# Estimation of non-stationary Processes : Part II (31.10.2008)

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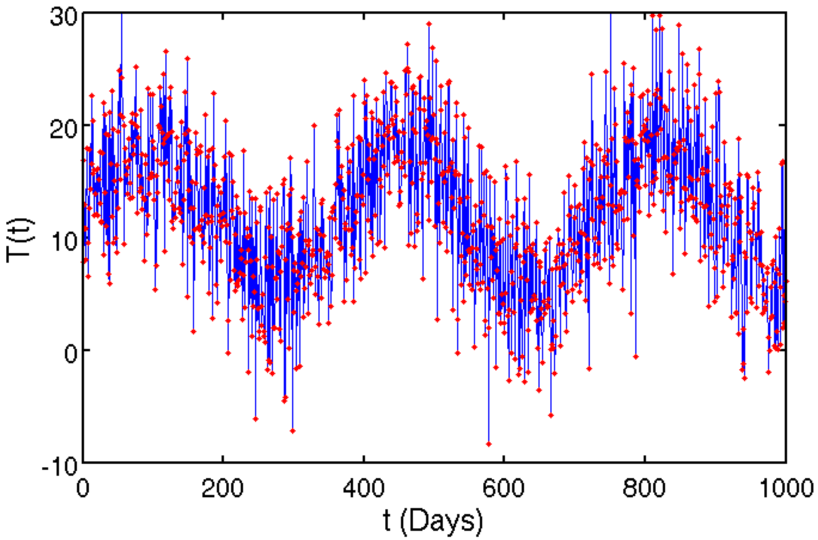
„Mathematics in key technologies"

# Memo III: Data-Interpolation




*Hypothesys:*

$$T(t) = C_0 + \sin\left(\frac{2\pi}{365.4}t\right) * (C_1 t + C_2) + C_3\epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0,1)$$

$$\{C_0, C_1, C_2, C_3\} - ?$$

*In General:*

*given* $(x_i, y_i) \quad i = 1, 2, \ldots, N$, *identify the “optimal“ parameters of a certain function* $y = F(x, \theta)$ *such that:*

$$\sum_{i=1}^{N} \|y_i - F(x_i, \theta)\| \to \min_{\theta}$$

$$\theta \in \Theta$$

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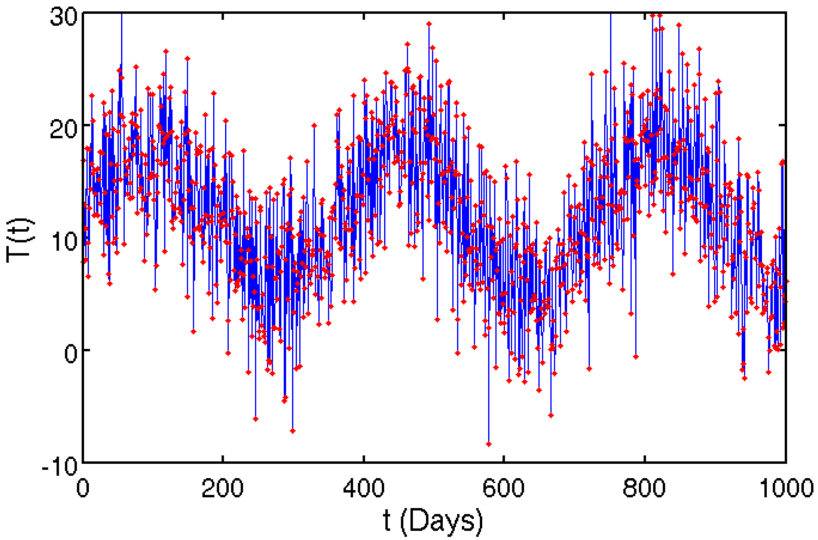


