# Chladni Towers

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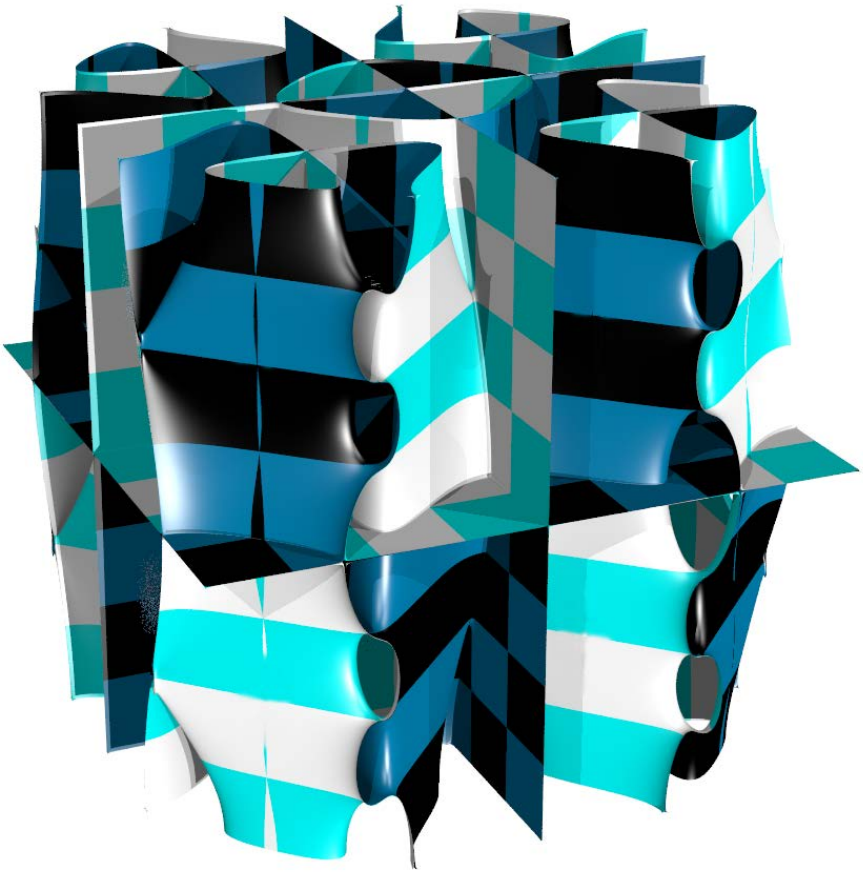

**Title Image:** “Chladni Towers”
**Keywords:** Vibration Modes, Acoustics, Chladni Figures

## 1 Introduction

In 1802, Ernst Florens Friedrich Chladni (November 30th, 1756 – April 3rd, 1827, see portrait in Figure 1) published his book "Acoustics". The book describes amongst other things an experiment by which different modes of vibration can be visualized. Namely, sand is distributed over a thin metal plate. A violin bow is then struck alongside the plate, causing it to oscillate, see Figure 1 for an illustration. Chladni found that the sand grains form different patterns corresponding to the varying vibration modes of the plate. His book contains a table of patterns he was able to create in his experiments, see Figure 1.

Since its first description, several other scientists like Margaret Watts-Hughes, Henry Holbrook, or Hans Jenny have further developed the Chladni experiment [1]. Despite their elegance and artistic value, Chladni figures have applications in the construction of musical instruments as illustrated in Figure 2.

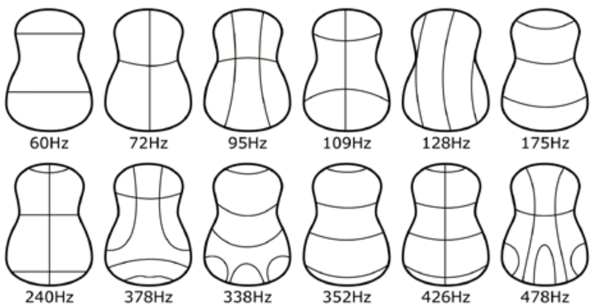


Figure 2: Application of Chladni Figures in building musical instruments.

We generalize Chladni’s concept to the third dimension. In terms of a corresponding physical experiment, this consists of distributing light particles in some viscose fluid. For example, speakers are then used to stir the fluid and the particles form stable patterns, just as in the two-dimensional case.

However, we do not build a physical experiment, but simulate the outcome of it, to print it via a 3D-printer. Applications of our method can be found for example in Architecture, where the printed model gives an impression of the acoustics of a given room.

