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SFB TRR 109 Discretization in Geometry and Dynamics | Berlin Mathematical School

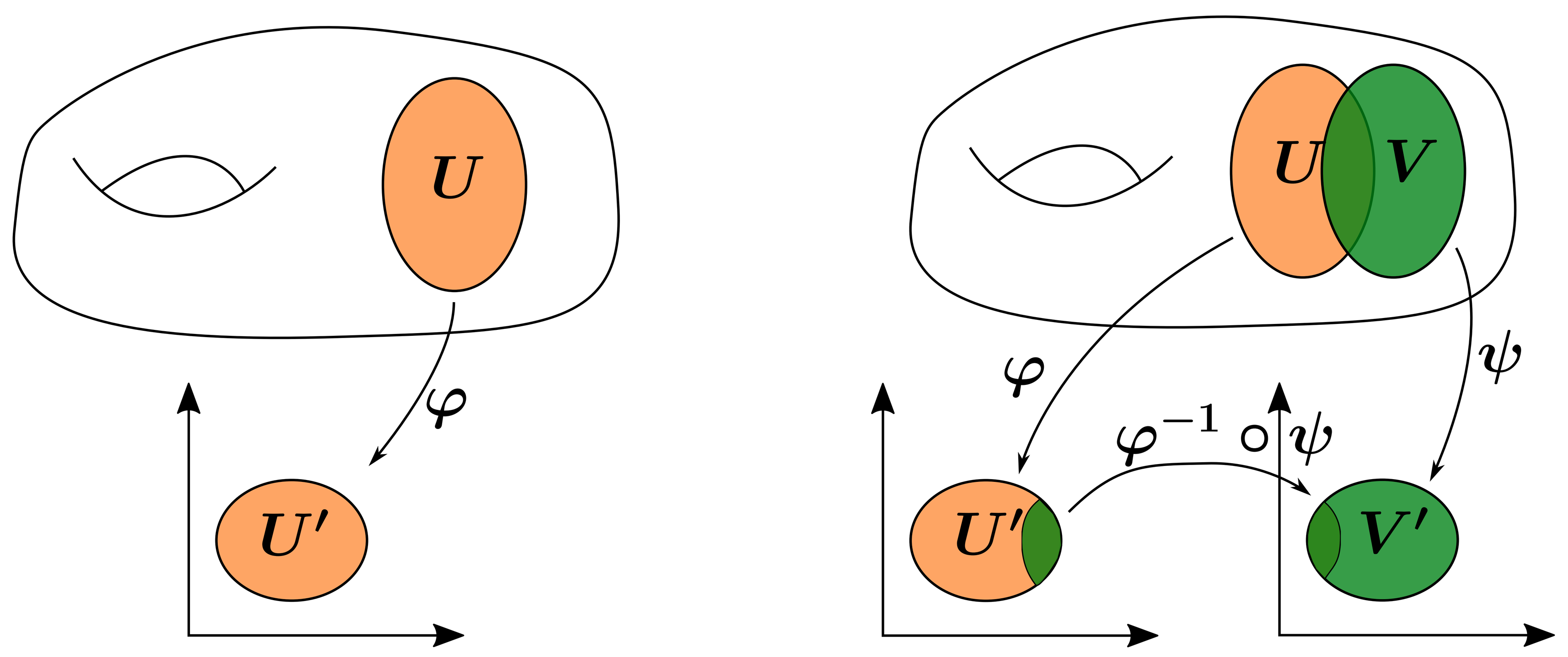
Computational and Structural Aspects of Point Set Surfaces (1)

Overview

Based on previous, well-established, and successfully used discretization schemes of differential geometric structures and operators on triangulated meshes, we take the next step and transfer this calculus to point set surface data. These arise naturally in 3D acquisition processes. We aim at a theoretical framework, which mimics a many features as possible of smooth manifolds in the setting of point sets.

Coordinate Charts and Transition Maps on Point Sets

Goal: Consistent definition of local coordinate charts and transition maps between them.
Recall the smooth theory of manifold surfaces:



A two-dimensional manifold surface (possibly embedded in $\mathbb{R}^d$) with local coordinate charts φ, ψ and transition maps $\varphi \circ \psi^{-1}$, $\psi \circ \varphi^{-1}$.

