Problem 1 Computing the intersections of two convex polygons 10 points

Let $P$ and $Q$ be two convex polygons with $n$ and $m$ vertices respectively; each polygon is given as a list of its vertices sorted in counter-clockwise (or clock-wise) order. Give a sweep-line algorithm that computes all intersections between $P$ and $Q$ in $O(n + m)$ time.

Problem 2 Triangulations for Polygons with Holes 10 points

Let $P$ be a polygon with $n$ vertices and $h$ holes.

(a) Give a reasonable definition for a triangulation of $P$.
(b) Show that $P$ has a triangulation.
(c) Find a formula for the number of triangles in any triangulation of $T$, and prove that it is correct.

Problem 3 The Dual of a Triangulation 10 points

Let $P$ be a simple polygon with $n$ vertices, and let $T$ be a triangulation of $P$. The dual graph of $T$, $T^*$, is the graph whose vertices are the triangles of $T$ in which two triangles are adjacent if and only if they share a diagonal.

(a) Show that $T^*$ is a tree.
(b) Use $T^*$ to give an alternative proof that $T$ is 3-colorable.
(c) Suppose that $n \geq 4$. An ear of $T$ is a triangle in $T$ that has two polygon edges as sides. Show that $T$ contains at least two ears.
(d) Let $n \geq 4$. Show that $P$ has a diagonal that partitions $P$ into two simple polygons with at least $\frac{n-3}{3} + 2$ vertices.