A performability model for a hybrid reactor system

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Abstract

In this paper a hybrid stochastic model is proposed for performance and reliability analysis of a hybrid manufacturing system. The system consists in a set of metal pieces that are heated up on a heater and then placed in a water tank by a robot. This system can be seen again as a model of a nuclear reactor and problems arising in nuclear power plants can be studied by regarding this laboratory set up. In this paper we assume the system has only one work pieces which helps us to design a simple model based on a hybrid Petri net formalism (FSPN). From it performance characteristics such as the utilization of the robot and temperature evolution of both the work pieces and the water tank can be obtained.

1 Introduction

Hybrid systems have gained much attention over the last years. Formal methods for their verification, performance and reliability analysis have been investigated intensively. For the latter a number of extensions to stochastic Petri nets have been proposed [HKNT98, GSB99, WZ00].

Stochastic models often suffer from not having closed-form solutions, so numerical methods or simulation techniques have to be applied [NM95, CNT99, BGG$^+$99]. The solution of the models, however, is not subject of the present paper.

In this work we use an earlier developed tool for the analysis of a real-world hybrid application. We study the behaviour of a hybrid manufacturing system that exists at the University of Hamburg-Harburg [NL00] and that probably is meant to be seen as representing a nuclear power plant. In a nuclear power plant sticks are heated by the nuclear reaction and they have to be put into water for cooling down. One question that arises in designing these systems is whether there is sufficient cold water so that no dangerous heat-up of the whole reactor can happen. Also important is whether there are enough transportation facilities to remove and replace the hot metal sticks as quickly as necessary. Other aspects concern more performance of the reactor: is the transportation utilized as wanted or should the system be dimensioned differently.

Here a very simple model will be studied, in which only one metal piece is used. The model has two continuous variables: one for the temperature of the metal piece and the other one for the temperature of the water in the tank. Two continuous variables can be conveniently handled by our solution algorithm. Increasing the number of metal pieces immediately increases the model complexity. An arbitrary number of continuous variables to represent the temperatures of an arbitrary number of work pieces can be handled by no existing software tool. In consequence the models have to be formulated differently. We are currently working on a more general formulation of the model, which will allow us to flexibly choose the number of work pieces used. This more general model will be included in a follow-up on this paper or in a later version.

The paper is organized as follows: in the next section the system we model is described in detail, Section 3 introduces briefly the modelling formalism we use, Section 4 holds the model and its solution and Section 5 concludes the paper.

2 The System

The system studied in this paper is a hybrid manufacturing system in which metal pieces are heated and later cooled down again in a water tank. The metal pieces are the discrete items in the system, whereas the temperatures of both the water in the tank and the metal pieces are continuous variables. The control of the system switches between discrete states, depending on the discrete state and the values of the continuous variables.

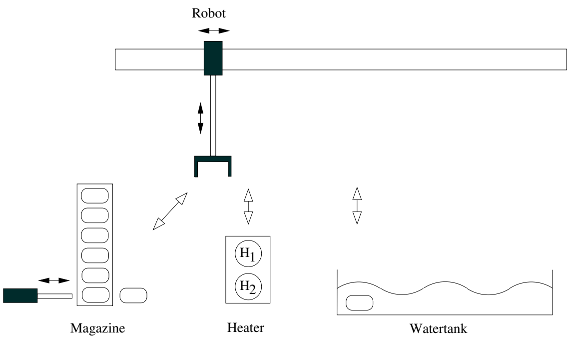

Figure 1: Sketch of the hybrid manufacturing system

The elements of the considered system are a magazine, holding small metal pieces, a heater on which those pieces are warmed up, a water tank in which the pieces cool down again - while increasing the temperature of the water - and a robot that is used for transportation. The energy of the heater represents the heating power of a nuclear reaction.

The system performs the following control sequence: the robot carries a piece to the heater, where this item stays until some temperature is reached, then the robot picks the work piece up from the heater and carries it to the water tank in which the metal piece is placed. When the metal piece has cooled down to some preset temperature it is removed from the water tank and placed back on the heater. Figure 1 shows a sketch of the hybrid system.

Fetching the robot to the position where it is needed (either the magazine, the heater or the tank) takes on the average 10 seconds, as does transportation of a work piece from one location to the next. The control sequence creates an action loop in the system, which will in a real nuclear power plant be ended after a known number of iterations. In our model system there is no limit to the number of iterations.

