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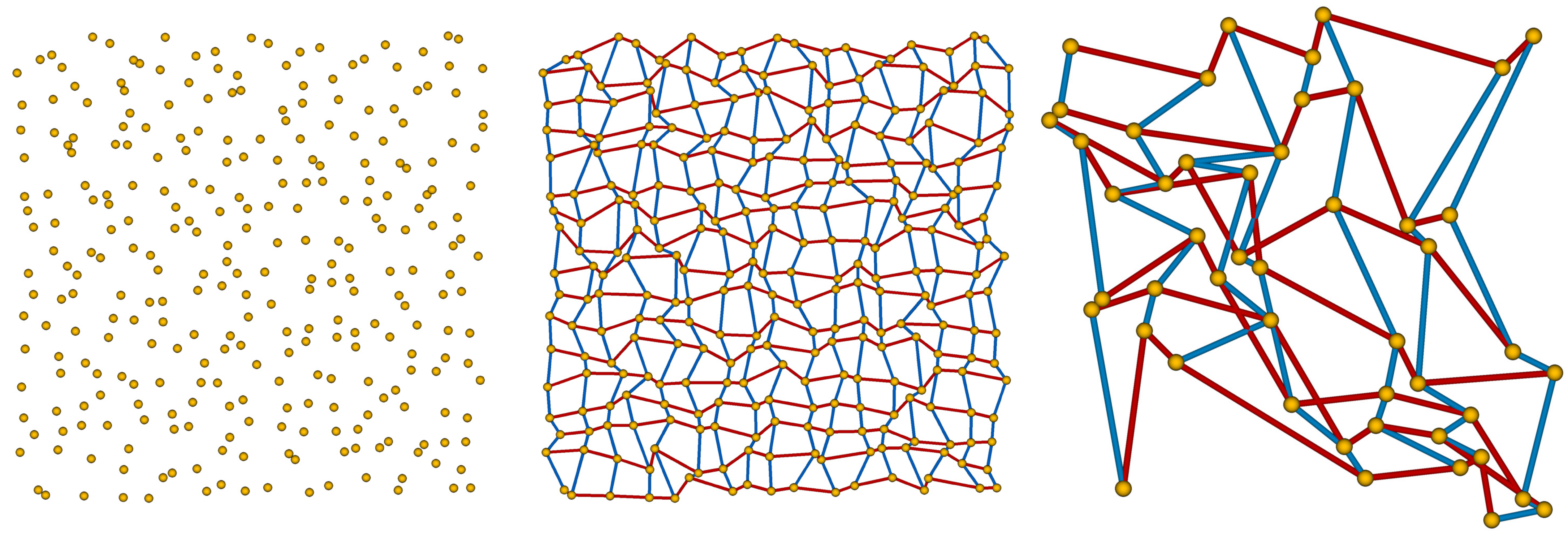
Asymptotical & Combinatorial Results on the Neighborhood Grid Data Structure

The Neighborhood Grid

The Neighborhood Grid approximates neighborhood information. A (quadratic) matrix contains the coordinates of the points such that in each row the x-values are increasing while in each column the y-values are increasing.

For the algorithm, the order of the points suffices, the exact coordinates are irrelevant. If the above ordering is given, we call it a "stable state".

Illustrating the Neighborhood Grid



From left to right: A raw point cloud, the corresponding structure induced by the grid, and an example where the neighborhood is not faithfully recovered.

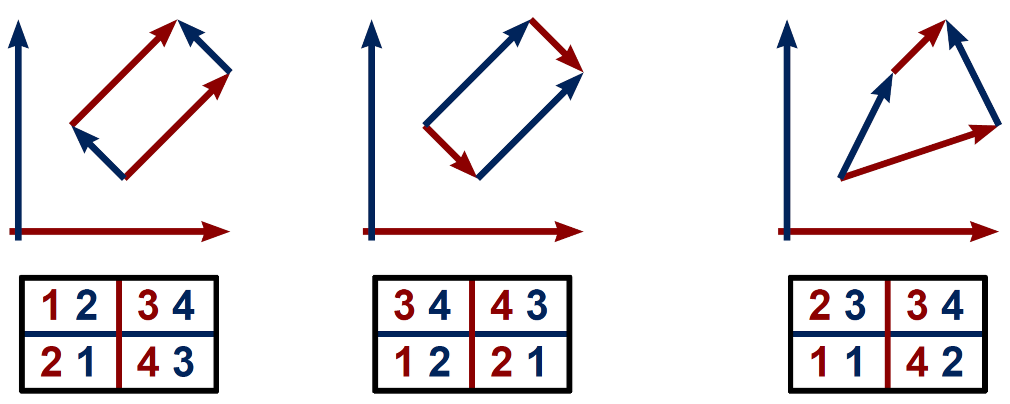
Upper Bound on building a Stable State

A stable state for $n^2 = N \in \mathbb{N}$ points can be computed in $\mathcal{O}(N\log(N))$ via sorting all points by their x-values in one sequence first and then sort $\sqrt{N}$ blocks by their y-values. Each block then gives a column of the matrix which is in a stable state.

