STORIES ABOUT SPETSES

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A group of Lie type over a finite field $\mathbb{F}_q$ of cardinal q (like $\mathrm{GL}_n(q)$, or $\mathrm{Sp}_{2n}(q)$) appears often as the specialization at $x = q$ of "something" depending on an indeterminate x.

For example,

- the order of $\mathrm{GL}_n(q)$ is the evaluation at $x = q$ of the polynomial $\mathrm{O}_n(x) := x^{\binom{n}{2}} \prod_{i=1}^{n}(x^i - 1)$,
- the degrees of its unipotent characters χ_λ (for $\lambda \vdash n$) are also evaluations at $x = q$ of some polynomials $\mathrm{Deg}_\lambda(x)$, which do divide the order $\mathrm{O}_n(x)$,
- etc...

This is still a real mystery : there is no field $\mathbb{F}_x$ of "cardinal x" which would allow us to speak of $\mathrm{GL}_n(x)$.

Although $\mathrm{GL}_n(-q)$ makes sense : $\mathrm{GL}_n(-q) = \mathrm{U}_n(q) \ldots$

But the mystery became thicker when we realized, 23 years ago – it was during a congress on the Greek island named Spetses –, that there are more general objects depending on x, looking like Lie type things, which behave like $\mathrm{GL}_n(x)$ but which do **not** specialize to a group or whatever for $x = \pm q$. This was the beginning of the "*Spetses story*".

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