All parts have an initial temperature of 20° Celsius. Throughout this paper temperature is measured in degree Celsius. Transportation of items between locations and movements of the robot in general are assumed to be exponentially distributed with mean value of 10 seconds - that corresponds to a rate $\lambda = 0.1$, the time unit is one second, the metal pieces are assumed to be round disks with 25 mm height and a diameter of 40 mm. The water tank holds 0.5 $dm^3 = 0.5$ liters.

3 The FSPN Formalism

For the definition of FSPNs the common notations [ABCD95, HKNT98] are taken over and extended where it is needed.

An FSPN is formally defined as a 8-tuple FSPN = $(\mathcal{P}, \mathcal{T}, \mathcal{A}, m_0, g, \lambda, r, w)$ where the set of places ($\mathcal{P} = \mathcal{P}_c \cup \mathcal{P}_d$) is divided into the *fluid (continuous)* and the *discrete* places. Discrete places (the elements of $\mathcal{P}_d$) are drawn as single lined circles and hold an integer number of *tokens*, whereas the continuous, or fluid, places (the elements of $\mathcal{P}_d$) are drawn as two concentrical circles and they hold a real-valued amount of *fluid*. The set of transitions $\mathcal{T} = \mathcal{T}_E \cup \mathcal{T}_I$ is composed of the exponentially distributed, and the immediate transitions, respectively. Exponentially distributed transitions are drawn as empty rectangles and the immediate transitions as thin bars.

The set of arcs $\mathcal{A} = \mathcal{A}_d \cup \mathcal{A}_c$ is divided into two subsets, the discrete arcs (the elements of $\mathcal{A}_d$), and the continuous, or fluid, arcs ($\mathcal{A}_c$). The discrete arcs are drawn as single-lined arrows, whereas the fluid double-lined arcs are drawn like pipes.

The *marking* $\mathbf{m} = (s, \mathbf{z}) \in \mathcal{M}$ consists of a discrete part $s = (\#p_i, i \in \mathcal{P}_d)$, where $\#p_i$ denotes the number of discrete tokens in the i-th discrete place p_i, and a continuous part, a vector representing the fluid level in each fluid place, $\mathbf{z} = (z_k, k \in \mathcal{P}_c) \in \mathcal{Z}$. The initial marking is $\mathbf{m}_0 = (s_0, \mathbf{z}_0)$. The total number of discrete states is S, the set of all discrete states is denoted $\mathcal{S}$.

$\mathrm{r}_{t,p}$, $\mathrm{r}_{p,t} : \mathcal{A}_c \times \mathcal{M} \to \mathrm{I\!R}^2$ is the flow rate function along the fluid arc connecting the timed transition t and the fluid place p or vice versa. The flow rate $r = (\mu, \sigma^2)$ is a normally distributed random variable specified by expectation μ and variance σ^2. If it is specified by only one parameter, that one is the expectation and the variance is equal to zero.

Let Z_k be the fluid level in place $k \in \mathcal{P}_c$. The change rate of the fluid level is again a normally distributed random variable with

$$\frac{dZ_k}{dt} = \sum_{t \text{ enabled in } \mathbf{m}} r_{t,p}(\mathbf{m}) - \sum_{t \text{ enabled in } \mathbf{m}} r_{p,t}(\mathbf{m}).$$

Reflecting boundaries assure, that the fluid level in each place stays within its range $[z^{\min}, z^{\max}]$.

$g_t : \mathcal{M} \rightarrow \mathbb{B}$ is the guard of transition t that can be a function of the discrete and the continuous state. $\lambda_t : \mathcal{M} \rightarrow \mathbb{R} \cup \{\infty\}$ is a function of both the continuous and the discrete marking. Immediate transitions have firing rate $\lambda_t = \infty$.

The weight function $w_t : \mathcal{S} \rightarrow \mathbb{R}$ is defined for immediate transitions. The firing probability of each of the enabled immediate transitions in a vanishing state is $w_t(s) / \sum_{t_i \text{enabled in } \mathbf{m}} w_{t_i}(s)$.

The enabling rules for the transitions are the same as in discrete Petri net models. The fluid arcs act only on the fluid places and do not influence the enabling conditions of the transitions.

