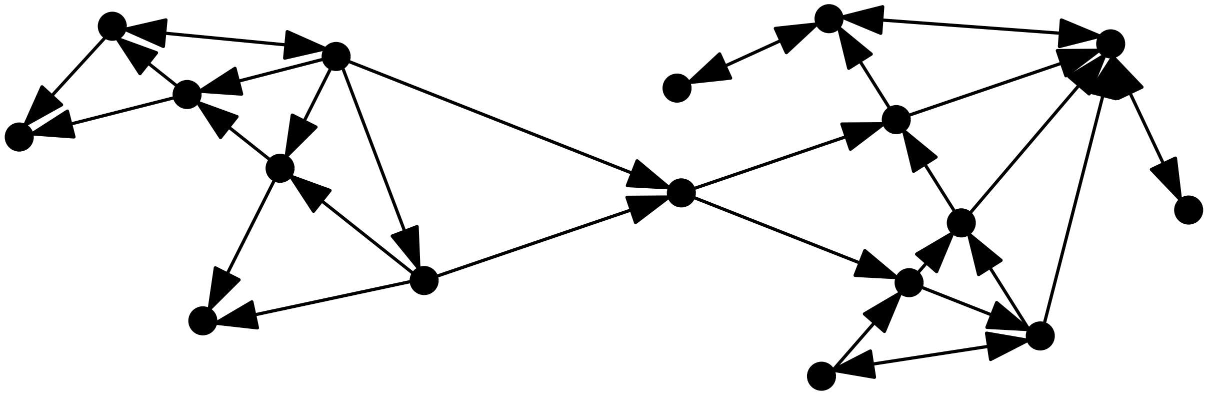
- $\bar{H}$: planar, $O(n)$ edges, same reachability as G



Reachability Oracles: $\Psi < \sqrt{3}$

- G transmission graph

$\Downarrow$ sparsification

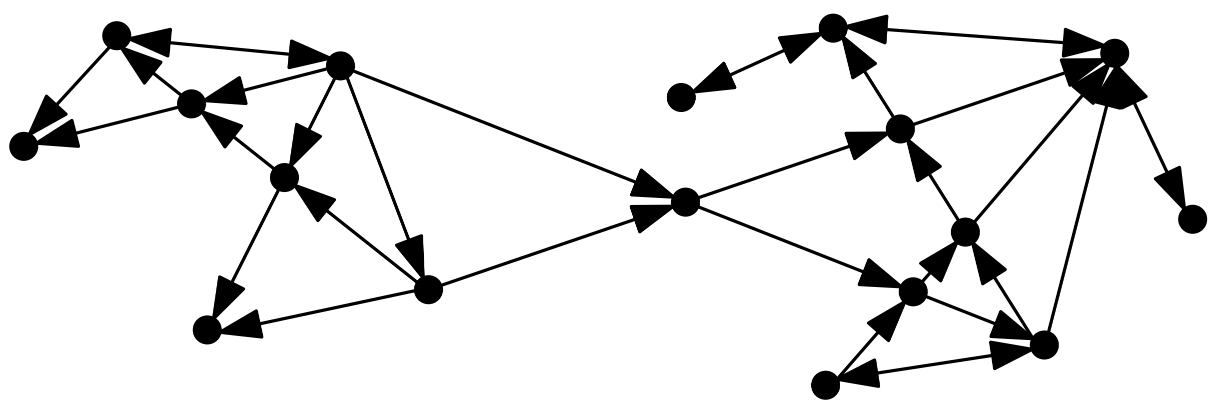
- $H \subseteq G$ with $O(n)$ edges and $O(n)$ crossings

$\Downarrow$ resolve crossings

- $\bar{H}$: planar, $O(n)$ edges, same reachability as G

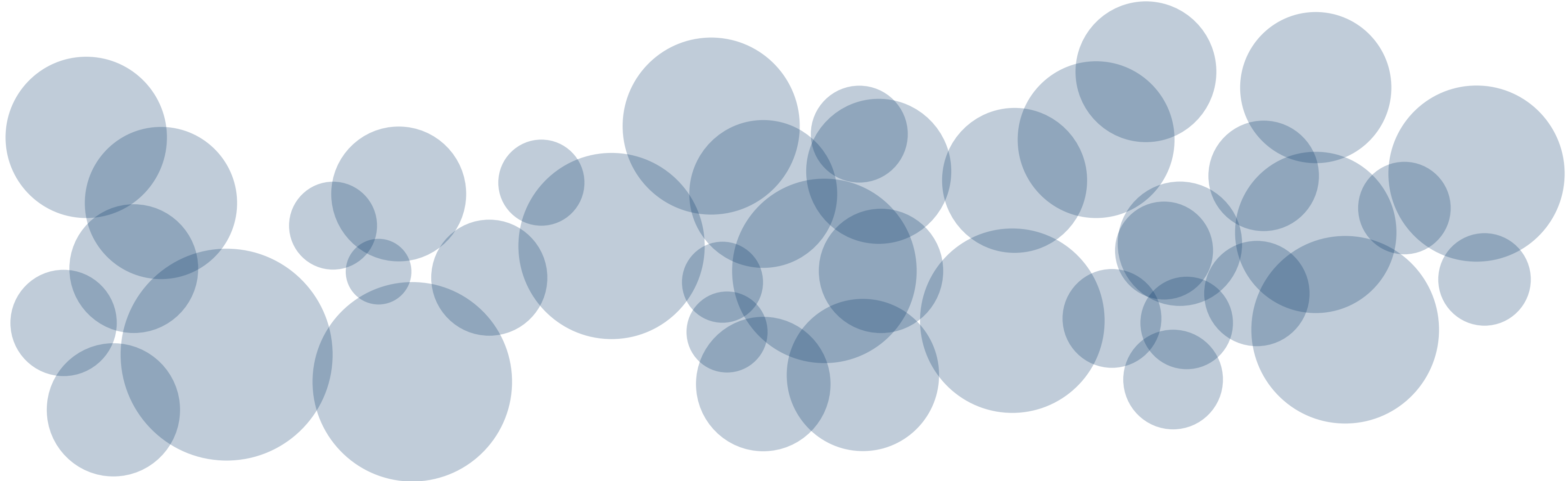
apply →

Theorem (Holm, Rotenberg, Thorup): We can compute a reachability oracle for planar graphs with $S(n) = O(n)$ and $Q(n) = O(1)$.



Reachability Oracles: Ψ small

Theorem (Alber, Fiala): Let $\mathcal{D}$ be a set of disks. We can find a seperating set $\mathcal{S} \subseteq \mathcal{D}$ with area $A(\mathcal{S}) = O(\Psi^2 \sqrt{A(\mathcal{D})})$



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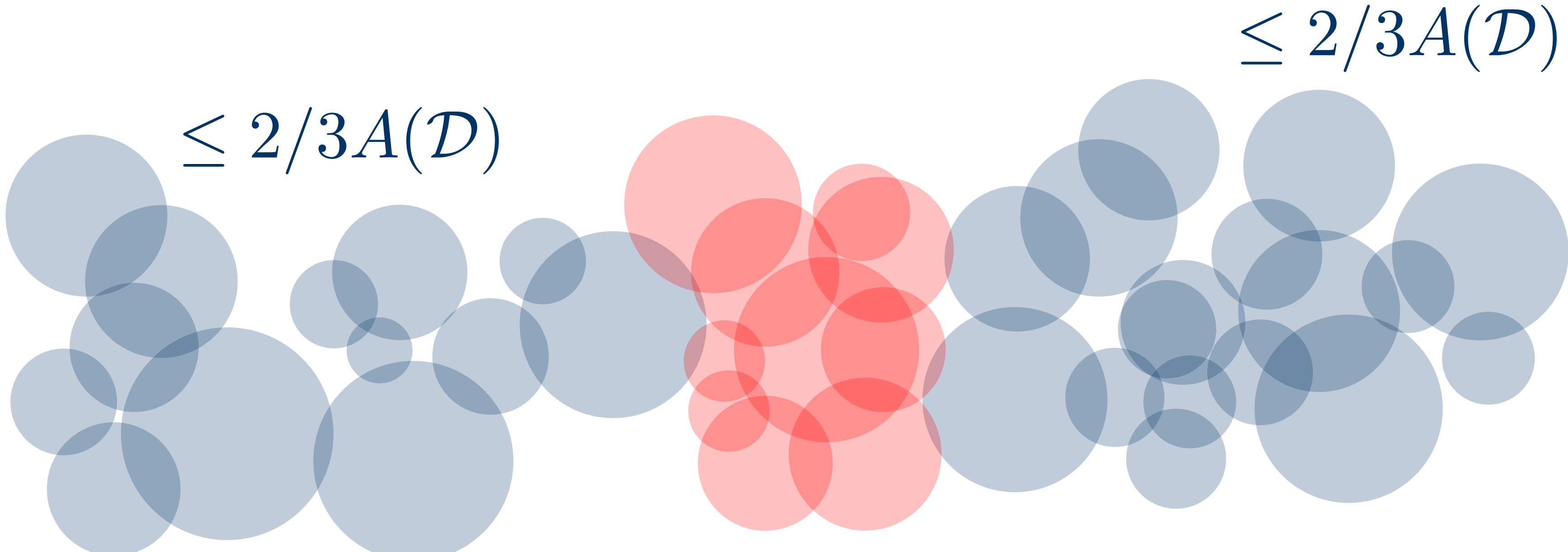

