Min-Symposium 1

Numerical modeling and solution methods for the multi-domain multi-scale models

Organizer: Fraçois-Xavier Roux

Speakers:

- Hachmi Bendhia
- Christian Rey
- Frédéric Feyel,
- David Dureissex
- François-Xavier Roux

The Arlequin methodology

Hachmi Ben Dhia
LMSSMat, Ecole Centrale Paris, UMR 8579

Abstract:

The Arlequin method [1], [2] is a general numerical modeling methodology that superposes models and glues them to each others while partitioning the enrgies. Thus, by construction, this approach allows for local multimodel and multiscale analyses of existant global coarse numerical models. This multiscale methodology will be explained, analyzed [3], [4] and exemplified (e.g. [5]). As a matter of fact, the Arlequin method which is actually a local multimodel partitioning framework leads to discrete problems that are rather similar to the ones obtained when using Domain Decomposition Methods to solve monomodel problems derived from PDE's. Thus one can take advantage of the development made in the latter field to solve the discrete Lagrangian or augmented lagrangian Arlequin problems as done in [6].

References:

[1] H. Ben Dhia, Multiscale mechanical problems : the Arlequin method, Comptes Rendus de l'Acad'mie des Sciences, S'rie IIb, 326, (1998) 899-904.
[2] H. Ben Dhia, Numerical modeling of multiscale problems: the Arlequin method, in Proceedings of the First European Conferenceon Computational Mechanics, Muenchen, (1999).
[3] H. Ben Dhia, G. Rateau, Mathematical analysis of the mixed Arlequin method, Comptes Rendus de l'Acad'mie des Sciences Paris S'rie I, 332, (2001) 649-654.
[4] H. Ben Dhia, Global-Local approaches: the Arlequin framework, European Journal of Computational Mechanics, 15, (1-3), (2006) 67-80.
[5] H. Ben-Dhia, G. Rateau, The Arlequin method as a flexible engineering design tool, International Journal for Numerical Methods in Engineering, 62, (2005) 1442-1462.
[6] H. Ben Dhia, N. Elkhodja, F-X Roux, Multimodeling of multi-alterated structures in the Arlequin framework. Solution with a Domain-Decomposition solver (selected in European Journal of Computational Mechanics, 2007)

Domain decomposition and non linear relocalization with or without overlapping

Chritian Rey, Julien Pebrel, Pierre Gosselet
Ecole Normale Supérieure de Cachan
christian.rey@lmt.ens-cachan.fr

Abstract:

We consider the evolution of large structure undergoing localized nonlinear phenomenon (plasticity, damage, cracking, microbuckling …). In that case, even if most of the structure remains elastic, the convergence of the nonlinear solver (typically Newton-Raphson algorithm) is driven (and then slowed) by

the local phenomenon. We propose a strategy intending to solve the nonlinearity at a local scale. The method relies on a domain decomposition introduced by a partition of the unity applied to the energy of the system. The connection is insured using augmented Lagrange multipliers, so that the method can be interpreted as finding "good" mixed conditions on the interface.
Then a classical algorithm (FETI, BDD) is employed, with local treatment of the non-linearity: in order to find the exact interface conditions, the nonlinear problem is condensed on the interface, then it is linearized leading to the resolution of a succession of linear interface problems (global) and nonlinear subdomain problems (local). In our presentation, the method will be first exposed and related to existing strategies, then assessed on academic problems (and if possible more complex situations).

FE² and similar methods: overview and link with domain decomposition

Frédéric Feyel
ONERA – MMSD
frederic.feyel@onera.fr

Abstract:

The FE² term has been introduced some years ago to designate multiscale models where the mechanical behavior of the representative volume element is computed, when needed, using a direct finite element computation over a unit cell.

Such models lead to a numerical framework in which the macroscopic behavior is computed as usual using the finite element method. During this macroscopic computation when one has to compute the material response, an embedded finite element computation is done over the unit cell. It thus results in simultaneous finite element computations at both scales. Domain decomposition methods are easily used at the macroscale to speed-up the computation.

Some interesting questions have been raised about the multiscale status of these FE² methods, and more generally about what are multiscale methods, from both a computational point of view and a mechanical point of view. Some insights will be given during the talk around this question.

The talk will be organized as a travel based on the size of the material heterogeneities. For each order of magnitude of this representative material size, we will try to propose methods able to compute heterogenous structures for each zone, including FE² models.