The reachability graph, for our system represented in Figure 3, gives a graphical representation of the stochastic process underlying the model. Augmenting it with continuous variables, one for each fluid place, gives a description of the full stochastic process which is formally defined as

$$\{(S(t), Z_1(t), Z_2(t)), t \in \mathbb{R}_0^+\},$$

if the model has two fluid places. $S(t)$ is a random variable for the discrete marking s at time t, and $Z_k(t)$ is a random variable, representing the fluid level in fluid place k at time t. This stochastic process again can be mapped on a system of partial differential equations with appropriate initial and boundary conditions. For these equations an efficient and stable discretization method has been implemented which is used for the analysis of the hybrid manufacturing system.

4 The Model

Figure 2 shows an FSPN model of the hybrid system. The transitions *get_roboti* represent the time it takes to position the robot where it is needed. We study here a model with only one piece in the magazine, but we are in the process of developing a more general model that can contain an arbitrary number of work pieces. Initially this work piece is in the magazine and the robot is idle. After the robot has been summoned to the magazine it picks up the metal piece and carries it to the heater. Here the work piece is put down and the robot is idle again. While the piece lies on the heater transition *heat* is enabled and *temp_of_item*, the temperature of the metal piece increases as specified by rate *r1*. For this rate, as for both the other rates, only the expectation is given and the variance is assumed to be

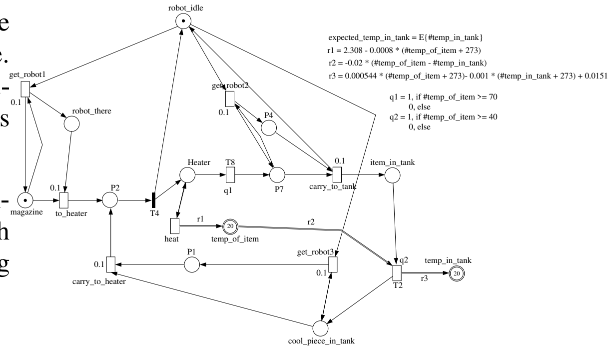

Figure 2: FSPN model of the hybrid manufacturing system

zero. Assuming a constant increase or decrease of the temperature might not be realistic, but to specify the variance of the rates is a problem we have not yet addressed. The expectation of the rates can be computed from the system description [NL00], but nothing is known about the nature of the variances. Transition *T8* has a fluid-dependent firing rate *q1* and is activated only if the temperature of the item on the heater exceeds 70°. We model here only one heater position although the original system has two, for one item can only lie in one position. Once the heating process is finished the robot is called to the heater again. It picks up the item and carries it to the water tank, in which it is placed. The temperature of the metal piece goes down (as represented by *r2* since *T2* is enabled) and the temperature of the water increases (through *r3*). Transition *T2* again has a fluid-depended firing rate and is enabled only if the item has cooled down to at most 40°. It will then still cool down a bit while the robot moves to the tank. The item is picked up and carried back to the heater to be warmed up once more.

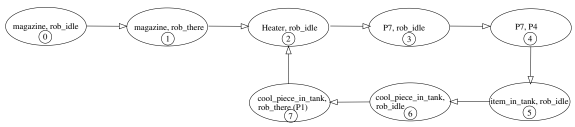

Figure 3: Reachability graph to the FSPN model

The rates *r1, r2* and *r3* are computed following the formula for temperature changes:

$$\frac{d\theta}{dt} = \frac{\dot{Q}_{in} - \dot{Q}_{out}}{mc_p} \tag{1}$$

where $\dot{Q}$ is the thermal input or output stream, m is the mass of the material whose temperature change shall be computed and c_p is the specific thermal capacity of the material. The change of temperature occurs where two substances of different temperature meet. It is proportional to the size of the contact area A and a coefficient of warmth transmission α. $\dot{Q}$ is defined as