Construction of Coordinate Charts and Transition Maps:

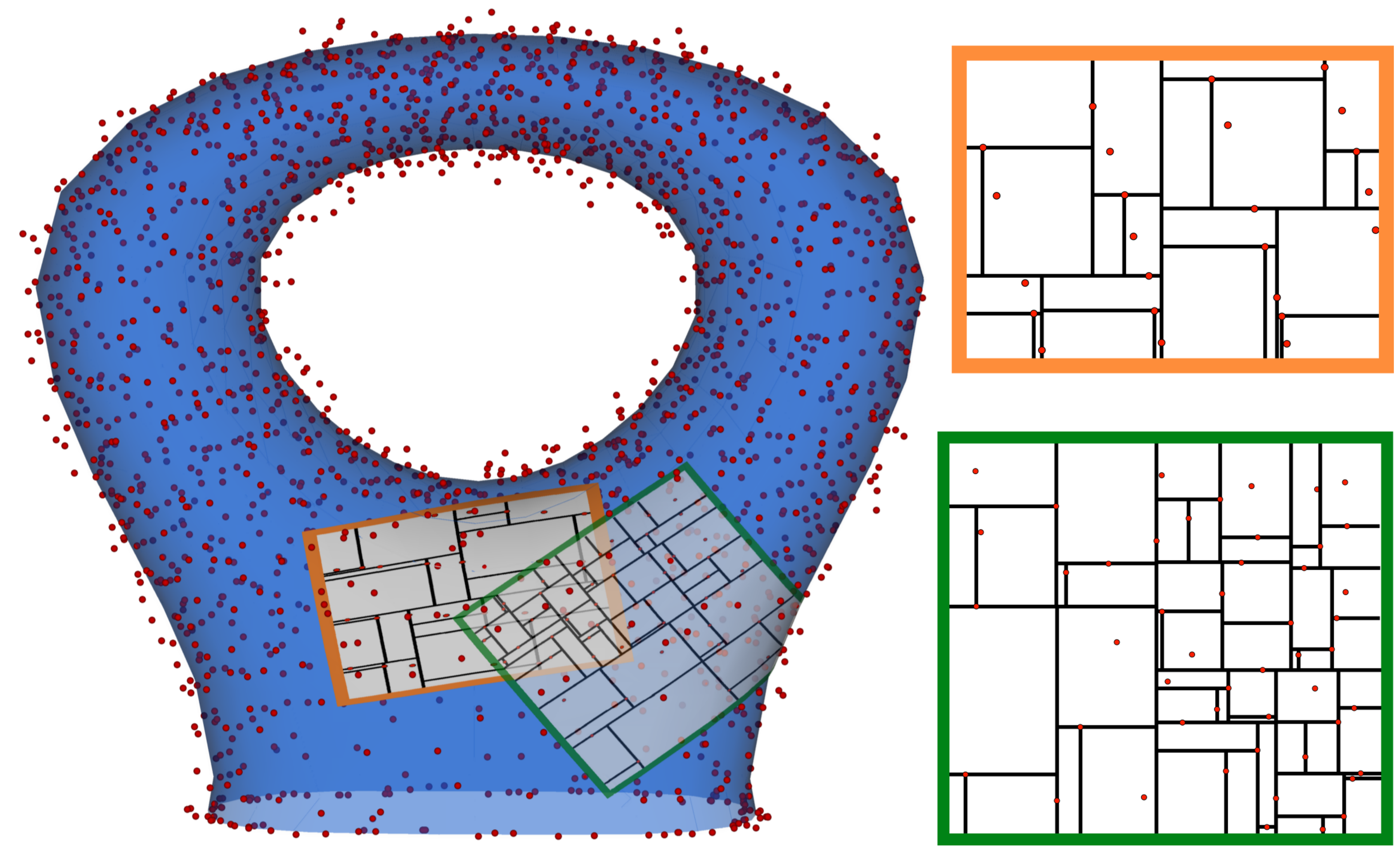
$$\begin{tikzcd}
p,q \in P \arrow[dr, "\text{Manifold Reconstruction } \mathcal{P}"] & & \\
& \tilde{p}, \tilde{q} \in \tilde{M} \arrow[dl, bend right, "\text{Map to } T_p,\ \varphi"'] \arrow[dr, bend left, "\text{Map to } T_q,\ \psi"] & \\
p' \in T_p \arrow[ur, "\text{Evaluate } \mathcal{P}"] \arrow[rr, leftrightarrow, "\psi \circ \varphi^{-1},\ \varphi \circ \psi^{-1}"'] & & q' \in T_q \arrow[ul]
\end{tikzcd}$$

Mimicing smooth transition maps with Manifold Reconstruction, e.g. [5,6].

