RESEARCH STATEMENT JOHANNES BUCHNER

My past and current research can roughly be grouped in 3 parts

(1) Computer Science / Artificial Intelligence
(2) Mathematics / Dynamical Systems
(3) Critical Mathematical Economics

As my main future research interest is Critical Mathematical Economics, I will outline in the following how my background in the first two fields will help me to do research in the latter.

1. Computer Science / Artificial Intelligence

I have been interested in computers since my high school time, and, supported by a friend, developed my own chess program "JOX – Joe's own chess system" back then. I pursued this interest further during my studies of computer science (as a minor in my Bachelor in Mathematics), and my field of specialization was "Artificial Intelligence", where I also wrote my Bachelor thesis.

1.1. **Bachelor thesis on Computer Chess programming.** The aim of my Bachelor thesis [1] is to give an overview of the theory of efficient alpha-beta-searches in computer chess and to explain the practical strategies implemented in the FUSc# chess program ([2]). The main focus is put on two topics: the first one is efficient move generation with a technique called "rotated bitboards" ([3]), and the second one is how to carry out efficient alpha-beta-searches by optimizing the move-ordering with static and dynamic heuristics. Practical experiments with FUSc# are used to verify the theoretical results in both questions. They show that I was able to increase the performance of FUSc# significantly.

1.2. **Neuronal Networks and Computational Agent-Based Models in Economics.** FUSc# was also the basis for a series of publications in later years, e.g. dealing with learning mechanisms to automatically improve the playing strength after every game ([4]). This was done in the working group of Prof. Raúl Rojas, an expert on neuronal networks ([5]). In the future, I plan to build on these experiences and deal with computational agent-based models where agents dynamically learn about their environment, as Matheus Grasselli describes it in a recent paper ([16]). For more details, see section 3.2.2.

2. Mathematics / Dynamical Systems

I am currently finishing my PhD in Mathematics in the research group on "Non-linear Dynamics" of Prof. Fielder at FU Berlin. One specialty of our research group is to deal with the dynamics not only of Ordinary Differential Equations (ODEs), but also of Partial Differential Equations (PDEs) and other types of equations where the state-space becomes infinite-dimensional, like e.g. Delay Equations.

In addition to these topics, I have dealt with Analysis and Geometry on Manifolds (e.g. Differential Forms, see [10]) and Stochastic Processes - those were also the minors during my graduate studies at the Berlin Mathematical School (BMS). I also have some interest and background in General Relativity, e.g. by attending courses by Alan Renall from the Max-Plack Institute for Gravitational Physics. Consequently, my PhD builds upon a combination of these fields. More concretely, i am dealing with the "Dynamics of Cosmological Models" as part of a research project of the "Collaborative Research Center" 647 on "Space - Time - Matter" of the German Science Foundation (DFG). One of the central aims is to better understand the structure of the singularities of evolution equations, e.g. of black hole initial data for the Einstein equations ([7]), but also of the initial singularity, i.e. the "big bang".

3. Critical Mathematical Economics

3.1. **Theorem of Sonnenschein-Mantel-Debreu on stability and uniqueness of market equilibria.** The theorem of Sonnenschein-Mantel-Debreu could be seen as the "endpoint" of the neoclassical reseach program to show existence, uniqueness and dynamic stability of a market equilibria - this fails already in one of the simplest possible examples, namely in a pure exchange economy. While the existence of a market equilibrium can be proved without further assumptions (by using fixed-point theorems), the theorem of Sonnenschein-Mantel-Debreu is a "negative result" in the way that it shows the complete arbitrariness of the excess demand functions under the standard assumptions in microeconomics.

3.1.1. *Original Papers.* The now so-called "Sonnenschein-Mantel-Debreu-theorem" or "Sonnenschein-Mantel-Debreu-conditions" have their origin in a few papers from the seventies ([19, 20, 21, 22]). I have studied these papers to some extend some years ago, when preparing a talk for a seminar ([12]). I would also be interested in studying the historical origin of these results, as it seems (see e.g. [24]) that a similar result had been proved much earlier by William Gorman ([23]).