$$\dot{Q} = \alpha A(\theta_1 - \theta_2). \quad (2)$$

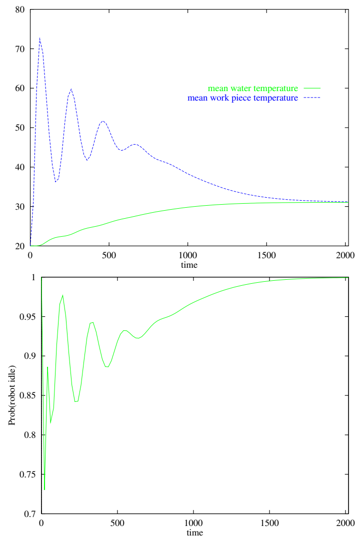

Figure 4: Mean item temperature and mean water temperature

The needed parameters for heating up the iron metal piece on the heater ($r1$), are $c_p = 0.46$, $A = 56.55cm^2$ and $\alpha = 13.85W/Km^2$. The heater has a heating power of 200 Watts (W), which is the warmth input to the metal piece. In the technical parameters temperature is always measured in Kelvin (K).

Analogously, $r2$ is the temperature change of the work piece in contact with the water and $r3$ is the temperature change of the water, when being in touch with a metal piece. All rates are computed following the same principles. Since the technical formulas always use Kelvin we have to add $^\circ Celsius + 273 = Kelvin$ to transform units in some places.

Figure 3 shows the reachability graph of this simple model. The states are labelled with the discrete marking of the net and in addition they are enumerated, for simplicity in referencing.

In Figure 4 some results are shown. The full solution of the model is a two-dimensional density at each point in time when summing over all discrete states. In each discrete state (one of which is shown in Figures 5 and 6) the curves are fractions of a two-dimensional density. Summing over the discrete states and then computing the mean values for each continuous variable yields a transient curve for the mean temperatures as shown in Figure 4 on the left. Those means converge to the same and fairly low value. This shows that the amount of water is sufficient for cooling down the metal piece.

The right curve in Figure 4 shows the robot utilization (that is Prob($\#rob\_idle = 0$)). The robot initially is used, but on the long-term average this usage is almost negligible. Most of the time the robot is idle. Consequently, the robot would be well capable of serving a system with more metal pieces. How many of them the system configuration can deal with well will be one of the questions addressed in the future.

Figures 5 and 6 show the evolution over time of the density in one selected discrete state, the state where the item is in the water tank and the robot is idle. At early times (Figure 5 upper graphs) the temperature of the item is high, while the temperature of the water is still low. Over time, the item continuously cools down while the water temperature slowly increases.

After 1000 time units, where each time unit corresponds to 1 second, the peak in the probability mass is already at the point where water and work piece temperature are almost equal. Their means (see Figure 4 on the left) are not yet equal because there is still some probability for the work piece being much warmer than the water, as can be seen even in Figure 6 on the bottom right.

The scaling of the z-axis in these graphs is not identical throughout all graphs, because the total probability mass in this discrete state is not constant. It varies between zero at the inital time and 0.88 after 1000 time units. After 2000 time units probability of the system being in this state is 0.99, so the system spends most of its time with the item in the water tank. The relative time the system spends in the discrete state with the item on the heater decreases from 0.6 in the beginning to 0.003 after 2000 time units. This shows that the heating process is very quick - due to the high power of the heater - compared with the cooling.