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$$\|\theta_k\| < C_i$$

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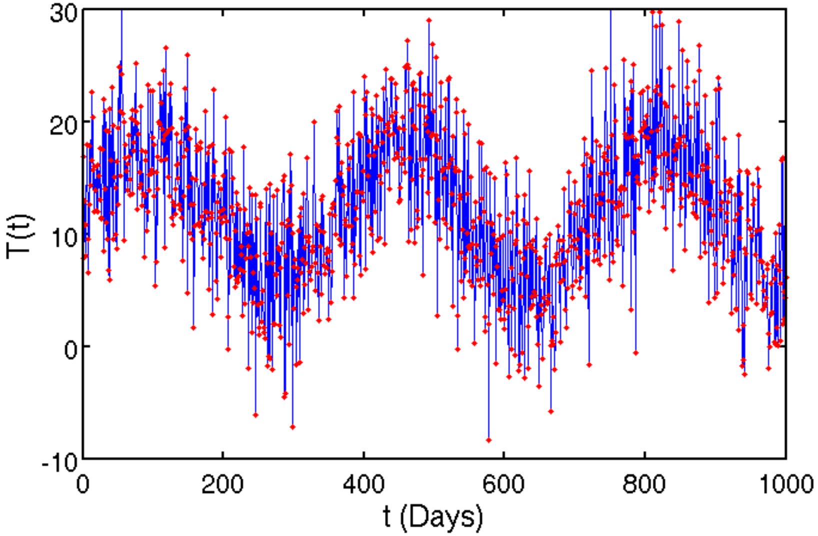


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*Tykhonov-Regularization:*

$$\mathcal{L}(\theta) = \sum_{i=1}^{N} \|y_i - F(x_i, \theta)\| + \epsilon \sum_k \|\theta_k\| \rightarrow \min_\theta$$

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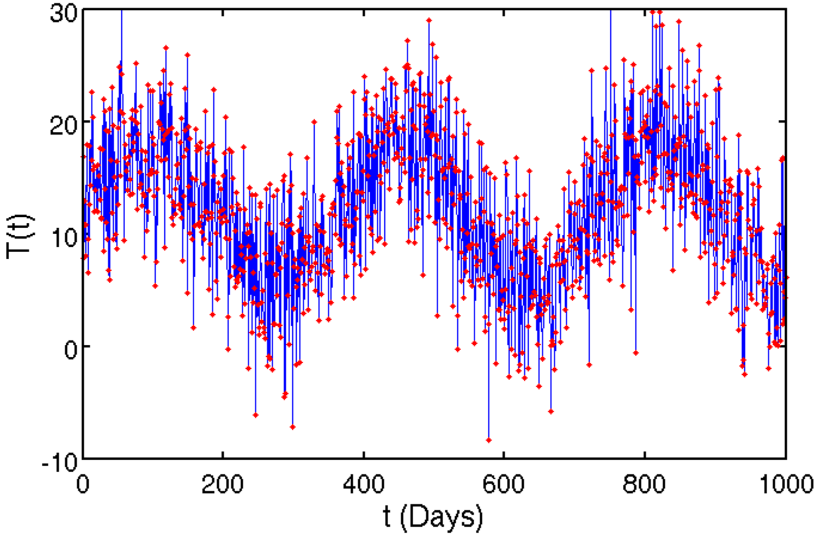


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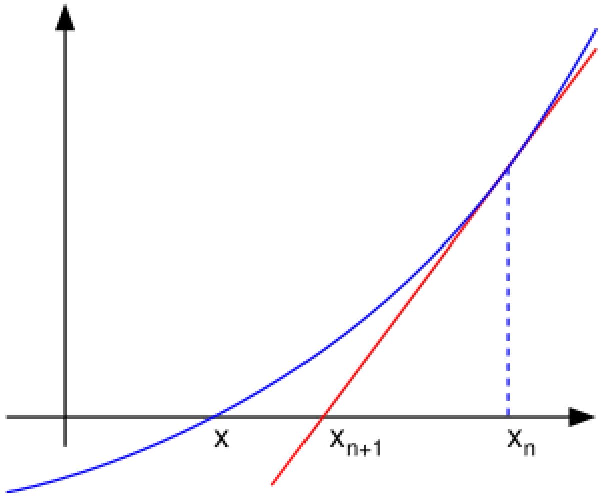
*Numerical Minimisation: Newton's-Method*