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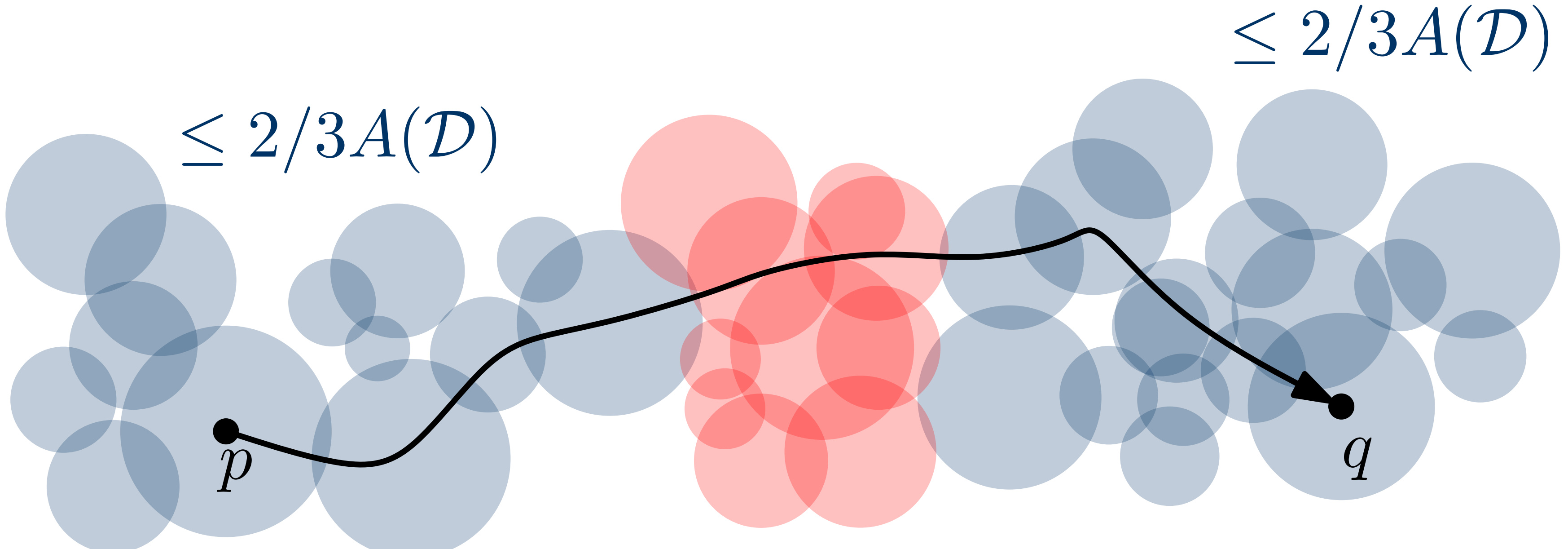

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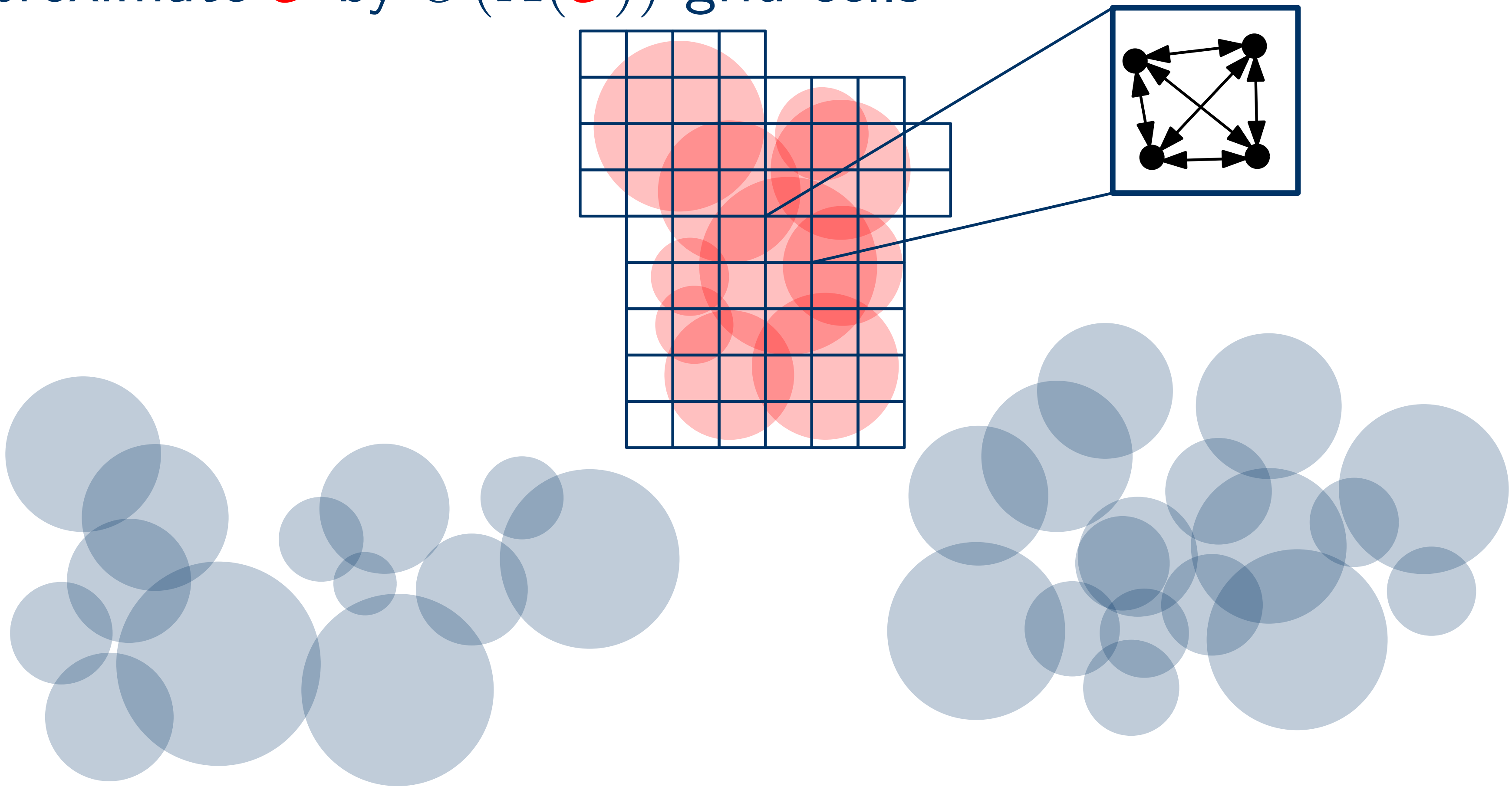

