# LEO-II

## A Higher-Order Theorem Prover

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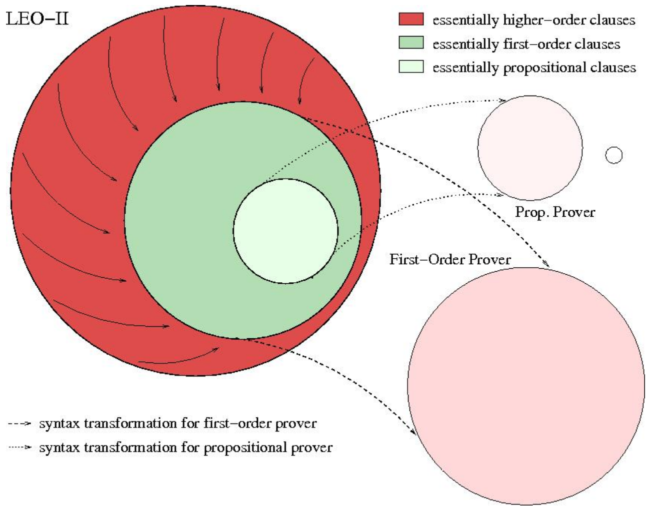
## Overview on LEO-II

LEO-II is a standalone, resolution-based higher-order theorem prover that is designed for fruitful cooperation with specialist provers for first-order and propositional logic. The idea is to combine the strengths of the different systems. On the other hand, LEO-II itself, as an external reasoner, wants to support interactive proof assistants such as Isabelle/HOL, HOL, and OMEGA by efficiently automating subproblems and thereby reducing user effort.

LEO-II predominantly addresses higher-order aspects in its reasoning process with the aim to quickly remove higher-order clauses from the search space and to turn them into essentially first-order clauses which can then be refuted with a first-order prover. Furthermore, the project investigates whether techniques that have proved very successful in automated first-order theorem proving, such as shared data structures and term indexing, can be lifted to the higher-order setting.

LEO-II also provides an interactive mode in which user and system can interact to produce resolution proofs in simple type theory.

LEO-II is implemented in Objective Caml; it is the successor of LEO, which was implemented in LISP and hardwired to the OMEGA proof assistant.




## Input Syntax: TPTP THF

```
thf(reflexiv,definition,
 (reflexive = (^[R:($i>($i>$o))]: (![X:$i]: ((R @ X) @ X))))).

thf(symmetric,definition,
 (symmetric =
  (^[R:($i>($i>$o))]: (![X:$i,Y:$i]:
   ((R @ X) @ Y) => ((R @ Y) @ X))))).

thf(transitive,definition,
  (transitive =
   (^[R:($i>($i>$o))]: (![X:$i,Y:$i,Z:$i]:
    ((((R @ X) @ Y) & ((R @ Y) @ Z)) => ((R @ X) @ Z)))))).

thf(equiv_rel,definition,
  (equiv_rel =
   (^[R:($i>($i>$o))]:
    (reflexive @ R) & (symmetric @ R) & (transitive @ R)))).

thf(test,conjecture,(?[R:($i>($i>$o))]: ~(equiv_rel @ R))).
```

## First Experiments with LEO-II

We evaluate LEO-II's performance on simple problems about sets, relations, and functions. The example problems are taken from the TPTP library and for LEO/Vampire and LEO-II/E they have been reformalized in higher-order logic.

Some examples:

| | |
|---|---|
| SET171+3 | $\forall X_{o\alpha}, Y_{o\alpha}, Z_{o\alpha}. X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$ |
| SET611+3 | $\forall X_{o\alpha}, Y_{o\alpha}. (X \cap Y = \emptyset) \Leftrightarrow (X \setminus Y = X)$ |
| SET624+3 | $\forall X_{o\alpha}, Y_{o\alpha}, Z_{o\alpha}.$ $\mathsf{Meets}(X, Y \cap Z) \Leftrightarrow \mathsf{Meets}(X, Y) \vee \mathsf{Meets}(X, Z)$ |
| SET646+3 | $\forall x_\alpha, y_\beta. \mathsf{Subrel}(\mathsf{Pair}(x,y), (\lambda u_\alpha.T) \times (\lambda v_\beta.T))$ |
| SET670+3 | $\forall Z_{o\alpha}, R_{o\beta\alpha}, X_{o\alpha}, Y_{o\beta}. \mathsf{IsRelOn}(R, X, Y) \Rightarrow$ $\mathsf{IsRelOn}(\mathsf{RestrictRDom}(R, Z), Z, Y)$ |