$$\frac{\partial}{\partial\theta}\mathcal{L}(\theta) = 0$$

# Memo IV: Newton‘s-Method



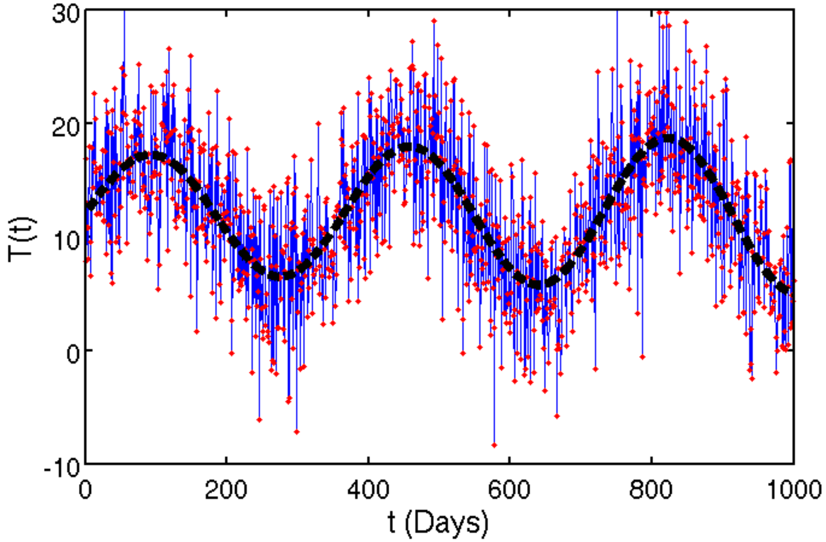

$$f'(x_n) = \frac{\text{rise}}{\text{run}} = \frac{\Delta \text{y}}{\Delta \text{x}} = \frac{f(x_n) - 0}{x_n - x_{n+1}}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

*In case of many variables:*

$$J_F(x_n)(x_{n+1} - x_n) = -F(x_n)$$

# Memo III: Data-Interpolation

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$$\epsilon_t \sim \mathcal{N}(0,1)$$

$$\{C_0, C_1, C_2, C_3\} - ?$$

*Fitted Trend Model:*

$$T(t) = 12 + \sin\left(\frac{2\pi}{365.4}t\right) * (0.002t + 5) + 5\epsilon_t$$

# Parametric Estimation

*H., submitted to Journal of Atmos. Sci. (2008)*

*Markovian Trend Model*:

$$P(t) \;=\; P^{(0)} + P^{(1)}\phi(t), \quad \phi : [1, T] \rightarrow (-\infty, +\infty)$$

# Parametric Estimation

*Markovian Trend Model*:

$$P(t) \;=\; P^{(0)} + P^{(1)}\phi\,(t)\,, \quad \phi : [1,T] \to (-\infty, +\infty)$$

*Log-Likelihood*:

$$\begin{aligned}
\sum_{j=1}^{m} \sum_{t \in \{t_{ij}\}} \log\left(P_{ij}^{(0)} + P_{ij}^{(1)}\phi\,(t)\right) \;&\to\; \max_{P^{(0)},P^{(1)}}\,,\\
\sum_{j=1}^{m} P_{ij}^{(0)} \;&=\; 1,\\
\sum_{j=1}^{m} P_{ij}^{(1)} \;&=\; 0,\\
P_{ij}^{(0)} + P_{ij}^{(1)} \sup_{t\in[1,T]} \phi(t) \;&\geq\; 0, \quad \text{for all } j,\\
P_{ij}^{(0)} + P_{ij}^{(1)} \inf_{t\in[1,T]} \phi(t) \;&\geq\; 0, \quad \text{for all } j.
\end{aligned}$$