A multiscale domain decomposition for the simulation of a non smooth structure, involving a numerical homogenization

Damien Iceta, Pierre Alart and David Dureisseix
Laboratoire de M´ecanique et G´enie Civil (LMGC)
University Montpellier 2 / CNRS UMR5508
Place E. Bataillon, F-34095 MONTPELLIER CEDEX 5, FRANCE
{iceta,alart,dureisseix}@lmgc.univ-montp2.fr

Abstract:

The simulation of a granular media behavior, with contact and friction between grains, is a typical example of a non smooth mechanical system [8]. A second example, for which no reconfiguration of connectivity occurs due to large displacements, is the dynamical evolution of a tensegrity structure [9]. This is and innovative civil engineering structure, composed with bars and cables. Non smoothness is diffuse over the whole structure, due to possible slackness of cables under overloading [10].
When a large scale structure is considered, a domain decomposition is used to solve the problem. Since the constitutive behavior is a non univoque and non smooth relation between primal and dual quantities, a mixed approach is used: both forces and displacements are the unknowns [5, 3]. With the proposed approach, non smoothness is localized within the subdomains, while the interfaces possess a perfect behavior. This choice is similar to the one used in [2], and somehow the dual of the one used in [7] where non linearities are isolated on the interfaces.
The splitting of mechanical fields arising on interfaces (forces and displacements) into a macroscopic part and a microscopic complement allows to build a multiscale approach, for which the relationship between macro forces and dis-placements leads to an homogenized behavior of the subdomains [6]. Thesefields constitute the coarse space of the multilevel domain decomposition [1].
The solver should deal with both the primal and dual unknowns. At least two different solvers can be used: (i) a dedicated version of the Large Time INcrement approach [5], similar to an augmented Lagrangian approach, and (ii) a derived version of the Non Smooth Contact Dynamics approach [4, 8], similar to a non smooth Jacobi/Gauss-Seidel solver.

References:

[1] P. Alart and D. Dureisseix. A scalable multiscale LATIN method adapted to nonsmooth discrete media. Computer Methods in Applied Mechanics and Engineering, 197(5):319–331, 2008.1
[2] M. Barboteu, P. Alart, and M. Vidrascu. A domain decomposition strategy for nonclassical frictional multi-contact problems. Computer Methods in Applied Mechanics and Engineering, 190:4785–4803, 2001.
[3] L. Champaney and D. Dureisseix. A mixed domain decomposition ap-proach. In F. Magoul`es, editor, Mesh Partitioning Techniques and Do- main Decomposition Methods. Civil-Comp Press / Saxe-Coburg Publica- tions, 2007. To appear.
[4] M. Jean. The non-smooth contact dynamics method. Computer Methods in Applied Mechanics and Engineering, 177:235–257, 1999.

[5] P. Ladev`eze. Nonlinear Computational Structural Mechanics — New Approaches and Non-incremental Methods of Calculation. Springer Verlag, 1999.
[6] P. Ladev`eze and D. Dureisseix. A micro / macro approach for parallel computing of heterogeneous structures. International Journal for Compu-tational Civil and Structural Engineering, 1:18–28, 2000.
[7] P. Ladev`eze, A. Nouy, and O. Loiseau. A multiscale computational ap-proach for contact problems. Computer Methods in Applied Mechanics and Engineering, 191:4869–4891, 2002.
[8] J. J. Moreau. Numerical aspects of sweeping process. Computer Methods in Applied Mechanics and Engineering, 177:329–349, 1999.
[9] R. Motro. Tensegrity. Hermes Science Publishing, London, 2003.
[10] S. Nineb, P. Alart, and D. Dureisseix. Domain decomposition approach for nonsmooth discrete problems, example of a tensegrity structure. Computers and Structures, 85(9):499–511, 2007.

Non conforming FETI-2LM method

François-Xavier Roux
ONERA and University of Paris 6
roux@onera.fr

Abstract:

For heterogeneous composite materials, it may appear relevant to use non conforming meshes in order to be able to have highly refined regions in the mesh.
In such a context, the FETI-2LM method, based on Robin interface matching conditions, that is very efficient for highly heterogeneous subdomains, appears to be the solution method of choice. In this paper, we present the extension of the FETI-2LM method using a recently introduced generalization of the mortar method for non conforming meshes.
This method will be compared to another approach using also the FETI-2LM method, consisting in localizing the non conforming interfaces by putting both sides of the interface inside one subdomain.

