Some Compactness Conditions on Locally Compact Groups

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The Schur-Zassenhaus Theorem in finite group theory states that any subgroup H of a finite group G such that the cardinality of H and the index $|G : H|$ are coprime admits a normal complement.

This result has analogues in profinite group theory (see e.g. Ribes-Zalesskii or J.S. Wilson's books) and in the theory of locally finite groups (see O.H. Kegel's book).

Recently the K.H. Hofmann, F.G. Russo, and the speaker observed that from these results one can provide a version of the Schur-Zassenhaus Theorem valid in a class we named *compactly ruled*: groups in this class are locally compact, totally disconnected, and every compact subset is contained in a compact subgroup.

A group in this class has the property that every topologically finitely generated subgroup is profinite. Therefore these groups are also called *topologically locally finite*.

This is an example of a *compactness condition*: Take a *finiteness condition* and appropriately formulate it for appropriate compact subsets of a topological group. Another example are the $\overline{FC}$-groups, introduced by Ušakov, topological groups in which each conjugacy class has compact closure.

During this talk I intend to review compactness conditions (touching also work of S.K. Grosser) with an emphasis on classification results of compactly ruled groups G with some extra condition – like containing a closed subgroup M whose elements $1 \neq x$ have centralizers $C_G(x) \subseteq M$.

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