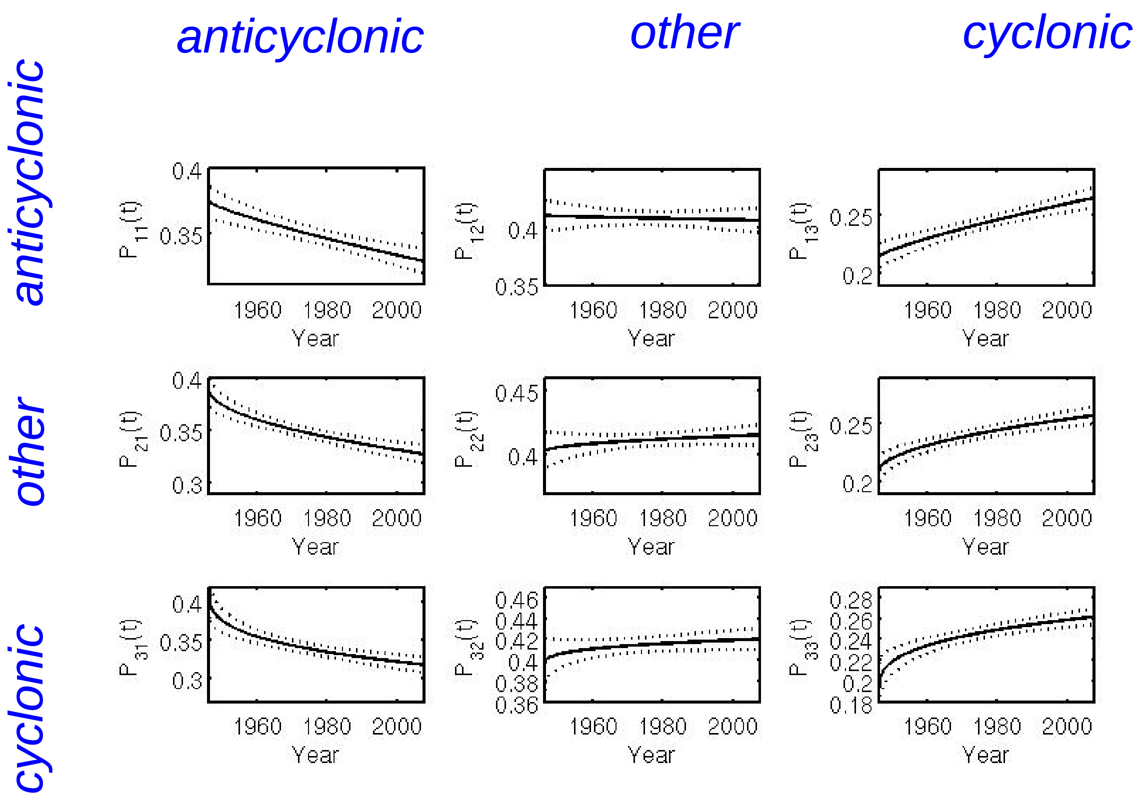
*Numerics: Nelder-Mead Optimization Algorithm*

# Circulation Patterns for UK(1945-2007)

*Historical Circulation Data: weather regimes*
*(Data from the Univ. of East Anglia)*
*3 atmospherical states considered*

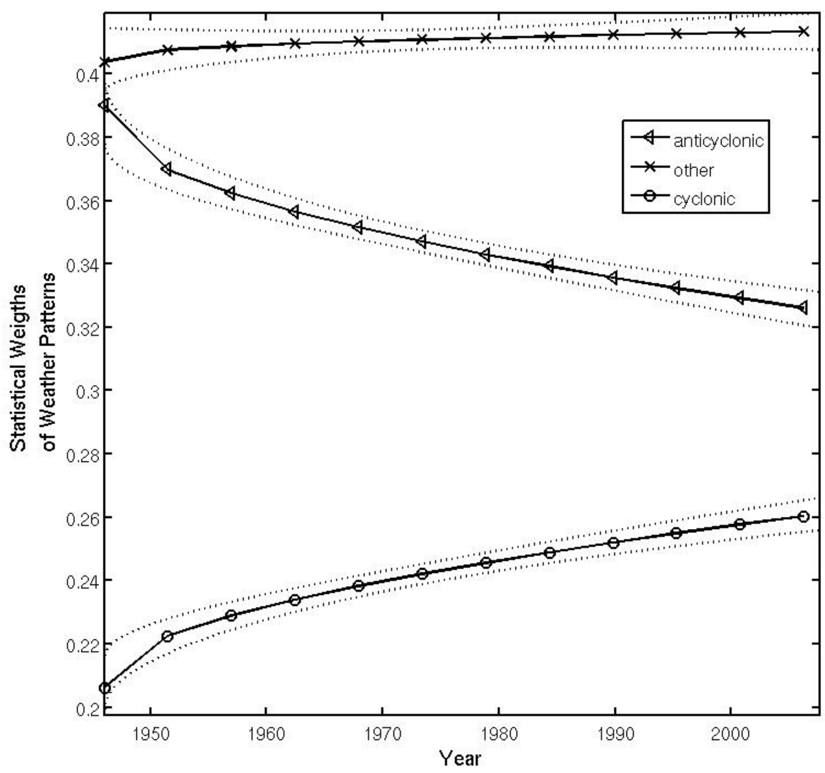
**Polynomial Trend Model**

*anticyclonic* | *other* | *cyclonic*



# Circulation Patterns for UK(1945-2007)

$$\pi(t)\,P(t) = \pi(t)$$



# Conclusion:

What kinds of Markov estimators do we know now?

- Stationary Estimator (standart)
- Locally Stationary Estimator (Gaussian window, non-parametric)
- Non-Stationary Single Trend Estimator (regression, parametric)


**Thank you for attention!**