Mini-Symposium 2a and 2b

DE-Constrained Optimization

Organizers:

- Hoppe
- Kornhuber
- Krause

Speakers:

- Grease
- Gross
- Hintermuller
- Hinze
- Schiela
- Tai
- Weiser
- Xu
- Zulehner
- +2 of the organizers

Global convergence of nonsmooth Newton methods for constrained minimization

Carsten Gräser
Freie Universität Berlin

Abstract:

Constrained minimization problems with inequality constraints can be formulated as nonsmooth equations in various ways. Nonsmooth Newton methods turned out to be an efficient approach for the solution of such problems. Unfortunately the particular algorithms converge only locally and rely on exact solution of the subproblems in general. Common approachs try to overcome this by the construction of artificial merit functions.

We show that natural problem inherent energies can be used as merit functions if the nonsmooth nonlinear strategy is chosen appropriately. The presented approachs lead to global convergence even in the case of very inexact solution of the linear subproblems. A Multiscale Approach for Convex and Non-convex Constrained Minimization.

C. Gross, R. Krause
Institute for Numerical Simulation, University of Bonn

Abstract:

In this talk, we present a solution strategy for convex and non-convex minimization problems subject to pointwise constraints. This strategy is based, on the one hand, on a non-linear multilevel algorithm and, on the other, on a trust-region strategy to solve the occuring non-linear minimization problems. Hence, the function is minimized iteratively by generating a sequence of admissible corrections. By imposing only slightly restrictive assumptions on the function and on the corrections, we are able to prove convergence to first- and second-order critical points, as well as, local quadratic convergence. We furthermore illustrate the effectiveness of our implementation of this algorithm by presenting results from three-dimensional non-linear elasticity.

A fully practical semi-smooth Newton method in variational-discrete pde constrained optimization

Michael Hinze
(Joint work with Morten Vierling)
Hamburg

Abstract:

We consider a semi-smooth Newton algorithm for variational discretizations of control constrained elliptic and parabolic optimal control problems. In this approach only the solution operator associated with the pde is discretized while the control itself is not. For this discrete approach we present a semismooth Newton method which operates in function space but is fully implementable as its iterates can be represented on refinements of the original triangulation allowing jumps along the border between active and inactive set.
We prove fast local convergence of the algorithm and propose a structure exploiting globalization strategy. Numerical tests for elliptic and parabolic optimal control problems with box constraints on the control confirm our analytical findings. We further present numerical results for state constrained elliptic optimal control problems based on the Lavrentiev relaxation.

Interior Point Methods in Function Space for State Constrained Optimal Control

Anton Schiela
ZIB Berlin

Abstract:
We consider a class of interior point methods that can be applied to PDE constrained optimization problems with state constraints. As a distinguishing feature, these methods are constructed and analysed in function space. This approach makes it easier to exploit the analytical structure of the optimal control problem. Moreover, implementations are solely based on quantities that are well defined in function space.

We study structure and convergence of homotopy paths generated by logarithmic and rational barrier functions. It turns out that a proper type of barrier function has to be chosen in order to guaranty existence and strict feasibility of solutions in function space.

For the right type of barrier functions, the convergence of corresponding primal Newton path-following schemes to the solution of the original state constrained problem can be established.

We propose, as an algorithmic modification, a pointwise damping strategy, which retains the theoretical properties of the pathfollowing scheme, but leads to significantly faster convergence in practice, compared to the pure primal Newton corrector. We conclude with numerical experiments that show the efficiency of this method.

Hyperthermia Treatment Planning with Interior Point Methods

Martin Weiser
ZIB, Berlin

Abstract:

The talk will review the problem of designing an optimal cancer treatment based on heating large tumors by microwave radiation. Modelling of heat distribution inside the human tissue is addressed. Specific aspects of the problem with strong impact on interior point methods used for the solution of the arising PDE-constrained semi-infinite program are discussed and illustrated with numerical examples.

Solving A Saddle Point Problem by Solving One Related Symmetric Positive Definite System

Jinchao Xu
PennState

Abstract:

In this talk, I will first explain how the solution of a saddle point problem can be reduced to that of a nearly singular symmetric positive definite system. I will then explain how these nearly singular systems can be solved in an optimal and robust fashion by using properly designed iterative procedure based on the method of subspace corrections.

