

Hans-Christian Hege
Konrad Polthier (Eds.)

Visualization and Mathematics III

Springer Verlag
Berlin Heidelberg New York

Preface

Mathematical Visualization aims at an abstract framework for fundamental objects appearing in visualization and at the application of the manifold visualization techniques to problems in geometry, topology and numerical mathematics. The articles in this volume report on new research results in this field, on the development of software and educational material and on mathematical applications.

The book grew out of the third international workshop “Visualization and Mathematics”, which was held from May 22-25, 2002 in Berlin (Germany). The workshop was funded by the DFG-Sonderforschungsbereich 288 “Differential Geometry and Quantum Physics” at Technische Universität Berlin and supported by the Zuse Institute Berlin (ZIB) and the DFG research center “Mathematics for Key Technologies” (FZT 86) in Berlin. Five keynote lectures, eight invited presentations and several contributed talks created a stimulating atmosphere with many scientific discussions.

The themes of this book cover important recent developments in the following fields:

- Geometry and Combinatorics of Meshes
- Discrete Vector Fields and Topology
- Geometric Modelling
- Image Based Visualization
- Software Environments and Applications
- Education and Communication

We hope that the research articles of this book will stimulate the readers’ own work and will further strengthen the development of the field of Mathematical Visualization.

We appreciate the thorough work of the authors and reviewers on each of the individual articles, and we thank you all. Beside the editors, the reviewers and members of the program committee were:

Helmut Alt	Ulrich Kortenkamp
Tom M. Apostol	Jens-Peer Kuska
James Arvo	Carsten Lange
Chandrajit Bajaj	Gregory Leibon
Thomas Banchoff	Nelson L. Max
Philippe Bekaert	Heinrich Müller
Werner Bengler	Gregory M. Nielson
Alexander Bobenko	Ronny Peikert
Alexander Bogomjakov	Ulrich Pinkall
Philip L. Bowers	Helmut Pottmann
Ken Brakke	Jürgen Richter-Gebert
Claude Bruter	Martin Rumpf
Matthieu Desbrun	Dietmar Saupe
Peter Deuffhard	Roberto Scopigno
Thomas Ertl	Hans-Peter Seidel
Gerald E. Farin	James Sethian
George Francis	Marc Stamminger
Hans Hagen	John Sullivan
Andrew J. Hanson	Nobuki Takayama
Joel Hass	Gabriel Taubin
David Hoffman	Daniel Weiskopf
Victoria Interrante	Rüdiger Westermann
Chris Johnson	Ross Whitaker
Michael Joswig	Luiz Velho
Alexander Keller	Jarke van Wijk
Leif Kobbelt	Günter M. Ziegler

Special thanks to Robert Staufenbiel for his help in compiling the manuscripts and creating the LaTeX source of this book.

Berlin, 2002
 Hans-Christian Hege
 Konrad Polthier

Table of Contents

Preface	V
---------------	---

Part I Geometry and Combinatorics of Meshes

Planar Conformal Mappings of Piecewise Flat Surfaces 3

Philip L. Bowers, Monica K. Hurdal

1 An Inversive Distance Primer	6
2 Piecewise Flat Surfaces and Circle Packings	12
3 Hexagonal Refinement	15
4 Proving Convergence and Conformality	17
5 A Gallery of Quadrilaterals	23
6 Implementation: Practical Experimental, Computational, and Theoretical Issues	31

Discrete Differential-Geometry Operators for Triangulated 2-Manifolds 35

Mark Meyer, Mathieu Desbrun, Peter Schröder, Alan H. Barr

1 Introduction	35
2 Defining Discrete Operators	38
3 Discrete Mean Curvature Normal	40
4 Discrete Gaussian Curvature	45
5 Discrete Principal Curvatures	46
6 Results and Applications	49
7 Discrete Operators in nD	53
8 Conclusion	54

Constructing Circle Patterns Using a New Functional 59

Boris A. Springborn

1 Introduction	59
2 Circle Patterns	60
3 Preliminaries	61
4 The Functional	62
5 Examples	64
6 Numerical Evaluation of the Functional	66

Constructing Hamiltonian Triangle Strips on Quadrilateral Meshes	69
<i>Gabriel Taubin</i>	
1 Introduction	70
2 Graphs and Meshes	71
3 Hamiltonian Paths	74
4 Basic Algorithm	77
5 Efficient Implementation	78
6 Diagonal Graph Structure	79
7 Transparent Vertex Caching	83
8 Implementation and Complexity	86
9 Subdivision	87
10 Borders	88
11 Conclusions and Future Work	89
<hr/>	
Part II Discrete Vector Fields and Topology	
<hr/>	
Visualizing Forman’s Discrete Vector Field	95
<i>Thomas Lewiner, Helio Lopes, Geovan Tavares</i>	
1 Introduction	95
2 Basic Concepts	96
3 Hypergraphs and Hypertrees	99
4 Algorithm	101
5 Applications	106
6 Future Works	109
Identifying Vector Field Singularities Using a Discrete Hodge Decomposition	113
<i>Konrad Polthier and Eike Preuß</i>	
1 Introduction and Related Work	113
2 Setup	116
3 Discrete Rotation	117
4 Discrete Divergence	120
5 Hodge Type Decomposition of Vector Fields	123
6 Decomposition Algorithm and Detecting Vector Field Singularities	127
7 Examples	129
8 Conclusions and Future Work	130
Searching for Knotted Spheres in 4-dimensional Space	135
<i>Dennis Roseman</i>	
1 Background for Random Knots	135
2 Generating Random Collections of Objects in \mathbb{R}^n	137
3 Methods of Generating Random Spheres in \mathbb{R}^4	138
4 Random Icosahedra in \mathbb{R}^4	141