Efficient Structures for Access of Coordinate Charts

Goal: Efficient Access to local parts of the point set for fast queries and alterations.

Concept: Equip every local tangent plane H_p with a Kd-Tree structure.



Local Kd-charts for efficient computations on a point set surface.

Benefits:

- Neighborhood query for k neighbors on a chart with m vertices can be performed in an average of $\log(m)$ [2].
- Deletion/Addition of a point only changes local charts, but not the global structure.
- Local charts behave like two-dimensional euclidean space, their height can be neglected.

MLS Procedure for Implicit Manifold Reconstruction

Follow [5,6]. For $P = \{p_i \in \mathbb{R}^3\}$, $p \in \mathbb{R}^3$ not necessary in P, perform:

1. Approximate local tangent plane T_p, by a normal $a \in \mathbb{R}^3$, $\|a\|_2 = 1$ and a distance $r \in \mathbb{R}$ of p to H_p by minimizing
$$I_p(a,r) = \sum_i \left(\langle a, p_i\rangle - \langle a, p + r\cdot a\rangle\right)^2 \cdot \theta\left(\|p_i - (p + r\cdot a)\|\right), \qquad (1)$$
for a and r, where θ is a monotonously falling function of local support.
2. Find a polynomial $\mathcal{P}$ of degree m on H_p that approximates the p_i well by solving
$$\arg\min_{\mathcal{P}} \sum_i \left(\mathcal{P}(p_i) - f_i\right)^2 \cdot \theta\left(\|p_i - (p + r\cdot a)\|\right)^2. \qquad (2)$$

Illustration MLS

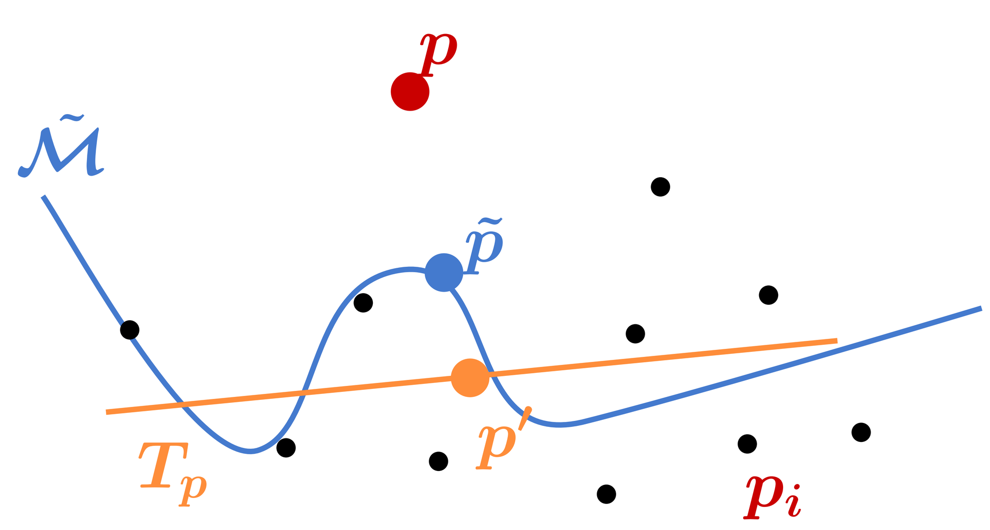


Illustration of the MLS Procedure as given in (1) and (2).

Applications

- Formulation of a Point Set Laplacian as in [1].
- Local re-sampling and geometric simplification [3].
- Feature detection and feature-preserving, anisotropic smoothing methods based on [4].
- Implicit surface reconstruction and surface description.

References

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[6] B. Sober and D. Levin, "Manifolds' Projective Approximation Using The Moving Least-Squares (MMLS)", 2016.

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