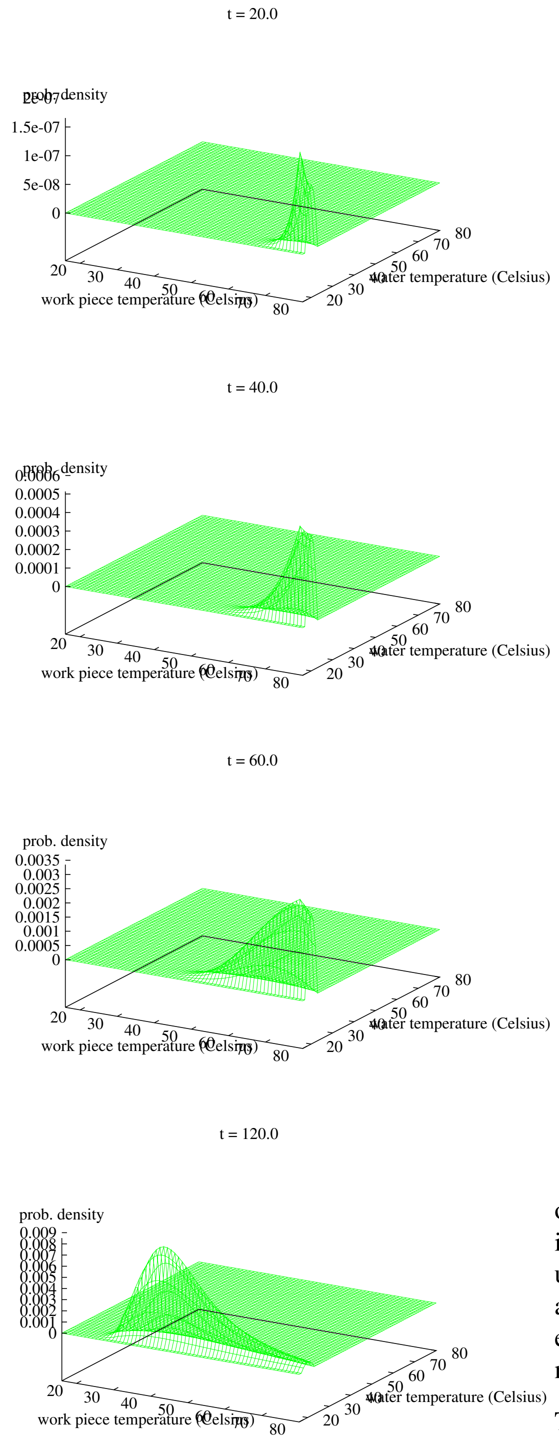

Figure 5: Density function in state 5

5 Conclusions

In this paper we have studied a hybrid manufacturing system in which temperatures are the continuous variables and the control as well as some items in the system have a discrete nature. We have set up a stochastic hybrid model for the system, which allows us to analyze utilization of the resources and enables us to come to conclusions about the performance of the system.

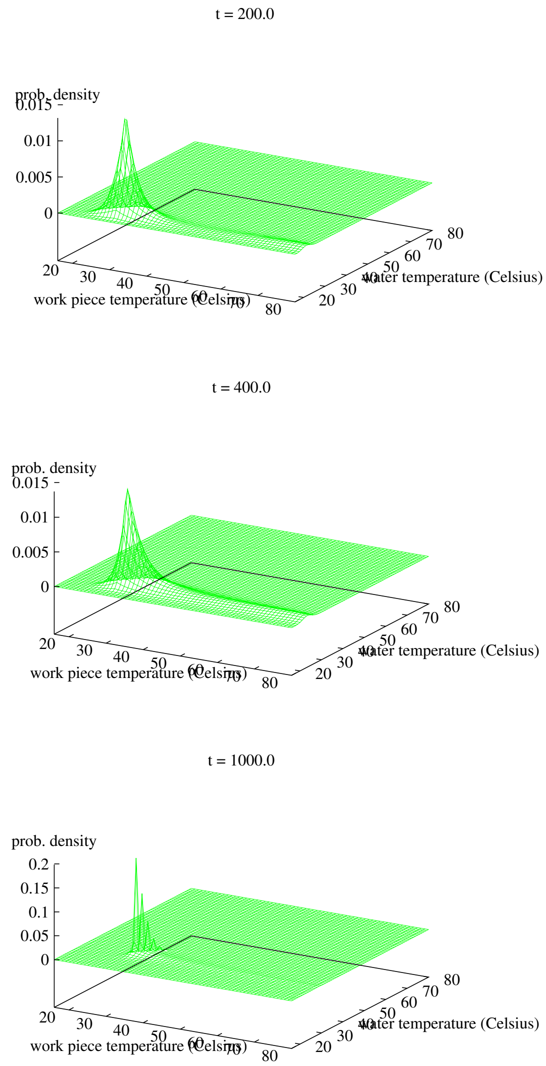

Figure 6: Density function in state 5 (cont.)

The solution of the model shows that the robot, the resource in the system, is able to deal with more work. At the same time we have seen that the water tank is sufficiently large for cooling the one work piece with the given power of the heater. The mean water temperature always stays below 35° Celsius, so no trouble from over-heating should be expected.

In the future we want to extend the model so that it can contain an arbitrary number of work pieces. We will then be able to see where the limit in the number of work pieces is for using just one robot and where the system will have bottle-necks. Furthermore we want to investigate different control strategies, such as whether the robot should give priority to taking hot items off the heater or whether it should first fill empty heater positions. Does either of these strategies heat up the water more than the other, and does either of the strategies lead to a higher throughput of the system?

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