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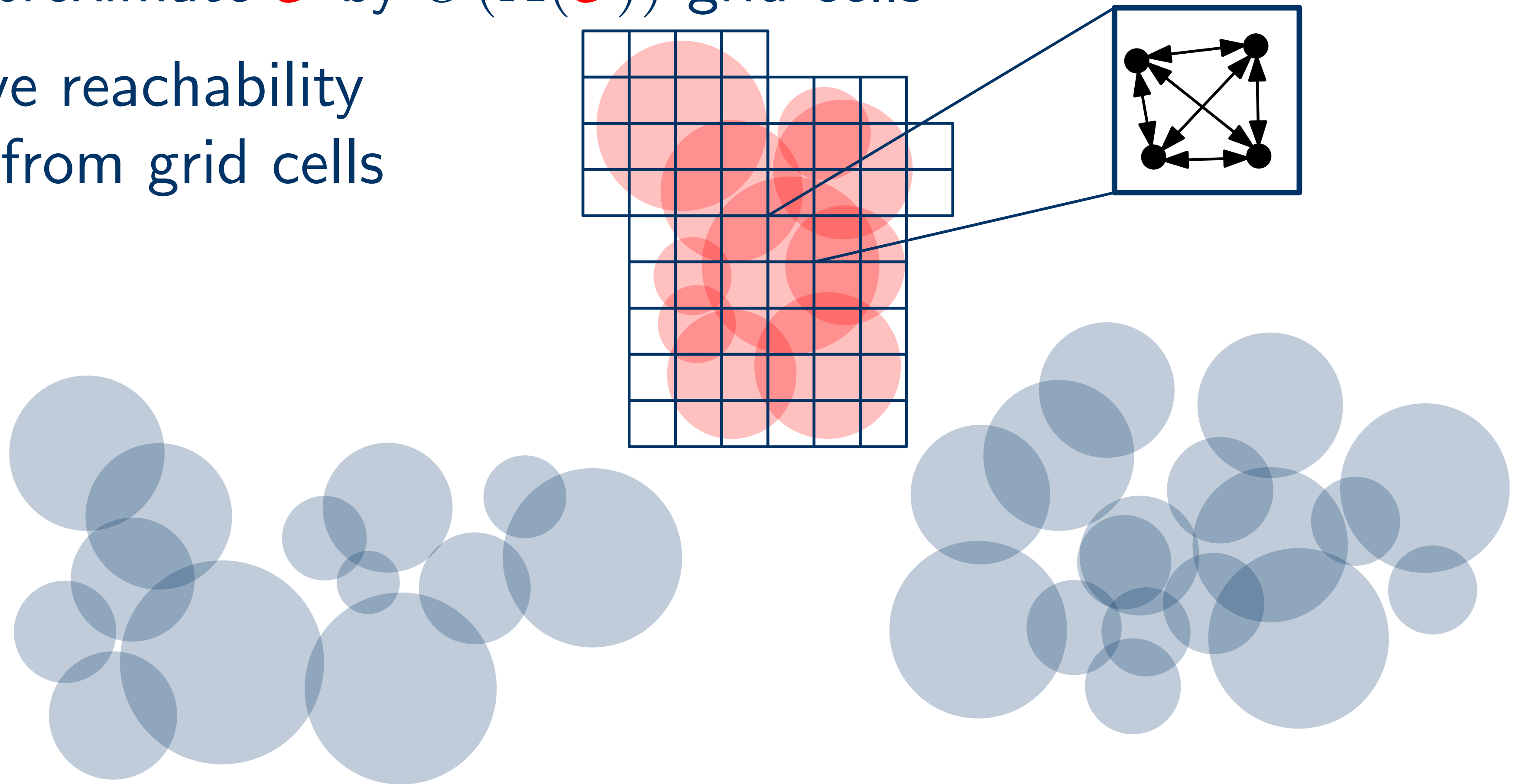

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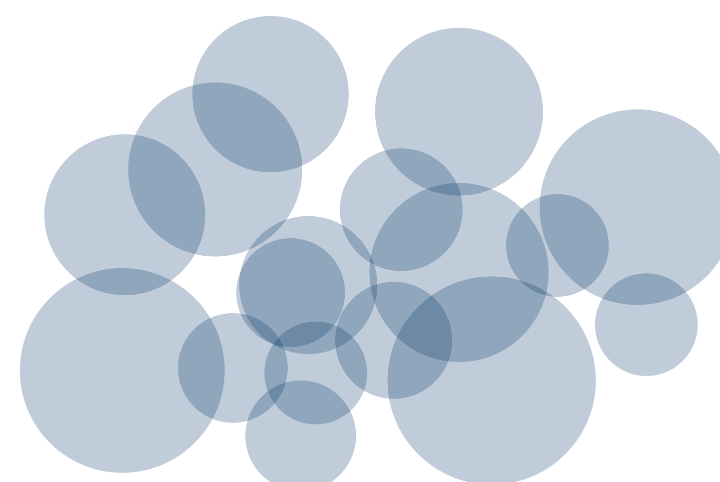

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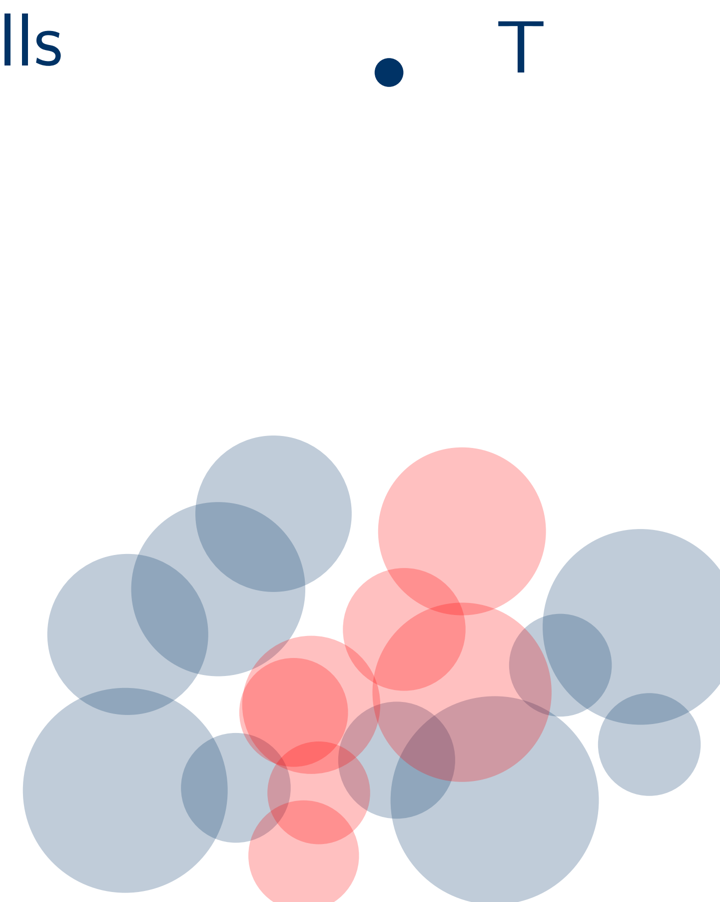

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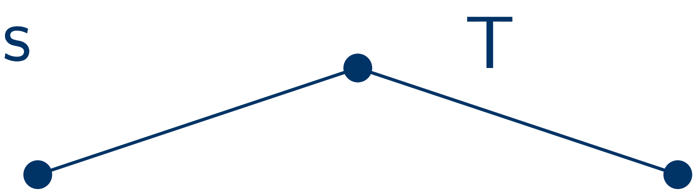

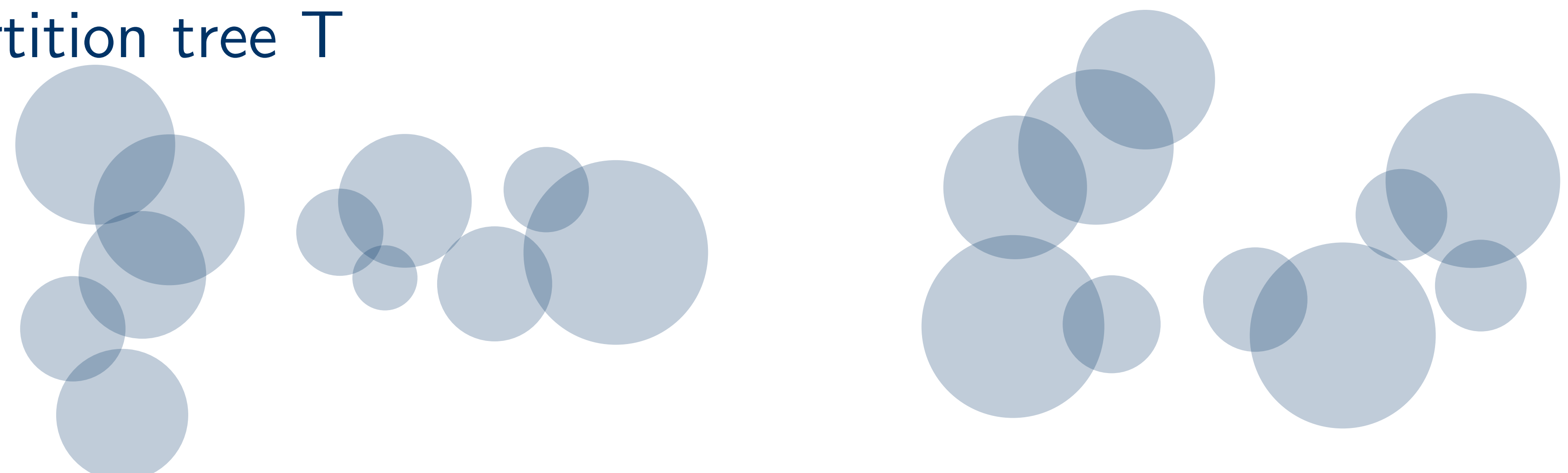

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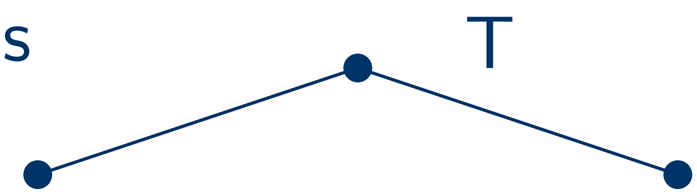