Patch smoothers for saddle point problems with applications to PDE constrained optimization problems

Renè Simon,
SFB 013, Johannes Kepler University Linz, Altenbergerstrasse 69, A-4040 Linz, Austria;
e-mail: rene.simon@sfb013.uni-linz.ac.at

Walter Zulehner*,
Institute of Computational Mathematics, Johannes Kepler University, Altenbergerstrasse 69, A-4040 Linz, Austria;
e-mail: zulehner@numa.uni-linz.ac.at

Abstract:

In this talk we consider a multigrid method for solving the discretized optimality system (Karush-Kuhn-Tucker system, in short: KKT system) of a PDE-constrained optimization problem. In particular, we discuss the construction of an additive Schwarz-type smoother for a certain class of elliptic optimal control problems. Each iteration step of the additive Schwarz method requires the solutions of several small local saddle point problems. Strategies for constructing the local problems are presented, which allow a rigorous multigrid convergence analysis.

Mini-Symposium 3

Computational and Programming techniques for Domain Decomposition.

Organizer: O. Pironneau.

Speakers:

- Pascal Have
- Frederic Hecht
- Alexei Lozinski
- Frederic Nataf

Some Domain decomposition methods in freefem++

Frédéric Hecht   Olivier Pironneau *
Universit_e Pierre et Marie Curie-Paris6, UMR 7598 Laboratoire Jacques-Louis Lions, Paris, F-75005 France

I will present an example of domain decomposition technique, one in in sequential and in parallel framework, the implementation of the discontinuous enrichment method (DEM) [3] with mesh adaptation.
First FreeFem++ is a free software to solve bidimensional PDE's and the characteristics of FreeFem++ are:

- Problem description (real or complex valued) by their variational formulations, with access to the internal vectors and matrices if needed.
- Multi-variables, multi-equations, bidimensional (or 3D axisymmetric) , static or time depen-dent, linear or nonlinear coupled systems; however the user is required to describe the iterative procedures which reduce the problem to a set of linear problems.
- Easy geometric input by analytic description of boundaries by pieces; however this software is not a CAD system; for instance when two boundaries intersect, the user must specify the intersection points.
- Automatic mesh generator, based on the Delaunay-Voronoi algorithm. Inner point density is proportional to the density of points on the boundary [?].
- Metric-based anisotropic mesh adaptation. The metric can be computed automatically from the Hessian of any FreeFem++ function [1].
- High level user friendly typed input language with an algebra of analytic and _nite element functions.
- Multiple finite element meshes within one application with automatic interpolation of data on different meshes and possible storage of the interpolation matrices.
- A large variety of triangular finite elements : linear and quadratic Lagrangian elements, dis-continuous P1 and Raviart-Thomas elements, elements of a non-scalar type, mini-element,. . . (but no quadrangles).
- Tools to define discontinuous Galerkin formulations via finite elements P0, P1dc, P2dc and
- keywords: jump, mean, intalledges.
- A large variety of linear direct and iterative solvers (LU, Cholesky, Crout, CG, GMRES, UMFPACK) and eigenvalue and eigenvector solvers.
- Near optimal execution speed (compared with compiled C++ implementations programmed directly).
- Online graphics, generation of ,.txt,.eps,.gnu, mesh _les for further manipulations of
- input and output data.
- Many examples and tutorials: elliptic, parabolic and hyperbolic problems, Navier-Stokes flows, elasticity, Fluid structure interactions, Schwarz's domain decomposition method, eigen-value problem, residual error indicator...

- An parallel version using mpi

1 The DEM method to solve a Poisson Problem

The Discrete Enrichment Method of Charbel Farhat et al solves

$$-\nabla\cdot\left(A\nabla u\right)= f \in\Omega, u|_{\Gamma}=0$$

by

$$a(u,w)+b(\lambda,w)= f,w),\quad b(\mu,u)=0\quad \forall w,\mu\in V_h\times\Lambda_H$$

where, $\sum=\bigcup E_k$ being the lines of discontinuity of w,

$$a(u,w)=\int_\Omega \nabla W^T A\nabla u,\quad b(\lambda,w)=\int_\Sigma [w]\lambda$$

Let T_H be a triangulation of where Ω is the approximate length of the edges E of the triangles T, and let E_K be the internal edges of T_H. Each triangle T of T_H is divided into smaller triangles of approximate size h independently. On each T if the value of u at the 3 vertices are known as well as its integrals on the 3 edges, then the values on the vertices of the small triangles can be found by a local problem. This is because, for any continuous piecewise linear U on T_H and any $\lambda\in\Lambda$ there is a unique solution u which is zero the vertices of T_H and has mean zero on the edges E and is solution of

> $a(u,w)=(f,w)+a(U,w)-b(\lambda,w)\quad\forall w$ Continuous piecewise linear on T, zero on ∂T

Furthermore this problem decomposes in N independent problems on each of the N triangles of T_H.