5	Sorting Knottings into Equivalence Classes	144
6	The Role of Visualization.....	148
7	Ordering Knots	148

3D Loop Detection and Visualization in Vector Fields 151

Thomas Wischgoll, Gerik Scheuermann

1	Introduction	151
2	Mathematical Background.....	152
3	Loop Detection	153
4	Results.....	158

Part III Geometric Modelling

Minkowski Geometric Algebra and the Stability of Characteristic Polynomials 163

Rida T. Farouki, Hwan Pyo Moon

1	Introduction	163
2	Minkowski Geometric Algebra	164
3	Families of Curves and Envelopes.....	168
4	Stability of Characteristic Polynomials	170
5	Complex Disk Polynomials	173
6	Hurwitz Stability of Disk Polynomials.....	175
7	Γ -Stability of Disk Polynomials	179
8	Robustness Margin of Disk Polynomials	183
9	Closure	185

Subdivision Invariant Polynomial Interpolation 189

Stefanie Hahmann, Georges-Pierre Bonneau, Alex Yvart

1	Introduction	189
2	Related Works	190
3	Subdivision Invariance	191
4	Subdivision Invariant G^1 Polynomial Triangular Interpolant	193
5	Results.....	197
6	Conclusion	198

Another Metascheme of Subdivision Surfaces 201

Heinrich Müller, Markus Rips

1	Introduction	201
2	Elementary Schemes Based on Vertex Assignment	203
3	Inversion	209
4	Selected Composed Subdivision Schemes	212
5	Subdivision Schemes of Higher Order	216
6	Concluding Remarks	219

Geometry of the Squared Distance Function to Curves and Surfaces	221
<i>Helmut Pottmann, Michael Hofer</i>	
1 Introduction	221
2 Graph Surface of the Squared Distance Function to a Planar Curve	223
3 Quadratic Approximations to d^2	226
4 Squared Distance Function to a Surface and its Second Order Approximants	231
5 Squared Distance Function to a Space Curve	234
6 Application to Geometric Optimization Problems	235
7 Future Research	240

Part IV Image Based Visualization

A Multiscale Fairing Method for Textured Surfaces	245
<i>Ulrich Clarenz, Udo Diewald, Martin Rumpf</i>	
1 Introduction	245
2 Image Processing Background	246
3 Anisotropic Geometric Diffusion	247
4 Coupling Anisotropic Texture and Surface Diffusion	250
5 Comparison and Conclusions	255
Generalized Block Iterative Methods	261
<i>Michel Leblond, François Rousselle, Christophe Renaud</i>	
1 Introduction	261
2 Generalized Block Partitioning of a Matrix	265
3 Generalized Block Iterative Methods	267
4 Application to Radiosity	273
5 Conclusion and Perspectives	280
6 Acknowledgements	283
Fast Difference Schemes for Edge Enhancing Beltrami Flow and Subjective Surfaces	287
<i>Ravi Malladi, Igor Ravve</i>	
1 Introduction	287
2 Implicit Scheme for Beltrami Flow	289
3 Simulation Results for Beltrami Flow	292
4 Implicit Scheme for Subjective Surfaces	294
5 Simulation Results for Completing Missing Boundaries	296
6 Closing Remarks	296

Part V Software Environments and Applications

ALICE on the Eightfold Way: Exploring Curved Spaces in an Enclosed Virtual Reality Theater	305
<i>George K. Francis, Camille M.A. Goudeseune, Henry J. Kaczmarek, Benjamin J. Schaeffer, John M. Sullivan</i>	
1 Introduction	305
2 Other Fully Enclosed Virtual Reality Theaters	307
3 Cluster Architecture	307
4 Mathematical Visualization of Three-dimensional Geometries	310
Computation and Visualisation in the NUMLAB Numerical Laboratory	317
<i>Joseph M.L. Maubach, Alexandru C. Telea</i>	
1 Introduction	317
2 The Mathematical Framework	321
3 The Software Framework	322
4 An Efficient NUMLAB Finite Element Implementation	325
5 Application Design and Use	331
6 Conclusions and Future Work	333
A Generic Programming Approach to Multiresolution Spatial Decompositions	337
<i>Vinicius Mello, Luiz Velho, Paulo Roma Cavalcanti, Cláudio T. Silva</i>	
1 Introduction	337
2 Background	338
3 Concepts	339
4 Models	350
5 Applications	351
6 Conclusion	355
Mathematical Modelling and Visualisation of Complex Three-dimensional Flows	361
<i>Alfio Quarteroni, Marzio Sala, M.L. Sawley, N. Parolini, G. Cowles</i>	
1 Introduction	361
2 Mathematical Formulation	364
3 Complexity	366
4 Visualisation Techniques	370
5 Conclusions	373
webMathematica	379
<i>Tom Wickham-Jones</i>	
1 Introduction	379
2 webMathematica Technology	380

3	Web Extension Technologies	382
4	web <i>Mathematica</i> Services.....	383
5	Summary.....	389
6	References.....	390

Part VI Education and Communication

Films: A Communicating Tool for Mathematics 393

Michele Emmer

1	Introduction	393
2	Mathematics: a Special Case?	395
3	The Mathematics and Art Project	396
4	Mathematics and Fiction Films	401
5	Final Comments	403

The Potentials of Math Visualization and their Impact on the Curriculum 407

Beau Janzen

1	Introduction	407
2	Palpability	409
3	Context	409
4	Implementation	412
5	Impact on the Nature of the Curriculum	416
6	Impact on the Content of the Curriculum	419
7	Conclusion	420

Appendix: Color Plates 423