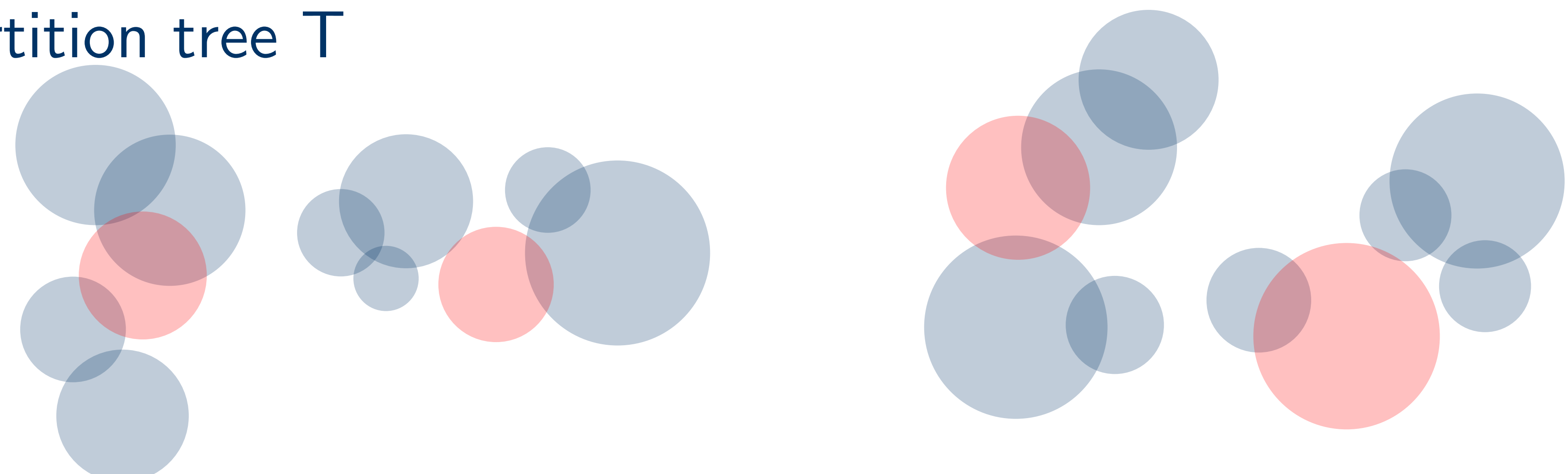

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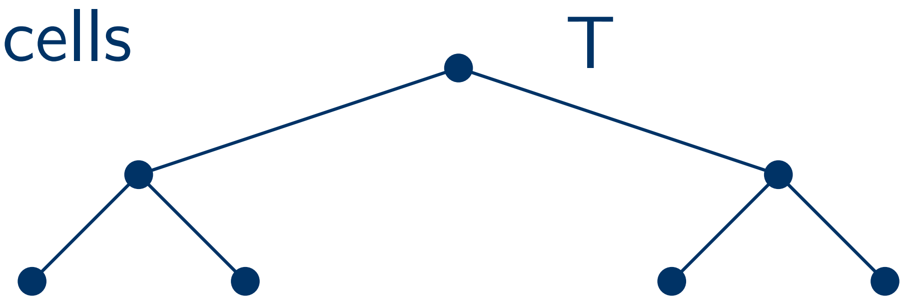

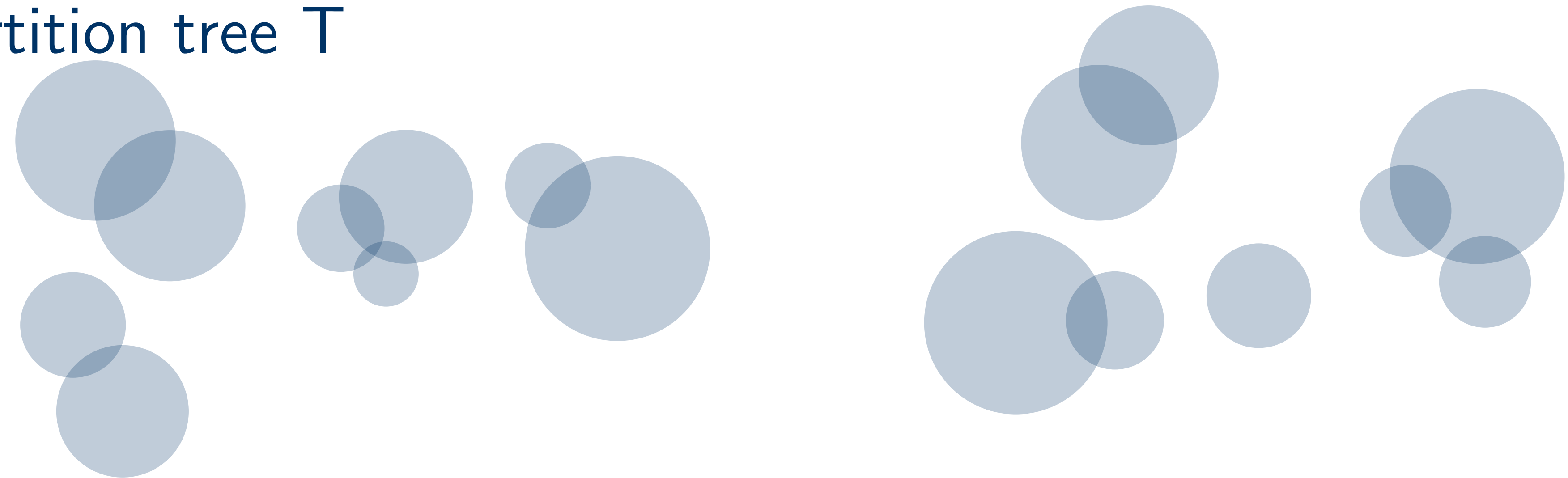

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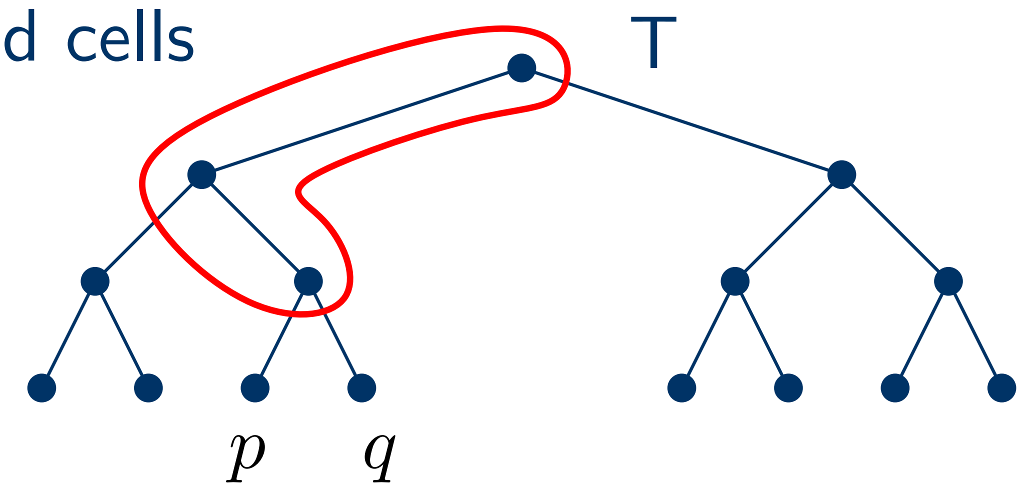
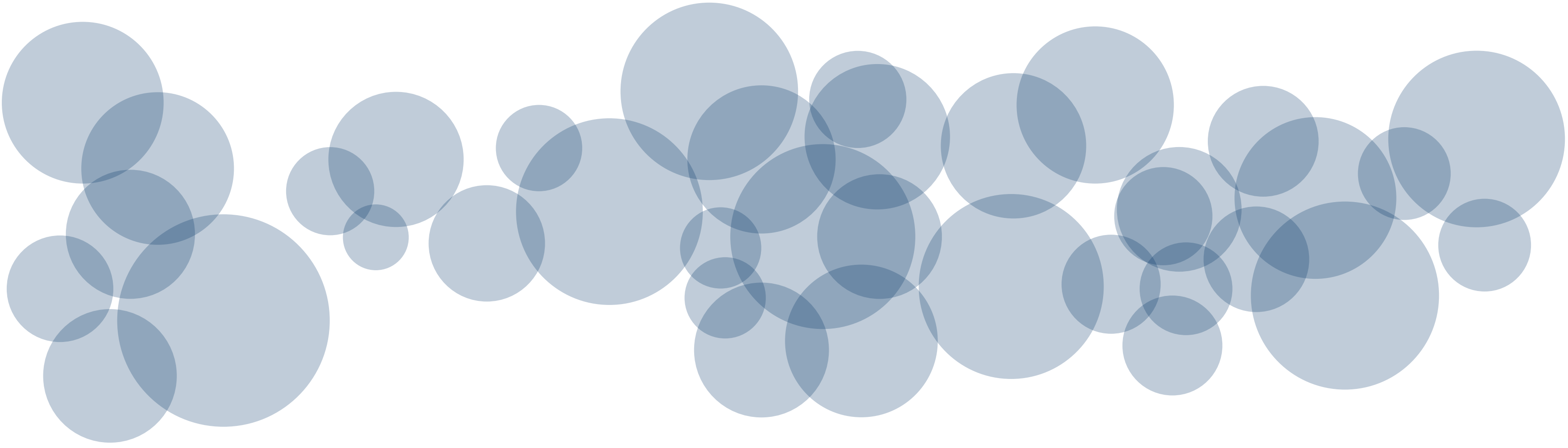