The interpolation space V_h is made of functions which are continuous at all the vertices of T_H and inside each T of T_H but discontinuous at the edges E. However we require that the mean jump $\int_E [u]=0$ on each edge of T_H and this constraint is enforced by a Lagrange multiplier λ constant on E. The set of piecewise polynomial functions of
degre k on the edges E is called Λ_H.

I will show how to implement this algorithms in freefem++ in sequential, in parallel,and a how to do mesh adaptation on each finite mesh of triangle $T\in T_H$.

References

[1] F. Hecht The mesh adapting software: bamg. INRIA report 1998. http://www-rocq.inria.fr/gamma/cdrom/www/bamg/eng.htm.
[2] F. Hecht, K. Ohtsuka, and O. Pironneau. FreeFem++ manual. Universite Pierre et Marie Curie, 2002{2005. on the web at http://www.freefem.org/ff++/index.htm.
[3] Charbel Farhat, Isaac Hararib, and Leopoldo P. Franca The discontinuous enrichment method Computer Methods in Applied Mechanics and Engineering Volume 190, Issue 48, 28 September 2001, Pages 6455-6479

Harmonic Finite Element Refinments

Alexei Lozinski

Institut de Mathématiques de Toulouse, Université Paul Sabatier

31062 Toulouse, France

Abstract:

We discuss a method for the numerical solution of elliptic problems with multi-scale data using multiple levels of not necessarily nested grids. The first version of the method was studied in [1]. It consists in calculating successive corrections to the solution in small sub-domains (patches) whose fine triangulations are not necessarily conforming to the coarse triangulation of the whole domain. While this method works fine in the case of nested triangulations, its convergence can be very slow if the fine grid is slightly shifted with respect to the coarse grid. We present here a new variant of the method (see [2]) whose convergence rate is essentially independent of the mutual placement of the two grids and which turns out to be significantly faster than the original one. The novelty of the method consists in restricting finite element functions on the coarse grid to be approximately harmonic inside the sub-domain where a finer triangulation is applied. We will discuss also alternative implementations of these methods that further reveal their connections with Schwarz domain decomposition algorithms without overlapping.

1. R. Glowinski, J. He, A. Lozinski, J. Rappaz and J. Wagner, Finite element approximation of multi-scale elliptic problems using patches of elements, *Numer. Math.* (2005) **101**(4), 663 – 687.
2. J. He, A. Lozinski and J. Rappaz, Accelerating the method of finite element patches using approximately harmonic functions, *Comptes rendus Mathematique* (2007) **345**(2), 107 – 112.

Mini-Symposium 4

Organizer: Laayouni-Gander

Speakers:

- Berninger
- Dubois
- Gander
- Japhet
- Laayouni/Gander
- Picasso
- Amik St. Cyr

Non-overlapping domain decomposition for the Richards equation via Nemytskij operators

Heiko Berninger*, Ralf Kornhuber and Oliver Sander
Freie Universität Berlin

Abstract:

We present new results on transmission problems related to the Richards equation in heterogeneous porous media. The Richards equation, which describes saturated-unsaturated groundwater flow, is discretized in time with an explicit treatment of the gravitational term. Then, different Kirchhoff transformations on the subdomains containing different soil-types lead to a coupling of local convex minimization problems across the interfaces. The nonlinear coupling is provided by Nemytskij operators acting on the trace space. The corresponding transmission conditions give rise to nonlinear Dirichlet--Neumann or Robin methods for which convergence results have been obtained in one space dimension ([1], [2]). We solve the local problems efficiently and robustly by monotone multigrid [3]. For the domain decomposition iterations, too, no further linearization is applied.
Our numerical results provide a detailed comparison of the Dirichlet-Neumann method and the Robin method for problems related to the stationary Richards equation in 2D. Furthermore, we present a numerical example in 2D wherein we apply the Robin method to the Richards equation in four different soils with surface water and realistic hydrological data.

References:

[1] H.~Berninger. {\em Domain Decomposition Methods for Elliptic Problems with Jumping Nonlinearities and Application to the Richards Equation.} Dissertation, FU Berlin, October 2007.
[2] H.~Berninger, R.~Kornhuber and O.~Sander. On nonlinear {D}irichlet--{N}eumann algorithms for jumping nonlinearities. In: O.B.~Widlund and D.E.~Keyes, editors, {\em Domain Decomposition Methods in Science and Engineering~XVI}, pp.\ 483--490, Springer, 2007.
[3] R.~Kornhuber. On constrained Newton linearization and multigrid for variational inequalities. {\em Numer.\ Math.}, 91:699--721, 2002.