with

$$\_ \in \_ \;=\; \lambda x_\alpha, A_{o\alpha}.[A x]$$
$$\emptyset \;=\; [\lambda x_\alpha. F]$$
$$\_ \cap \_ \;=\; \lambda A_{o\alpha}, B_{o\alpha}.[\lambda x_\alpha. x \in A \wedge x \in B]$$
$$\_ \cup \_ \;=\; \lambda A_{o\alpha}, B_{o\alpha}.[\lambda x_\alpha. x \in A \vee x \in B]$$
$$\_ \setminus \_ \;=\; \lambda A_{o\alpha}, B_{o\alpha}.[\lambda x_\alpha. x \in A \vee x \notin B]$$
$$\mathsf{Meets}(\_,\_) \;=\; \lambda A_{o\alpha}, B_{o\alpha}.[\exists x_\alpha. x \in A \wedge x \in B]$$
$$\mathsf{Pair}(\_,\_) \;=\; \lambda x_\alpha, y_\beta.[\lambda u_\alpha, v_\beta. u = x \wedge v = y]$$
$$\_ \times \_ \;=\; \lambda A_{o\alpha}, B_{o\beta}.[\lambda u_\alpha, v_\beta. u \in A \wedge v \in B]$$
$$\mathsf{Subrel}(\_,\_) \;=\; \lambda R_{o\beta\alpha}, Q_{o\beta\alpha}.[\forall x_\alpha, y_\beta. Rxy \Rightarrow Qxy]$$
$$\mathsf{IsRelOn}(\_,\_,\_) \;=\; \lambda R_{o\beta\alpha}, A_{o\alpha}, B_{o\beta}.[\forall x_\alpha, y_\beta. Rxy \Rightarrow x \in A \wedge y \in B]$$
$$\mathsf{RestrictRDom}(\_,\_) \;=\; \lambda R_{o\beta\alpha}, A_{o\alpha}.[\lambda x_\alpha, y_\beta. x \in A \wedge Rxy]$$

## Cooperation with Other Provers

Provers supported (so far): E, SPASS

Translations supported so far

[Kerber94] $(V^0_{\iota\to\iota\to o}\ V^1_\iota\ V^1_\iota)$ translates to
$@_{(\iota\to o)\to\iota\to o}(@_{(\iota\to\iota\to o)\to\iota\to(\iota\to o)}(V^0, V^1), V^1)$

[Hurd02] $(V^0_{\iota\to\iota\to o}\ V^1_\iota\ V^1_\iota)$ translates to
$ti(@(ti(@(ti(V^0, \iota\to\iota\to o), ti(V^1,\iota)), \iota\to o), ti(V^1,\iota)), o)$

## Results

| Problem | Vampire 9.0[1] | LEO/Vamp.[2] | LEO-II/E[3] |
|---|---|---|---|
| SET014+4 | 114.5 | 2.60 | 0.300 |
| SET017+1 | 1.0 | 5.05 | 0.059 |
| SET066+1 | – | 3.73 | 0.029 |
| SET067+1 | 4.6 | 0.10 | 0.040 |
| SET076+1 | 51.3 | 0.97 | 0.031 |
| SET086+1 | 0.1 | 0.01 | 0.028 |
| SET096+1 | 5.9 | 7.29 | 0.033 |
| SET143+3 | 0.1 | 0.31 | 0.034 |
| SET171+3 | 68.6 | 0.38 | 0.030 |
| SET580+3 | 0.0 | 0.23 | 0.078 |
| SET601+3 | 1.6 | 1.18 | 0.089 |
| SET606+3 | 0.1 | 0.27 | 0.033 |
| SET607+3 | 1.2 | 0.26 | 0.036 |
| SET609+3 | 145.2 | 0.49 | 0.039 |
| SET611+3 | 0.3 | 4.00 | 0.125 |
| SET612+3 | 111.9 | 0.46 | 0.030 |
| SET614+3 | 3.7 | 0.41 | 0.060 |
| SET615+3 | 103.9 | 0.47 | 0.035 |
| SET623+3 | – | 2.27 | 0.282 |
| SET624+3 | 3.8 | 3.29 | 0.047 |
| SET630+3 | 0.1 | 0.05 | 0.025 |
| SET640+3 | 1.1 | 0.01 | 0.033 |
| SET646+3 | 84.4 | 0.01 | 0.032 |
| SET647+3 | 98.2 | 0.12 | 0.037 |
| SET648+3 | 98.2 | 0.12 | 0.037 |
| SET649+3 | 117.5 | 0.25 | 0.037 |
| SET651+3 | 117.5 | 0.09 | 0.029 |
| SET657+3 | 146.6 | 0.01 | 0.028 |
| SET669+3 | 83.1 | 0.20 | 0.041 |
| SET670+3 | – | 0.14 | 0.067 |
| SET671+3 | 214.9 | 0.47 | 0.038 |
| SET672+3 | – | 0.23 | 0.034 |
| SET673+3 | 217.1 | 0.47 | 0.042 |
| SET680+3 | 146.3 | 2.38 | 0.035 |
| SET683+3 | 0.3 | 0.27 | 0.053 |
| SET684+3 | – | 3.39 | 0.039 |
| SET716+4 | – | 0.40 | 0.033 |
| SET724+4 | – | 1.91 | 0.032 |
| SET741+4 | – | 3.70 | 0.042 |
| SET747+4 | – | 1.18 | 0.028 |
| SET752+4 | – | 516.00 | 0.086 |
| SET753+4 | – | 1.64 | 0.037 |
| SET764+4 | 0.1 | 0.01 | 0.032 |
| SET770+4 | 145.0 | – | – |

[1] Intel(R) Pentium(R) 4 CPU 2.80GHz, 1GB, Linux, CPULimit 600s
[2] Intel(R) Xeon(TM) 4 CPU 2.40GHz, 4GB, Linux, CPULimit 120s
[3] Intel(R) Pentium(R) 1 CPU 1.60GHz, 1GB, Linux, CPULimit 60s