Lower Bound

Given a comparison-based algorithm $\mathcal{A}$, its decision tree has depth $\log((N)!)$, but every placement π of points is stable for exactly $\frac{(n^2)!}{(n!)^n}$ point sets. Thus, the algorithm has to traverse $\log((n^2)!) - \log(\frac{(n^2)!}{(n!)^n}) = \mathcal{O}(N\log(N))$, see [4].

(Non)Unique Stable States



Combinatorial Results

Given a point set P with n^2 points, it can have several stable states. Overall, there are $(n^2)!$ such sets with a total of $\frac{(n^2)!}{(n!)^n}$ stable states. The "identity" has $N!/\left(\prod_{i=1}^{n}\prod_{j=1}^{n}(2n-i-j+1)\right)$ stable states.

Results

size n	points n^2	unique	min	max	avg.
1	1	1	1	1	1.0
2	4	12	1	2	1.5
3	9	966	1	42	7.777
4	16	0	≤ 8	$\geq$24,024	190.077

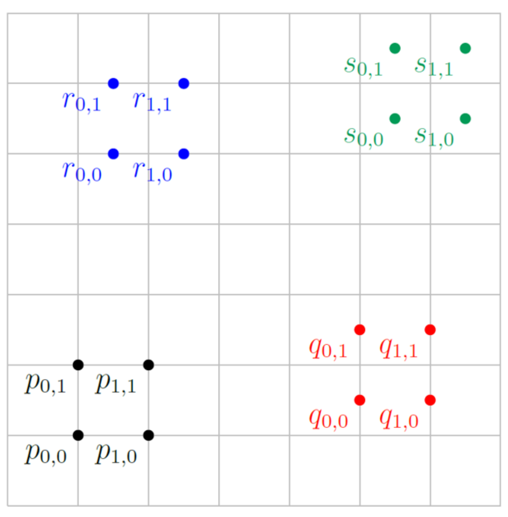
Open Questions

Combinatorial For a given n, ...

- ... what is a point set P with minimal number of stable states?
- ... does the "identity" $P = \{(1,1),\dots,(n^2,n^2)\}$ have the maximal number of stable states?

Geometrical

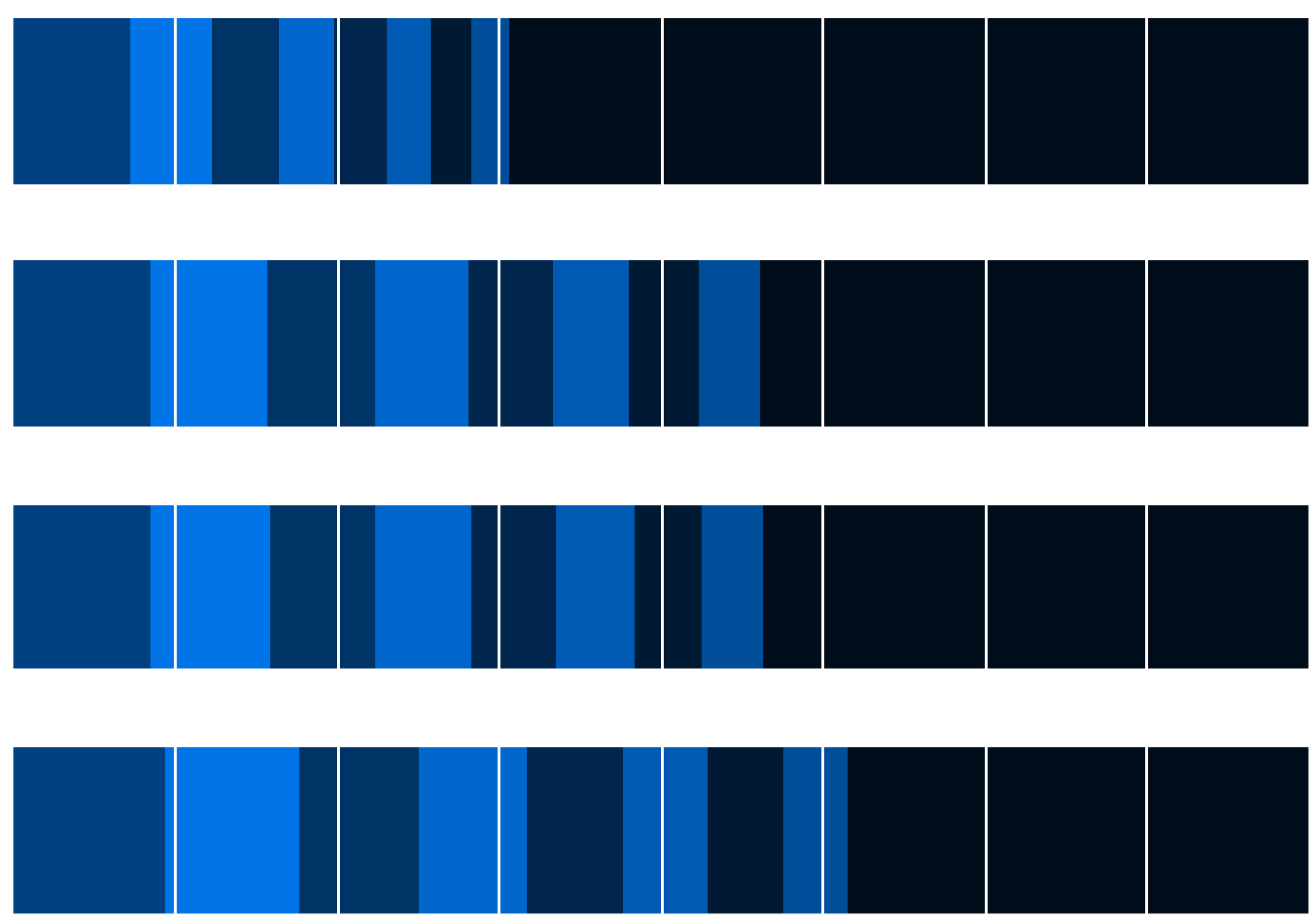
- What is the average neighborhood quality of a random stable state of a random point set?