Behavior of optimized Schwarz methods for multiple subdomains and coarse space corrections.

Olivier Dubois
Institute for Mathematics and its Applications, University of Minnesota

Abstract:

Optimized transmission conditions are known to greatly improve the convergence of the Schwarz iteration. In the derivation of these transmission conditions, the convergence factor is optimized for a model problem with two subdomains. In this work, we study optimized Schwarz methods with Robin transmission conditions when applied to multiple subdomains. We experiment with several choices of coarse space corrections, and compare the performance with the classical two-level additive Schwarz preconditioner. We demonstrate in particular that, given a method for coarse space correction, the best parameters for the two-level Schwarz iteration may be very different from the best parameters for the one-level Schwarz iteration. Numerical experiments in one and two dimensions will be presented.

Discontinuous Galerkin and nonconforming in time optimized Schwarz waveform relaxation for coupling heterogeneous problems

Laurence Halpern, Caroline Japhet*

Laboratoire d'Analyse, G´eom´etrie et Applications, Universit´e Paris XIII, Avenue J-B Cl´ement, 93430 Villetaneuse, France

Jérémie Szeftel

Department of Mathematics, Princeton University, and C.N.R.S., Mathématiques Appliquées de Bordeaux, Université Bordeaux

Abstract:

In the context of long time computations in highly discontinuous media such as far field simulations of underground nuclear waste disposal or in climate modeling, it is of importance to split the computation into subproblems for which robust and fast solvers can be used. Optimized Schwarz waveform relaxation algorithms have been described for linear convection-diffusion-reaction problems in [1], and provide an efficient approach even with rotating velocity fields [4]. These algorithms are global in time, and thus allow for the use of non conforming space-time discretizations. They are therefore well adapted to coupling models with very different spatial and time scales, as in ocean-atmosphere coupling or nuclear waste disposal simulations [3]. Our final objective is to propose efficient algorithms with a high degree of accuracy, for heterogeneous advection-diffusion problems with discontinuous coefficients. The goal is to split the time interval into time windows, and to perform, in each window, a small number of iterations of an optimized Schwarz waveform relaxation algorithm, using second order optimized transmission conditions . The subdomain solver is the discontinuous Galerkin method in time, and classical finite elements in space. The coupling between the subdomains in done by a simple projection procedure, where no composite grid on the boundary is needed. This approach has been introduced in [2], with promising one dimensional numerical results, and we extend here the analysis in higher dimension. The nonconforming domain decomposition method is proved to be well posed, and the iterative solver to converge for simple geometries. We present two-dimensional numerical results to illustrate the performances of the method.

References:

1. D. Bennequin, M J. Gander, and L. Halpern A Homographic Best Approximation Problem with Application to Optimized Schwarz Waveform Relaxation, To appear in Math. of Comp.

2. E. Blayo, L. Halpern and C. Japhet. Optimized Schwarz waveform relaxation algorithms with nonconforming time discretization for coupling convection-diffusion problems with discontinuous coefficients. Domain Decomposition Methods in Science and Engineering XVI

3. M. Gander, L. Halpern and M. Kern. A Schwarz Waveform Relaxation Method for Advection–Diffusion–Reaction Problems with Discontinuous Coefficients and non-Matching Grids. Domain Decomposition Methods in Science and Engineering XVI

4. V. Martin, An optimized Schwarz waveform relaxation method for the unsteady convection diffusion equation in two dimensions, Appl. Numer. Math. 52 (2005), no. 4, 401–428.

From Schwarz's idea to a New Era of Optimized Domain Decomposition Methods

Laayouni-Gander

Abstract:

The classical Schwarz algorithm is one of the very well known iterative schemes for solving partial differential equations. The original idea was introduced by Schwarz in 1870 to prove existence and uniqueness of solution of poisson's problem on irregular domains. The advances in computer performance and multiprocessor architectures have permitted the Schwarz method to become one of the popular iterative algorithms amongst other solution methods.Different investigations have been devoted to improving the slow convergence of the classical Schwarz method, in particular for nonoverlapping decompositions. A remedy and a novel idea was to modify the transmission conditions, thus changing the information that is communicated between the subdomains. A new class of domain decomposition methods was then introduced, known as optimal and optimized domain decomposition methods.Although computer architectures have evolved significantly, we still need efficient and optimal iterative algorithms. Optimized Schwarz methods have shown to be efficient in solving several differential models.
This minisymposium will focus on the evolution progress of the original idea of Schwarz to the optimized Schwarz era. The minisymposium will also consider some technical issues and challenges, including the extension of Optimized Schwarz methods to more complex systems, the treatment of periodic problems, the design of coarse space corrections for multiple subdomains, and the convergence analysis for more complicated geometries.

