Hereditarily minimal topological groups

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In [3] we have studied locally compact groups having all subgroups minimal. We call such groups *hereditarily minimal* (HM for short). In [1] Prodanov proved that the infinite hereditarily minimal compact abelian groups are precisely the groups $\mathbb{Z}_p$ of p-adic integers. We extend Prodanov's theorem to the non-abelian case at several levels. For infinite hypercentral (in particular, nilpotent) locally compact groups we show that the hereditarily minimal ones remain the same as in the abelian case. On the other hand, we classify completely the locally compact solvable HM groups, showing that in particular they are always compact and metabelian.

The proofs involve the (hereditarily) locally minimal groups, introduced similarly. In particular, we have proved a conjecture stated in [2], showing that the group $\mathbb{Q}_p \rtimes \mathbb{Q}_p^*$ is hereditarily locally minimal, where $\mathbb{Q}_p^*$ is the multiplicative group of non-zero p-adic numbers acting on the first component by multiplication. It turns out that the locally compact solvable HM groups are closely related to this group.

References

[1] Iv. Prodanov, *Precompact minimal group topologies and p-adic numbers*, Annuaire Univ. Sofia Fac. Math. Méc. 66 (1971/72), 249–266.

[2] W. Xi, D. Dikranjan, W. He, Z. Xiao, *Locally Compact Groups and Locally Minimal Group Topologies*, Fundam. Math. 244 (2019), no. 2, 109–124.

[3] W. Xi, D. Dikranjan, M. Shlossberg, D. Toller, *Hereditarily minimal topological groups*, Forum Mathematicum 31 (2019), no. 3, 619–646.

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