Reachability Oracles: $\Psi = O(n^c)$

Long paths:

Short paths:



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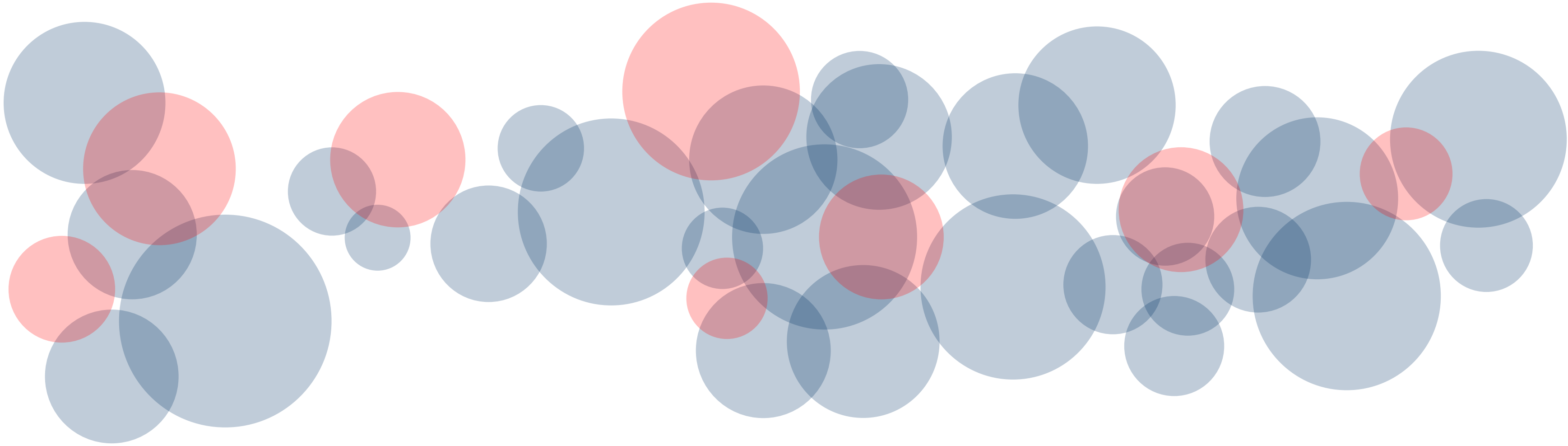

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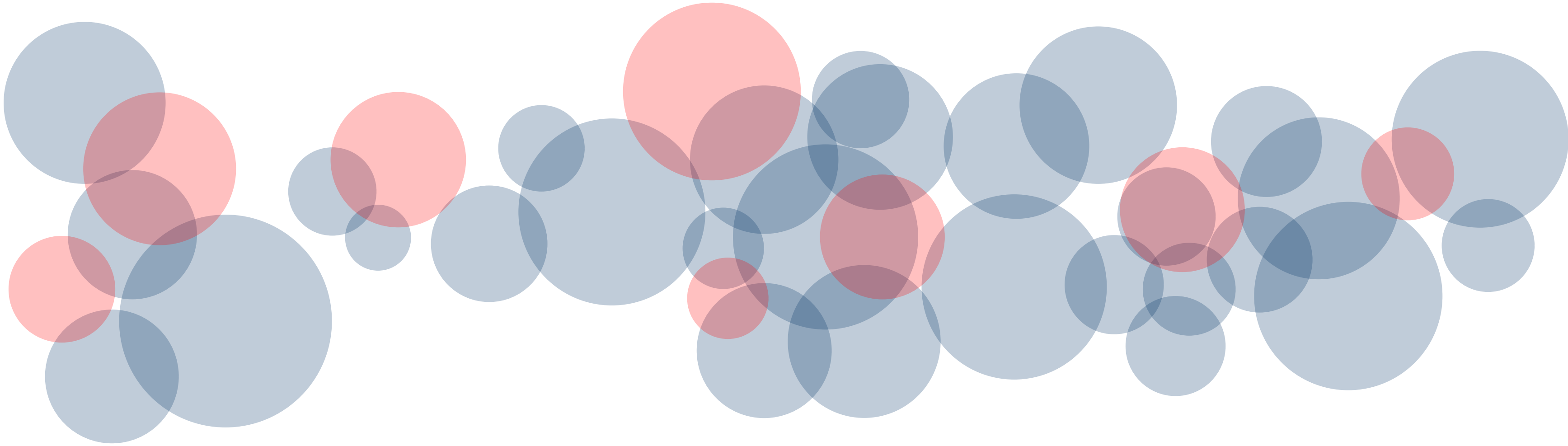

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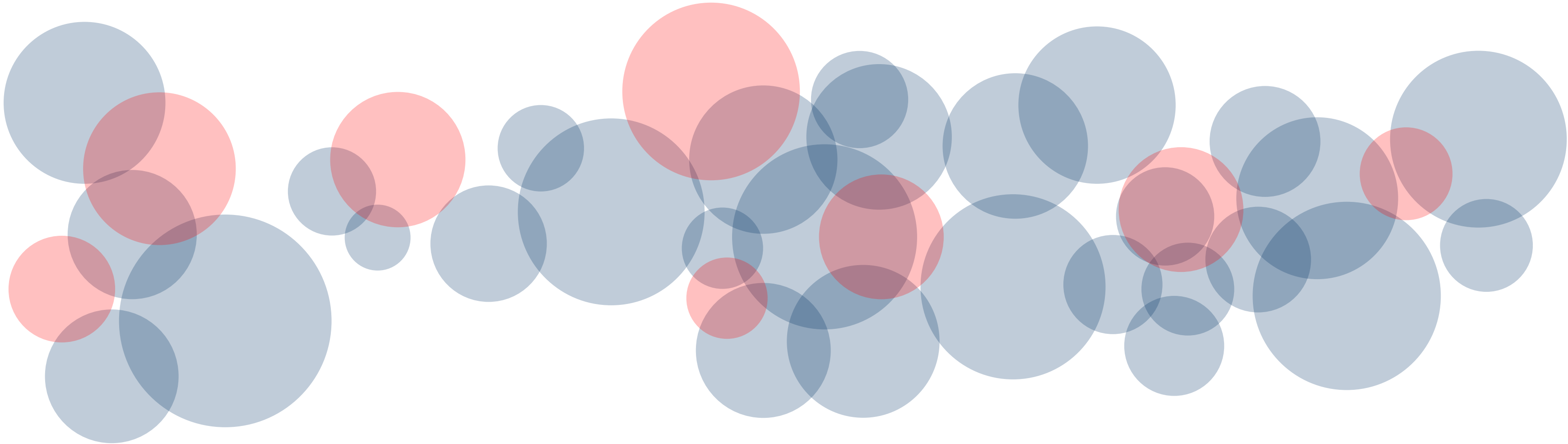