Finite elements with patches

Marco Picasso[*], Jacques Rappaz, Vittora Rezzonico

Ecole Polytechnique Fédérale de Lausanne

Abstract:

We develop a discretization and solution technique for elliptic problems whose solutions may present strong variations, singularities, boundary layers and oscillations in localized regions. We start with a coarse finite element discretization with a mesh size H, and we superpose to it local patches of finite elements with finer mesh size $h<<H$ to capture local behaviour of the solution. The two meshes (coarse and patch) are not necessarily compatible. The corresponding linear system is solved using a subspace correction method. Similarly to mesh adaptation methods, the location of the fine patches is identified by an a posteriori error estimator. Unlike mesh adaptation, no remeshing is involved. We discuss the implementation and illustrate the method on academic and industrial examples.

Optimized Schwarz preconditioning for spectral elements based magnetohydrodynamics.

Amik St. Cyr
Institute for Mathematics Applied to Geosciences,
National Center for Atmospheric Research

Duane Rosenberg
Institute for Mathematics Applied to Geosciences, National Center for Atmospheric Research

Sang Dong Kim
Department of Mathematics, Kyungpook National University

Abstract:

A recent theoretical result on optimized Schwarz algorithm presented in [SISC, 29(6),pp 2402–2425] enables the modification of an existing Schwarz procedure to its optimized counterpart. In this work, it is shown how to modify a bilinear FEM based Schwarz preconditioning strategy to its optimized version. The latter is then employed to precondition the pseudo Laplacian operator arising from the spectral element discretization of the magnetohydrodynamic equations. In order to yield resolution independence in the Krylov iteration count various experiments are performed with a coarse solver based on radial basis functions.

Mini-Symposium 5

Milestones in the Development of Domain Decomposition Methods: a Historical Perspective

Organizer:

- Martin J. Gander
- David E. Keyes

Speakers:

- Olof Widlund
- Petter Bjorstad
- Roland Glowinski
- Jinchao Xu
- David Keyes
- Francois-Xavier Roux
- Frederic Nataf
- Xiao-Chuan Cai
- Laurence Halpern

Coarse Space Components of Domain Decomposition Algorithms

Olof Widlund

Abstact:

An historical overview of the development of some quite exotic coarse spaces, i.e., global low-dimensional components of domain decomposition preconditioners will be given. Some highlights:

The beginning, there was a realization that a second level is required in order to obtain scalability of these iterative methods, i.e., bounds on their convergence rates which are independent of the number of subdomains into which the region of the elliptic problem has been divided. By the time of DD1, a series of four important papers by Bramble, Pasciak, and Schatz had been written; they turned out to be very influential both in terms of algorithm development and new technical tools. With the development of relatively exotic coarse spaces, bounds on the convergence rates could be localized to individual subdomains, which allowed for bounds which are independent of even large jumps in material properties across the interfaces between subdomains.

Soon thereafter, the two-level overlapping Schwarz methods were introduced and the null space property was formulated.

Another important milestone was the introduction of the balancing domain decomposition methods, where the original algorithms soon were augmented by a coarse component. Already in these algorithms, the coarse component is defined in an implicit way and the same is true for the one-level FETI methods and the BDDC and FETI-DP algorithms, which now represent the state of the art.

The coarse component of a domain decomposition method can also serve purposes other than just providing some global interchange of information in each step of the iteration; almost incompressible elasticity will serve as an example. In these cases, the coarse spaces need to be enriched beyond the point when the null space condition is met.

To Overlap or not to Overlap

Petter Bjorstad

Abstract:

In this talk, we provide a historical context and motivation for overlapping as well as non-overlapping domain decomposition algorithms. We will further, by way of a simple example, show some of the relationships between these methods that were discovered quite early. The talk will further discuss some trade-off issues, then end with a look ahead as we (again) need to adopt to massively parallel computer systems.

On Fictitious Domain Methods

Roland Glowinski

Abstract:

During this lecture (complementary to the one dedicated to Moshe Israeli) the author would like to discuss various approaches for the direct numerical simulation of particulate flow and providethe rational for the particularfictitious domain methodhe and his collaborators have been advocating.This presentation will be illustrated by the visualization (including movies)of the results of various numerical experimentsfor 2 and 3-dimensional particulate flow.

