Kirillov's orbit method and the polynomiality of the faithful dimension of p-groups

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The faithful dimension of a finite group G is defined as the least dimension of a faithful complex representation of G. When G is a finite p-group, the faithful dimension of G is also intimately related to the notion of essential dimension, introduced by Buhler and Reichstein.

The problem of determining the faithful dimension for families of p-groups arising from $\mathbb{F}_p$-points of a nilpotent algebraic group defined over the field of rational numbers has been studied in some special cases, e.g. the Heisenberg and the full upper-triangular unipotent group. In this talk, I will explain how a variant of Kirillov's orbit method for finite groups can be used to to address this question for a large family of groups in a uniform fashion. Among other things, it will be shown that the dependence on the prime p is always a piecewise polynomial along Frobenian sets, while in some specific natural cases, the function is a single polynomial in p. These results employ tools from number theory, combinatorics, and representation theory. This talk is based on two joint works with Mohammad Bardestani and Hadi Salmasian [1], and Dzmitry Rumiantsau [2].

References

[1] Bardestani, M. Mallahi-Karai, K., Salmasian, H., "Kirillov's orbit method and polynomiality of the faithful dimension of p-groups", Accepted for publication in Compositio Mathematica.

[2] Mallahi-Karai, K., Rumiantsau, D., "Faithful dimension of free nilpotent metabelian p-groups", in preparation.

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