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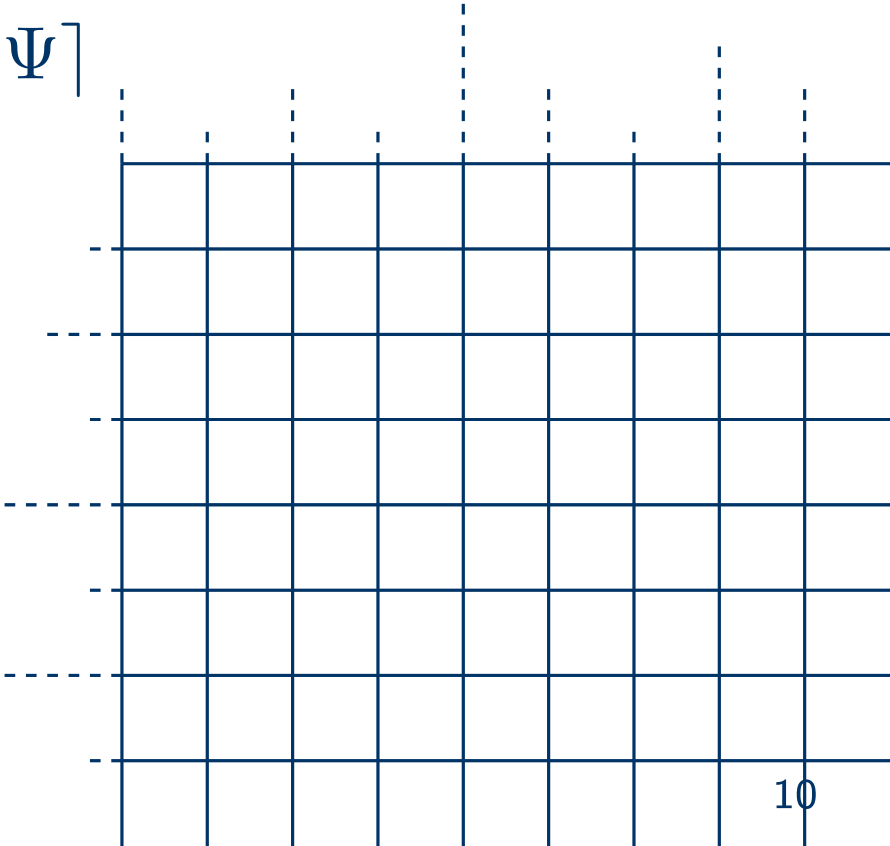

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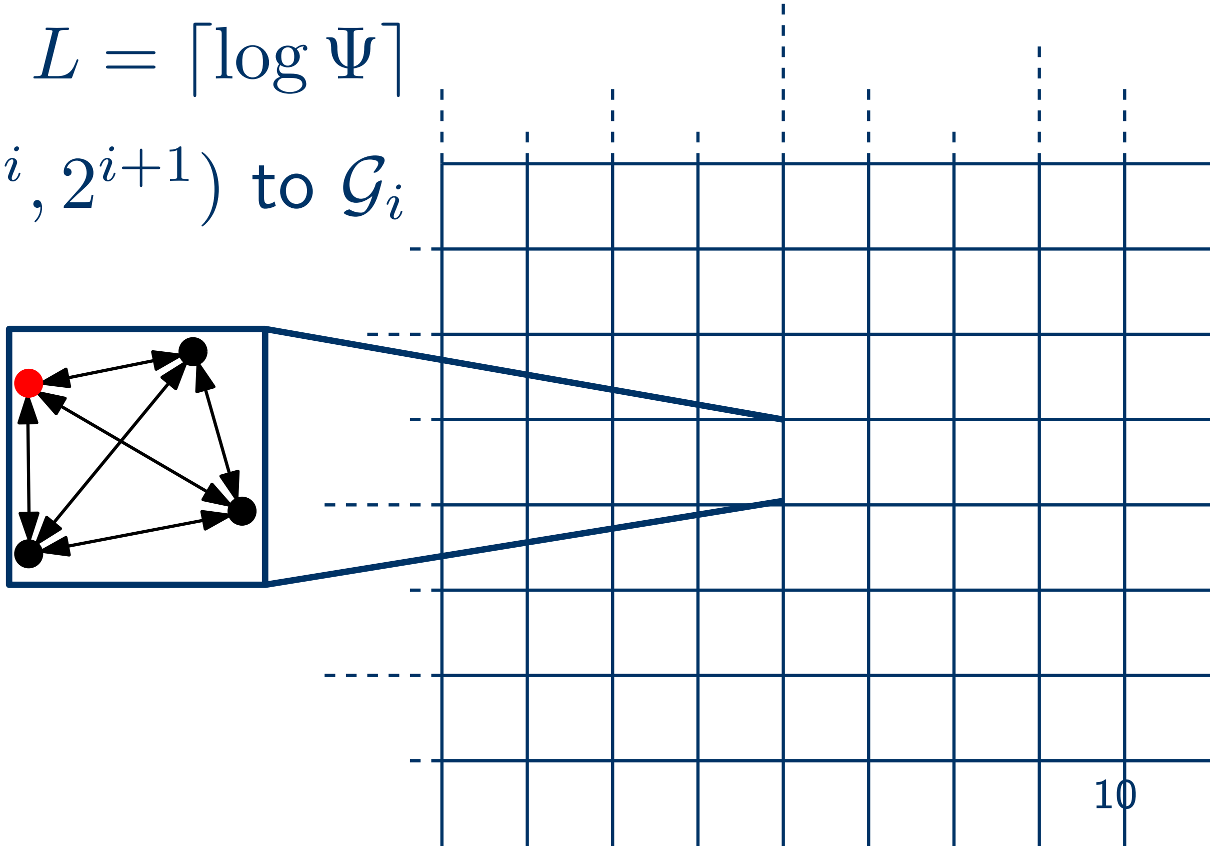

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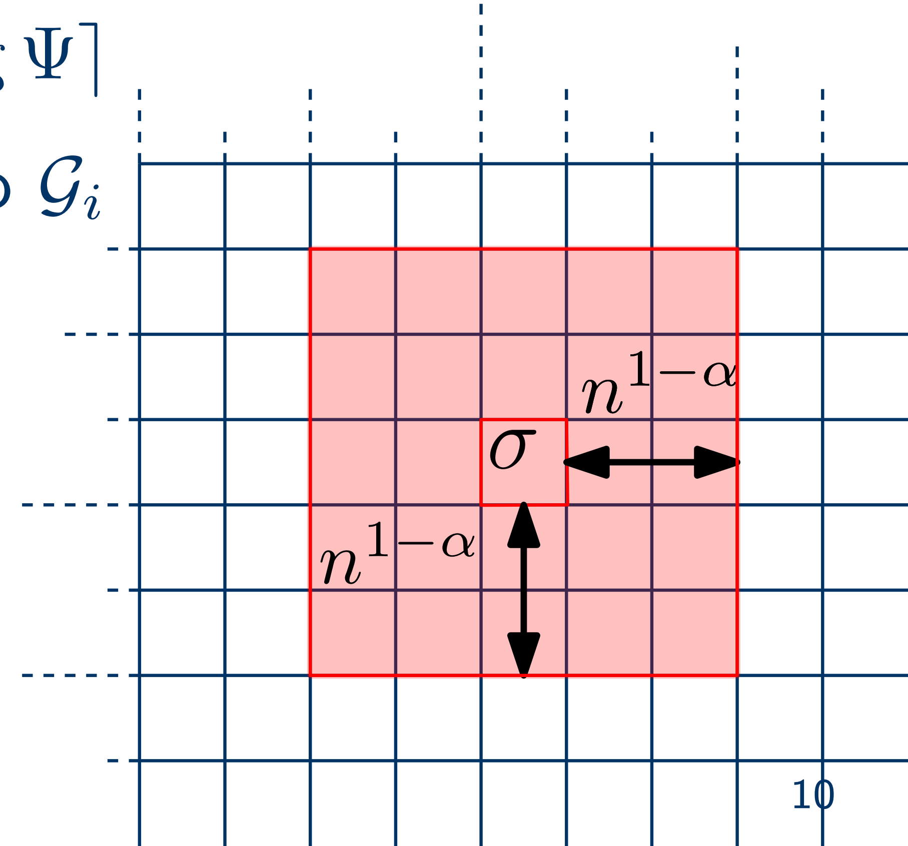