On the method of subspace corrections

Jinchao Xu

Abstract

In this talk, an overview will be given on the development of the method of subspace correction based on space decomposition. Main ideas will be explained on its basic algorithmic framework, relevant theoretical analysis and practical applications. Multigrid and domain decomposition methods will be presented as primary examples of this type of methods.

The FETI Method

Francois-Xavier Roux

Abstract:

The FETI method was the first domain decomposition method based on the use of Lagrange multipliers for enforcing interface continuity condition. With the FETI method, the interface variable is not the trace of the solution, but the Lagrange multiplier that is in the dual space and, so, the local problems are associated with Neumann boundary conditions.

The main specific feature of the FETI method is that it contains a built in "coarse grid" preconditioner. Since the local Neumann problems are ill posed in all subdomains that do not inherit Dirichlet boundary conditions from the global problem, the condensed interface problem of the FETI method is a mixed problem that is solved via a projected conjugate gradient algorithm. The projection phase consists in solving a global coarse problem whose

unknowns are, in the case of structural analysis, the rigid body motion coefficients of the floating subdomains.

The FETI solution method can be easily extended to the case of non matching grids on the interface, since the introduction of Lagrange multipliers for the weak continuity conditions is the natural way to tackle such problems. With the FETI-2 method, a very general and flexible way to introduce more sophisticated coarse grid preconditioners in the method has been designed.

For indefinite problems, like the Helmholtz equation, the FETI method has been extended to the case of Robin boundary conditions on the interface, with one (FETI-H) or two (FETI-2LM) Lagrange multipliers. The FETI-2LM method has given now a very general methodology to design domain decomposition methods for a wide range of discretizations of PDEs.

A variant of FETI, mixing the dual approach based on the introduction of Lagrange multipliers with the primal Schur complement method, called FETI-DP, has become more and more popular in the past few years. Another mixed variant has also been developed for mixed formulations of EDPs, like uncompressible elasticity.

This paper will present the panorama of what is now the family of FETI methods, and will give an insight of their various features as well as an idea on the criteria for choosing the right method for a given application.

Optimized Schwarz Methods

Frederic Nataf

Abstract:

The strategy of domain decomposition methods is to decompose the computational domain into smaller subdomains. Each subdomain is assigned to one processor. The equations are solved on each subdomain. In order to enforce the matching of local solutions, interface conditions have to be imposed on the boundary between subdomains. These conditions are enforced iteratively. The convergence rate of the associated algorithm is very sensitive to these interface conditions. The Schwarz method is based on the use of Dirichlet conditions. It is slow and requires overlapping decompositions. In order to improve the convergence and to be able to use non-overlapping decompositions, it has been proposed to use more general boundary conditions. It is even possible to optimize them with respect to the efficiency of the method. Theoretical and numerical results are given along with open problems.

Domain Decomposition Methods for Nonlinear Problems

Xiao-Chuan Cai

Abstract:

In this talk we review and discuss some overlapping domain decomposition methods for solving nonlinear systems of algebraic equations arising from the discretization of partial differential equations. These methods are well studied as linear preconditioners in the context of Newton-Krylov based nonlinear solvers, and the focus of the talk is on extensions of the methods for nonlinear problems, in particular, for improving the nonlinear convergence of Newton's methods. Several classes of equations will be discussed and the emphasis is on coupled systems of nonlinear partial differential equations with local high nonlinearities.

Space-Time parallel Methods

Laurence Halpern

Abstract:

Evolution problems have a particular direction, namely the time direction, which usually plays quite a different role from the spatial directions. This needs to be taken into account when one tries to solve such problems in parallel. Over the last decades, several different approaches for the parallelization of evolution problems have been proposed and analyzed: shooting methods, parallel predictor correctior methods, waveform relaxation methods, parallel time stepping methods, space-time multigrid methods, and very recently the parareal algorithm, which fits into the class of shooting methods.

After a historical overview of these approaches, we will focus on the class of optimized Schwarz waveform relaxation methods. These methods are based on a decomposition of the problem in space, like classical Schwarz methods, but they solve subdomain problems in both space and time. This approach allows us to use non-matching grids both in space and time, or even different space-time models in different subdomains. Rapid convergence is obtained using optimized transmission conditions between subdomains, like in optimized Schwarz methods. Such methods are also easy to use, if one has already a solver for the associated evolution problem.