Generalized Kneser Coloring Theorems with Combinatorial Proofs (Erratum)

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May, 2005

In [5], we presented a lower bound for the chromatic numbers of hypergraphs $\mathrm{KG}^r_{\boldsymbol{s}}\mathcal{S}$, "generalized r-uniform Kneser hypergraphs with intersection multiplicities $\boldsymbol{s}$." It generalized previous lower bounds by Kříž [1, 2] for the case $\boldsymbol{s} = (1, \ldots, 1)$ without intersection multiplicities, and by Sarkaria [4] for $\mathcal{S} = \binom{[n]}{k}$.

The following two problems that arise for intersection multiplicities $s_i > 1$ were noticed by Carsten Lange resp. by Karsten Vogel:

1. In the presence of intersection multiplicities, there are two different versions of a "Kneser hypergraph," depending on whether one admits hypergraph edges that are multisets rather than sets. It is shown in [3] that the chromatic numbers are substantially different for the two concepts of hypergraphs. The lower bounds of Sarkaria [4] and of [5, Thm. 5.1] apply only to the multiset version. The coloring and upper bound of [5, Lemma 3.1] is also valid for the multiset version. The coloring of [5, Example 7.2] refers to the set version.
2. The reductions to the case of prime r in the proof for [5, Thm. 5.1] works only if the intersection multiplicities are strictly smaller than the largest prime factor of r. (Specifically, a problem arises in the reduction of [5, pp. 679-680] if the auxiliary set system $\mathcal{T}$ is empty.) Currently we have no valid proof for the lower bound result in the other cases. This also applies to the special case $\mathcal{S} = \binom{[n]}{k}$ of Sarkaria [4].

Details will be presented in [3].

References

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*Partially supported by Deutsche Forschungs-Gemeinschaft (DFG)