The subgroup architecture of groups: something really new

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Abstract

This talk extends the validity of sandwich classification theorems of subgroups of a classical or classical-like group $G_n(R), n > 2$, over module finite rings R, which are normalized by the elementary subgroup $E_n(R)$ of $G_n(R)$. In cases when the $E_n(R)$-normal subgroups are normal, for example when R is a field, division ring,or commutative semilocal ring, then we get a sandwich classification of normal subgroups of $G_n(R)$ as well.

Our generalization of the sandwich classification theorem above is firstly to all rings R and then to arbitrary groups G and an arbitrary subgroup E of G where E is replacing $E_n(R)$ above. Our main result is:

Theorem Let G be an arbitrary group and E an arbitrary subgroup of G. Then there are subgroups B of G called E-basic subgroups and for each B there is a corresponding subgroup $C_G(E, B)$ called the E-cobasic subgroup of B such that: A subgroup H of G is normalized by E if and only if there is an upper sandwich $S_G(E, B) \subseteq Subgr(G)$ with largest subgroup $C_G(E, B)$ such that $H \in S_G(E, B)$. Moreover, B is unique and if E is perfect then $S_G(E, B)$ is a sandwich in the usual sense whose smallest subgroup is B.