3.1.2. *Mas-Colell's proof using index theory.* In his book ([13]), Andreu Mas-Collel proves a version of the Sonnenschein-Mantel-Debreu-theorem using topological index theory. For a regular economy, it turns out that to every equilibrium one can associate in a natural manner an index equal to plus one or minus one, in such a way that the sum of these indicies is one. In particular, this implies that an equilibrium exists because the number of equilibria must be odd. The key insight is, however, that it is not possible to derive any stronger theorem from the general hypotheses describing an exchange economy. To get restrictions on the equilibrium set beyond the ones yielded by the index theorem (like dynamic stability, for example), it is necessary to consider special classes of economies. The central result is the theorem below: It tells us that, except at the boundary, we are dealing with an arbitrary vector field $f(p)$, and in general, even for a unique equilibrium, it is possible for the dynamics not to approach this equilibrium from any initial point, e.g. because of the existence of a stable periodic orbit surrounding the (unstable) equilibrium.

Theorem. *Let $f : S \to \mathbb{R}^l$ be a C^3- vector field satisfying some boundary conditions. Then for any $\epsilon > 0$ we can find an economy E such that the excess demand function of E coincides with f on S_ϵ and p is an equilibrium for E if and only if $f(p) = 0$, i.e. no new equilibrium is added.*

The theorem and description above is taken (with some simplifications) from chapter 5 on exchange economies from [13]. I find the approach mathematically appealing and would be interested in studying this in detail, especially to check the genericity of "regular economies" that the author claims in later parts of the book. This might also be important in the discussion on the economic consequences of the results.

3.1.3. *Exterior Differential Calculus and Sonnenschein's problem.* As mentioned above, I am interested and have some background in differential forms, which some people call exterior differential calculus. I was very astonished to see the latter having applications in economics, and in the paper [18] it is used to address the problem of characterizing aggregate demand of a market economy, which the authors call "Sonnenschein's problem".

In the paper, it is stated that, from a mathematical standpoint, the ideas of maximization and aggregation have a natural translation in terms of combination of gradients. Specifically, this means that for a function $X : \mathbb{R}^n_+ \to \mathbb{R}^n$ representing aggregate behavior, the question is if it can be decomposed as a linear combination of gradients $D_p V^k(p)$, where $V^k, k = 1...K$ are functions defined on $\mathbb{R}^n_+$:

$$X(p) = \sum_{k=1}^{K} \lambda_k(p) D_p V^k(p)$$

A natural question, initially raised by (Sonnenschein, 1972) is the following: what does the above relation imply upon the form of the function X? In particular, are there testable necessary restrictions

on the aggregate function $X(p)$ that reflect its decomposability into individual maximizing behaviour? And is it possible to find sufficient conditions on $X(p)$ that guarantee the existence of a decomposition of the above type?

I find the mathematics used above interesting, although I am of course not an expert in Symplectic Geometry / Topology where the methods mentioned above seem to come from. But the lecturer of my classes on "Analysis and Geometry on Manifolds", Klaus Mohnke, has a specialization in Symplectic Geometry, and was teaching us some basic ideas of the field back then. Also, I did partly attend a lecture on Symplectic Geometry by H.G. Bothe at FU Berlin in the winter 2010/2011. For me, it would be interesting to check whether using the language of differential forms and symplectic geometry has an advantage to deal with questions of economic aggregation. The relevant question is if the approach presented above is more a neat mathematical way of putting well-known things, or if it substantially adds to the mathematical understanding of the economic questions asked by the Sonnenschein-Mantel-Debreu.

3.2. **Building a twenty-first-century economics.** Probably a twenty-first-century economics will be a melange of many heterodox schools of thought, like e.g. Post-Keynesian macroeconomics, evolutionary microeconomics, and complexity theory (see e.g. [24]). Below, I have collected some research ideas from these fields. I think it is necessary to get a broad overview of opportunities where substantial mathematics can be combined with critical economics. I would love to do research in such a direction, in order to contribute to building a coherent and appealing alternative to the mainstream.