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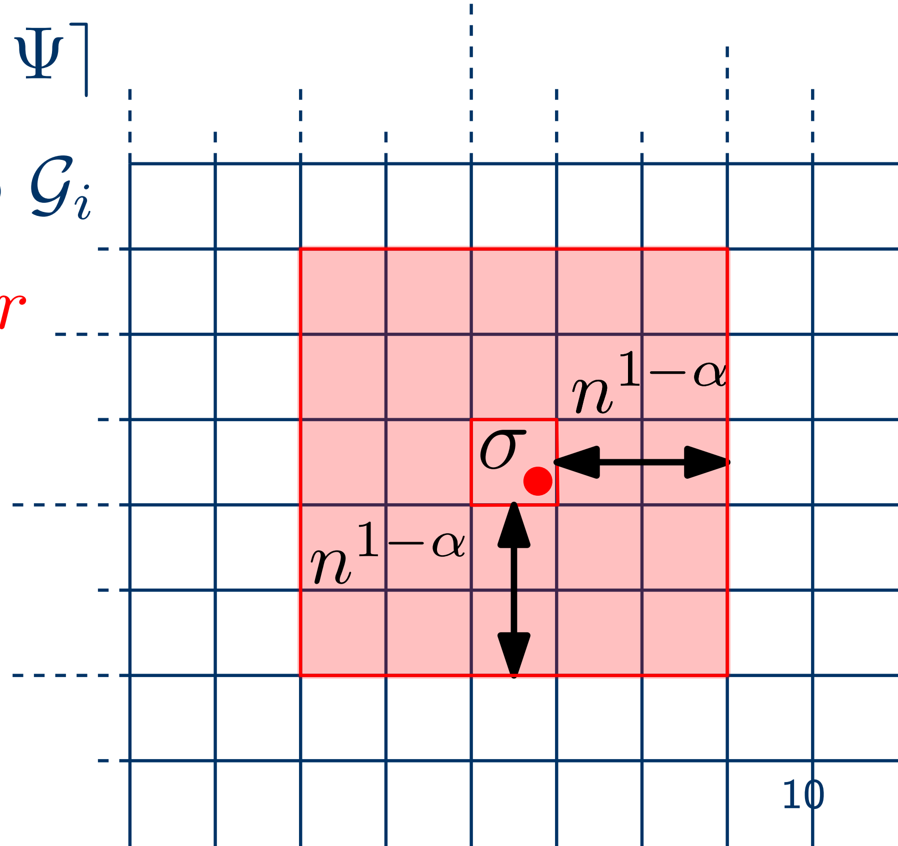

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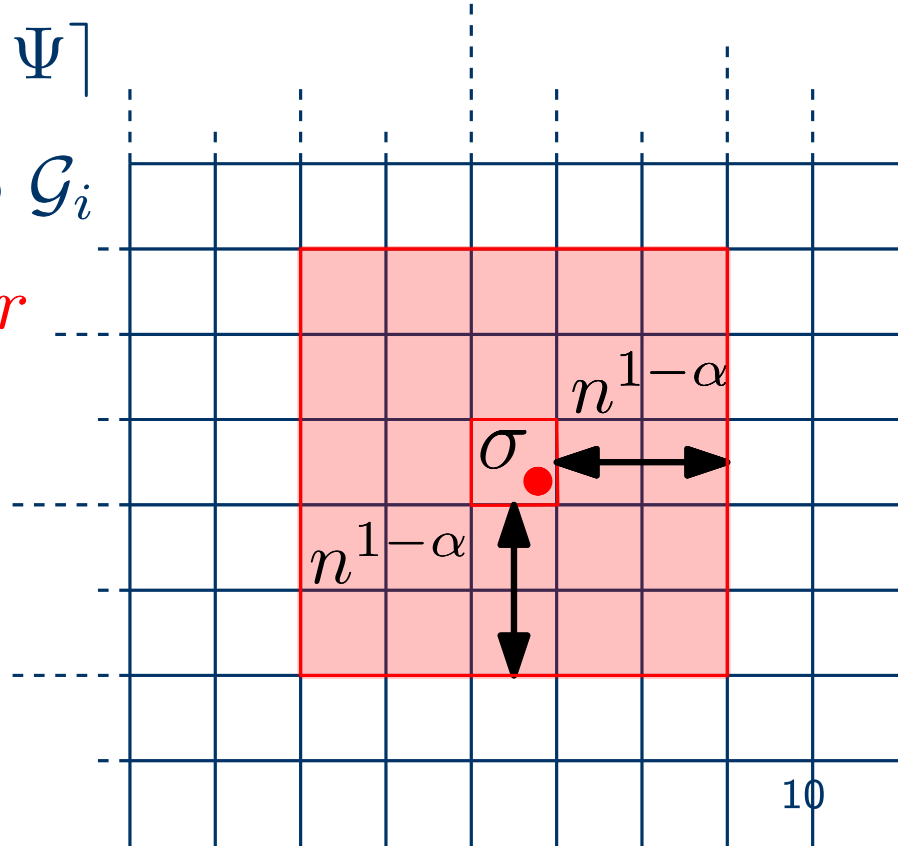
- requires $O(n^{3-2\alpha} \log \Psi)$ space



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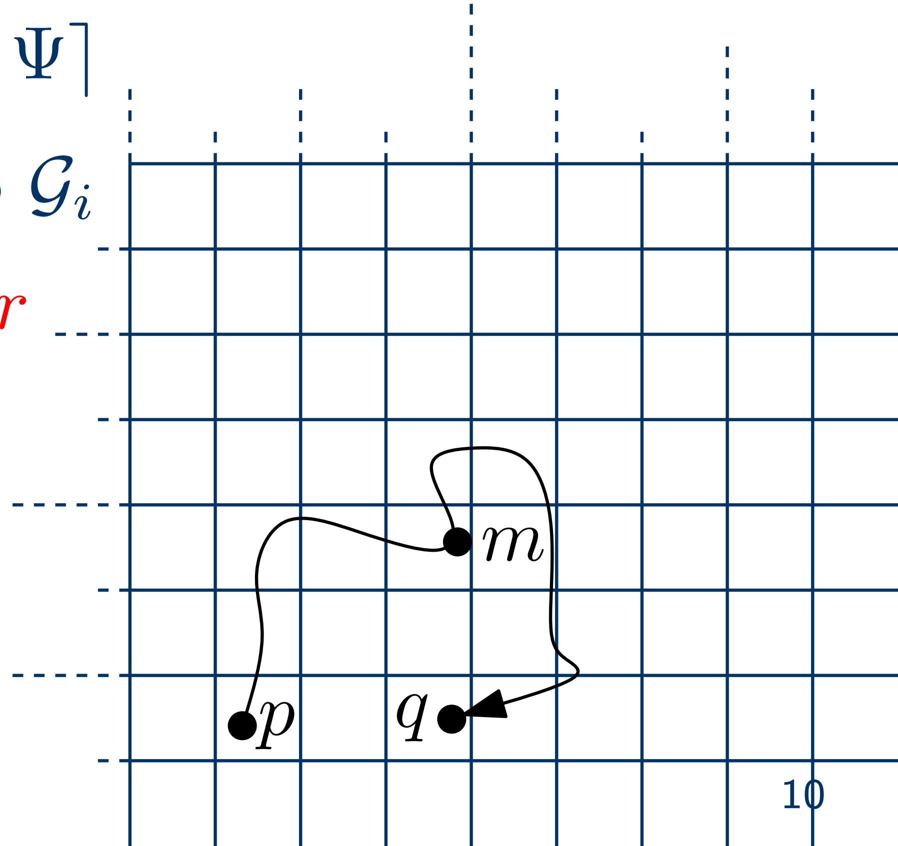

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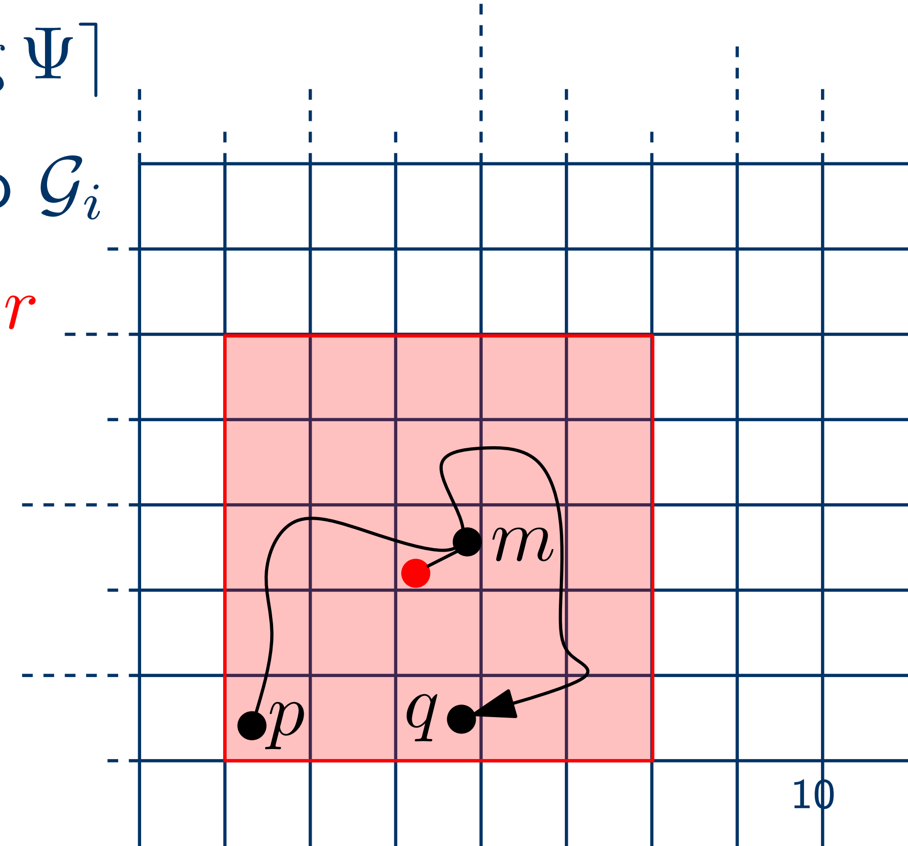

