Unipotent elements in irreducible representations of algebraic groups

Mikko Korhonen

Let G be a simple linear algebraic group over an algebraically closed field K of characteristic $p \geq 0$. In this talk, I will discuss the following question and some related problems.

Let $f : G \to I(V)$ be a rational irreducible representation, where $I(V) = \mathrm{SL}(V)$, $I(V) = \mathrm{Sp}(V)$, or $I(V) = \mathrm{SO}(V)$. For each unipotent element $u \in G$, what is the conjugacy class of $f(u)$ in $I(V)$?

Solutions to this question in specific cases have found many applications, one basic motivation being in the problem of determining the conjugacy classes of unipotent elements contained in maximal subgroups of simple algebraic groups.

In characteristic zero, there is a fairly good answer by results of Jacobson-Morozov-Kostant. I will focus on the case of positive characteristic $p > 0$, where much less is known and few general results are available. When G is simple of exceptional type, computations due to Lawther describe the conjugacy class of $f(u)$ in $\mathrm{SL}(V)$ in the case where V of minimal dimension (adjoint and minimal modules). I will discuss some recent results in the case where G is simple of classical type.

University of Manchester

email: `korhonen_mikko@hotmail.com`