$$M(P) = \begin{array}{|c|c|c|c|c|c|c|}
\hline
q_{0,\frac{n}{2}-1} & s_{0,\frac{n}{2}-1} & q_{1,\frac{n}{2}-1} & s_{1,\frac{n}{2}-1} & \cdots & q_{\frac{n}{2}-1,\frac{n}{2}-1} & s_{\frac{n}{2}-1,\frac{n}{2}-1} \\
\hline
p_{0,\frac{n}{2}-1} & r_{0,\frac{n}{2}-1} & p_{1,\frac{n}{2}-1} & r_{1,\frac{n}{2}-1} & \cdots & p_{\frac{n}{2}-1,\frac{n}{2}-1} & r_{\frac{n}{2}-1,\frac{n}{2}-1} \\
\hline
\vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots \\
\hline
q_{0,1} & s_{0,1} & q_{1,1} & s_{1,1} & \cdots & q_{\frac{n}{2}-1,1} & s_{\frac{n}{2}-1,1} \\
\hline
p_{0,1} & r_{0,1} & p_{1,1} & r_{1,1} & \cdots & p_{\frac{n}{2}-1,1} & r_{\frac{n}{2}-1,1} \\
\hline
q_{0,0} & s_{0,0} & q_{1,0} & s_{1,0} & \cdots & q_{\frac{n}{2}-1,0} & s_{\frac{n}{2}-1,0} \\
\hline
p_{0,0} & r_{0,0} & p_{1,0} & r_{1,0} & \cdots & p_{\frac{n}{2}-1,0} & r_{\frac{n}{2}-1,0} \\
\hline
\end{array}$$

Construction of a point set and a stable state s.t. no point has its nearest neighbor in the 1-ring.

Neighborhood Estimates



Top to Bottom: direct, rows/cols iteratively, odd/even, maximum growing energy; Left to Right: 1st, 2nd,...,8th nearest neighbor known.

Applications

- Crowd Simulation [1].
- Fluid Animation [2].
- Biological Cell Simulation [3].

References

[1] M. Joselli, E. B. Passos, M. Zamith, E. Clua, A. Montenegro, and B. Feijó. "A Neighborhood Grid Data Structure for Massive 3D Crowd Simulation on GPU", 2009.

[2] M. Joselli, J. R. da S. Junior, E. W. Clua, A. Montenegro, M. Lage, and P. Pagliosa. "Neighborhood grid: A novel data structure for fluids animation with GPU computing", 2015.

[3] M. de Geomensoro Malheiros and M. Walter. "Simple and Efficient Approximate Nearest Neighbor Search using Spatial Sorting", 2015.

[4] M. Skrodzki, R. Reitebuch, and K. Polthier. "Combinatorial and Asymptotical Results on the Neighborhood Grid", ArXiv, 2018.

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