## 2 Physical and Mathematical Background

For simplicity, we generalize Chladni's patterns from the square-shaped plate to a cubical model. In the following, our cube will be assumed to occupy space $[-1,1]^3$. Starting from the physical formulation of a damped oscillation of a string, we combine the solutions to the wave equation to obtain a formulation of a one-dimensional Chladni pattern as the zero set of an expression on the string. From this formulation, we can obtain a three-dimensional pattern as the zero-set of the following expression:

$$\begin{aligned} &A \cdot \sin(u \cdot \pi \cdot x) \cdot \sin(v \cdot \pi \cdot y) \cdot \sin(w \cdot \pi \cdot z) \\ &+B \cdot \sin(u \cdot \pi \cdot x) \cdot \sin(v \cdot \pi \cdot z) \cdot \sin(w \cdot \pi \cdot y) \\ &+C \cdot \sin(u \cdot \pi \cdot y) \cdot \sin(v \cdot \pi \cdot x) \cdot \sin(w \cdot \pi \cdot z) \\ &+D \cdot \sin(u \cdot \pi \cdot y) \cdot \sin(v \cdot \pi \cdot z) \cdot \sin(w \cdot \pi \cdot x) \\ &+E \cdot \sin(u \cdot \pi \cdot z) \cdot \sin(v \cdot \pi \cdot x) \cdot \sin(w \cdot \pi \cdot y) \\ &+F \cdot \sin(u \cdot \pi \cdot z) \cdot \sin(v \cdot \pi \cdot y) \cdot \sin(w \cdot \pi \cdot x). \end{aligned}$$

Note that the formulation permits two different types of figures: Those with sine (as shown) and those with cosine formulations. These correspond to different boundary conditions on the wave equation. For further details on the mathematical and physical background, as well as other visualizations, see [2].

The particular model generated for the Asian Digital Modeling Contest and shown as title image has been named "Chladni Towers". Its exact parameters are given in Table 1 and it corresponds to Dirichlet boundary conditions on the wave equation.

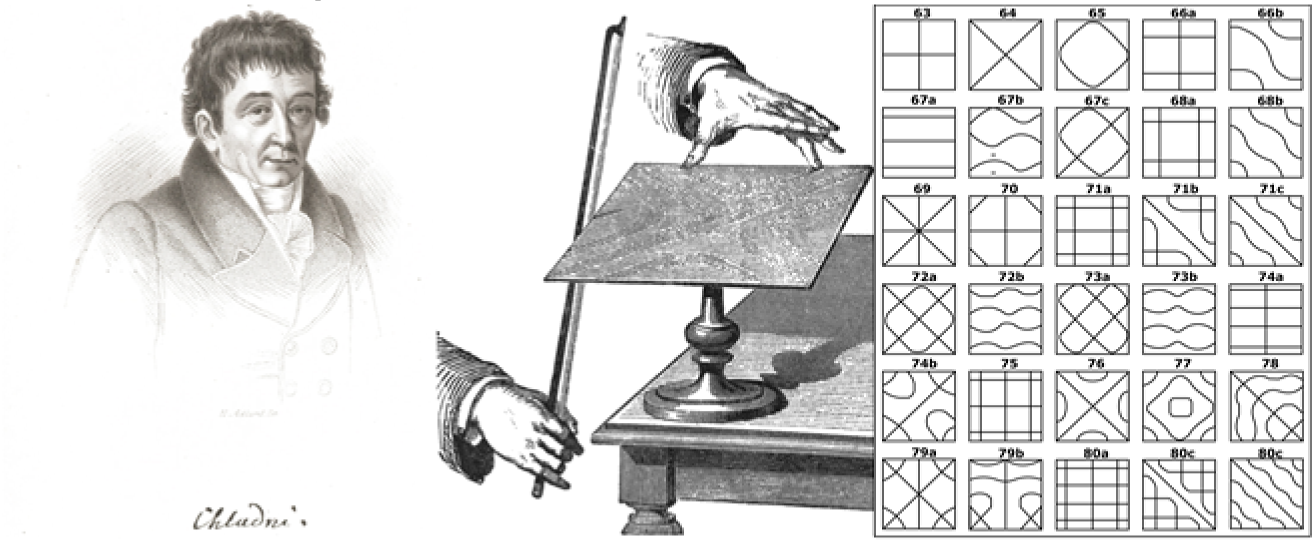


Figure 1: *Left*: Portrait of Ernst Florens Friedrich Chladni by H. Adlard, 19th Century. *Center*: Illustration of the violin bow experiment, taken from [3]. *Right:* Table of Chladnis figures from Chladni's book "Acoustics", 1802.

## 3 Software & System

Our approach consists of plotting the zero level set of the expression given above in a cubical bounding box. This corresponds to visualizing the isosurface given by the expression. To do so, we utilize the marching cubes algorithm [4]. The algorithm has been implemented in the JavaView Framework [5]. Within this framework, we visualize the zero set for different prescribed values of

$$A, \ldots, F \in \mathbb{R}, u, v, w \in \mathbb{Z}.$$

Given the first rough visualizations, more elaborate pictures like the title image are created with the PovRay Raytracer software [6]. Finally, from these, a model is chosen for 3D printing.

## References

[1] Findeiß. F., *Die Entdeckung der Chladnischen Klangfiguren. Ursprünge und Weiterentwicklung im historischen Kontext* (in German). Leuphana Universität Lüneburg, Germany, 2015.

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[4] Lorensen, W.E. and Harvey, E.C., *Marching cubes: A high resolution 3D surface construction algorithm.* ACM Siggraph Computer Graphics, Vol. 21, No. 4, pp. 163-196, 1987.

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[6] Persistence of Vision Pty. Ltd. Raytracer (version 3.7), http://www.povray.org

Table 1: Exact parameter values of the "Chladni Towers" Model

| Parameters | Values |
|---|---|
| Integral Parameters | u=1, v=2, w=4 |
| Real Parameters | A=0.5, B=0.0, C=0.5, D=8.0, E=0.5, F=8.0 |