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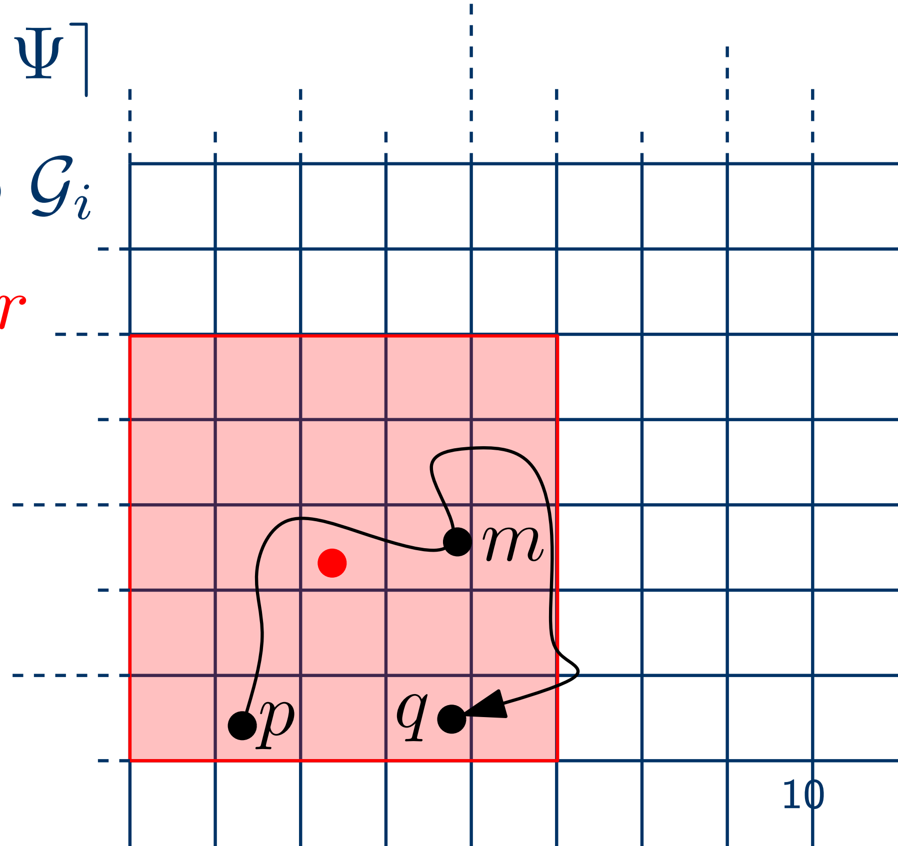

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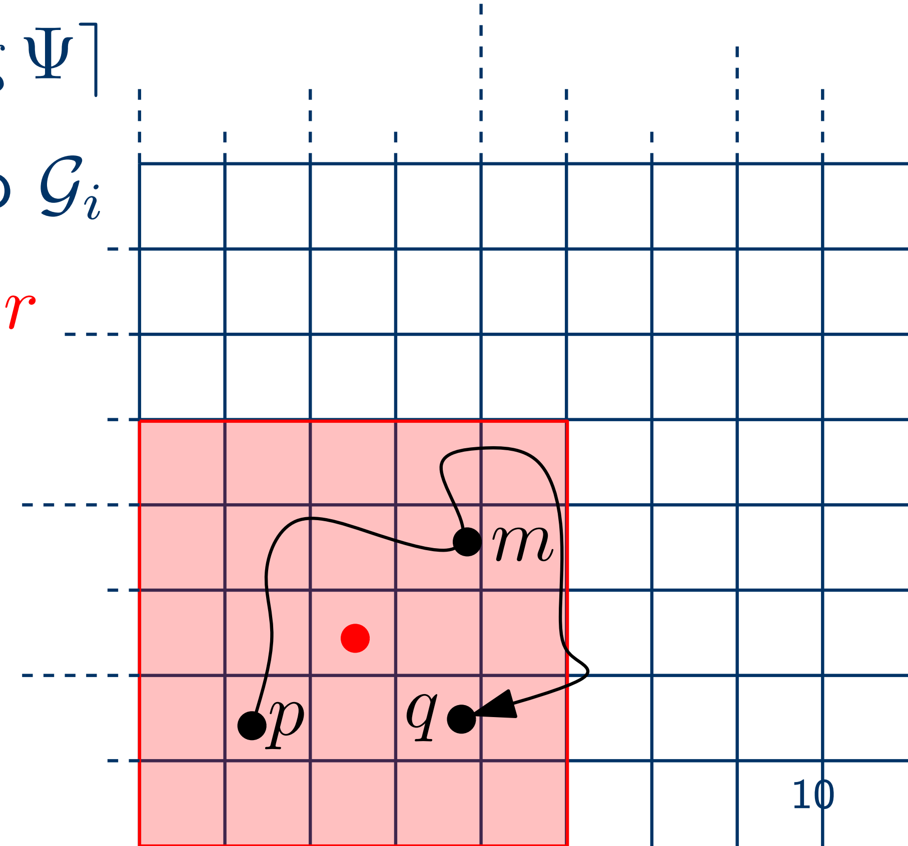

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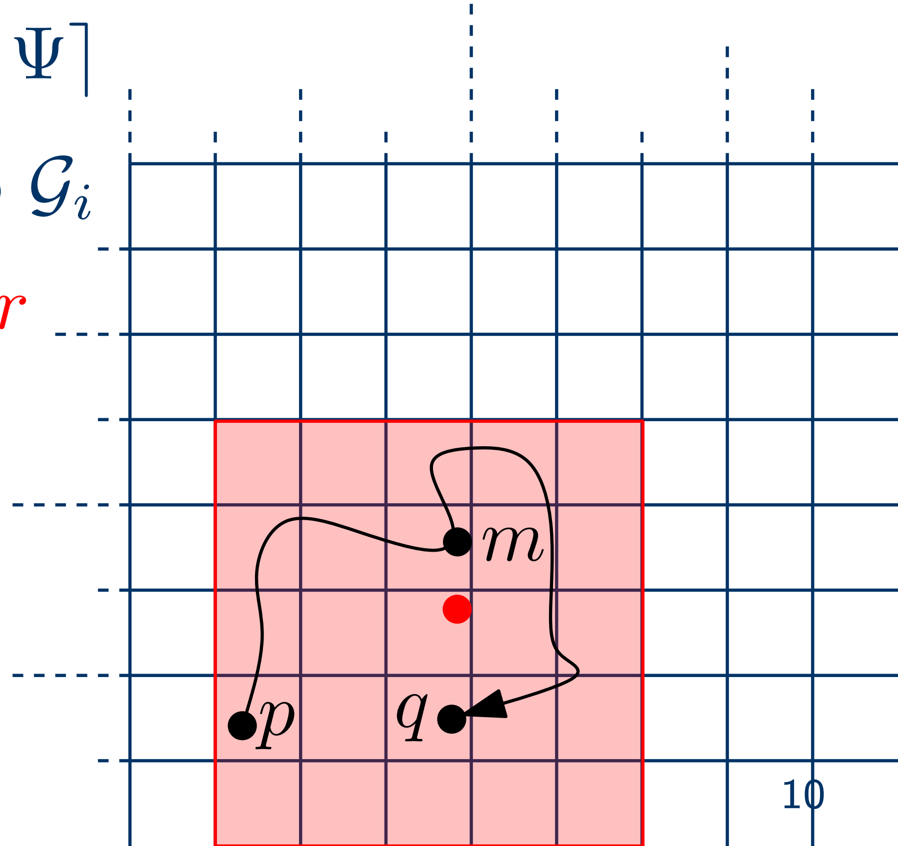

Summary

Let G be the transmission graph of $P \subset \mathbb{R}^2$, with $|P| = n$.

Spanner: For any $t > 1$, we can construct a t-spanner for G in time

- $O(n(\log n + \log \Phi))$
- $O(n(\log n + \log \Psi))$
- $O(n \log^6 n)$.

Oracles: We can construct a reachability oracle for G with

space	and query time
$O(n)$	$O(1)$ (if $\Psi < \sqrt{3}$)
$O(\Psi^5 n^{3/2})$	$O(\Psi^3 \sqrt{n})$
$O(n^{5/3} \log n)$	$O(n^{5/3} \log n)$ (if $\Psi = O(n^c)$).

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??? Questions ???