3.2.1. *Post-Keynesian macroeconomics, financial systems and stability.* Currently, the Post-Keynesians (see e.g. [17]) are viewed as "the most coherent alternative school of economic thought today", and they are also likely "to gain substantial credence in the recent financial crisis", given their "explicit monetary approach to economics" ([24], p.450). Two key features of the Post-Keynesian school are the concept of endogenous money, see [17, 15] and the distinction between risk and (fundamental) uncertainty ([17]). This has tremendous consequences on the policy recommendations e.g. in Europe, where Post-Keynesians have a very critical view on the austerity programs imposed on the weaker economies of the Euro-zone ([15]). As I have dealt with European economic policy intensively in the past, e.g. during my internship at the European Parliament in Brussels, I am very interested in such questions. One interesting project could be to study in detail which (macro-)economic assumption leads to which policy conclusion, in order to deconstruct the belief of the general public that the policy of austerity is backed up by an economic theory without alternatives, which is very prominent especially in Germany.

Another important feature of Post-Keynesian macroeconomics is Minsky's Financial Instability Hypothesis. In the paper [14], Grasselli and Lima describe that it "links the expansion of credit with the rise of asset prices and the inherent fragility of the financial system". They provide a mathematical analysis of the system of differential equations proposed by Keen to model the features of Minsky's hypothesis. In the end of the paper, they propose to "replace the deterministic interest rate by a combination of a base rate plus a risk premium that increases when the asset price crashes. Extended in this way, the model becomes fully stochastic" ([14], p. 209). I'm very interested in exploring the consequences of such an extension, both because it is economically relevant as well as because it would allow me to combine my knowledge in Dynamical Systems with some topics from Stochastic Analysis and Financial Mathematics, two areas of mathematics that I plan to study more intensively in the future.

Also, I would like to deal intensively with Dynamic Stochastic General Disequilibrium (DSGD)-models of the so-called "Bielefeld school", as for example discussed in the book [25]. These models can be viewed as a (mathematically) sophisticated formulation of certain Post-Keynesian elements, and they thus constitute an important source for further research in critical mathematical economics.

3.2.2. *Evolutionary Microeconomics and computational agent-based models.* One reason why Dynamic Stochastic General Equilibrium (DSGE)-models are so popular is because of their micro foundation on neo-classical General Equilibrium Theory. In the paper [16], Grasselli and Ismail articulate two main points of critique of such a microfoundation - they claim that "both 'rational expectations'

are an inherently inappropriate way to make forecasts under frequent unanticipated changes in the environment and that models based on 'representative agents' merely assume away the solution of the aggregation problem, entirely disregarding the powerful negative results on stability and uniqueness of equilibrium provided by the Sonnenschein-Mantel-Debreu theorems" ([16] p. 2, see [19, 21, 22] and section 3.1).

They propose to use an "agent-based computational model instead, which does not rely on any free-floating notion of equilibrium either, with the possible outcomes being the result of the interactive dynamics for agents, as is the case with most complex adaptive systems". In the paper, they develop such a model in the context of banking systems, which I find very interesting. They are confident that "agent-based computational models constitute an important new weapon in the arsenal of statistical, mathematical and economic methods deployed to understand and mitigate systemic risk in modern banking systems" ([16], p. 25).

As massive computation power is necessary for these simulations, the use of GPUs (Graphics Processing Units) might be an option for the future. I would like to try that, and for this project it could be useful that I have some background in "low-level-coding" and even in machine language, because my chess program "JOX" was written in the programming languages C and Assembler. I would also like to explore possibilities to use methods from artificial intelligence like automated learning algorithms in this context, and i can build on some experience and have good contacts in this field, see section 1.2.

Also, i have established contacts to the research group of Wolfram Elsner on Evolutionary Mircoeconomics in Bremen. They have published a what they call 'Non-Toxic' Textbook ([26]) on the "Microeconomics Of Interactive Economies", and we are discussing about applying for money from the German Science Foundation (DFG) together for future common research projects.

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