

WP1: Interpretation of Informal Mathematical Input

Robust Sentence-Level Analysis

- Processing “ill-formed” input (syntactic errors, incompleteness, out-of-grammar) through combination of deep and shallow methods:

In diesem Fall: z.B. $K(A) = \text{dem Begriff } K(A \cup B)$

- Extension from written-only to simultaneous written and spoken input, accompanied with simple pointing/selection on screen

Discourse Representation of Informal Mathematical Input

Und wenn $B \subseteq K(A)$ sein soll, muss es auch Element von $K(A)$ sein.

- **Coreference** of symbolic identifiers
- **Anaphoric reference** to parts of mathematical expressions
- **Discourse structure** as reflex of proof structure

Ontology-Based Domain-Specific Interpretation

- Informal and/or imprecise naming of domain concepts and relations:

*A muss in B sein
... B vollständig ausserhalb von A liegen muss ...
... dann sind A und B vollkommen verschieden*

- Semantically complex operators:

Wenn alle A in $K(B)$ enthalten sind und dies auch umgekehrt gilt, ...

WP2: Proof Management and Proof Step Evaluation (PSE)

Abstract-level Proof Representation

- Required for PSE:
cognitive oriented proof representation

PSE example scenario

Assertions already introduced
(A1) $A \wedge B$.
(A2) $A \Rightarrow C$.
(A3) $C \Rightarrow D$.
(A4) $F \Rightarrow B$.
(G) $D \vee E$.

Alternative proof step directives.
(a) From the context follows D .
(b) B holds.
(c) It is sufficient to show D .
(d) We show E .

PSE: Novel Theorem Proving Application

Criterion	Task (first approach)	Requirements for theorem prover
Soundness	$E \vdash_C^? D \vee E$	'Yes' or 'No' answer; any theorem prover resp. calculus C
Granularity	proof-steps($E \vdash_C^? D \vee E$)	adequate abstract-level theorem prover resp. calculus C; measure 'shortest' proof; take tutorial constraints into account; proof planning or assertion level reasoning?
Relevance	$A \wedge B$ $A \Rightarrow C$ $C \Rightarrow D \vdash_C^? E$ $F \Rightarrow B$	recognize detours; compare with other 'shorter' proofs; take tutorial constraints into account; forward case more challenging

WP3: Domain Reasoning for Ambiguity Resolution

An example

Discourse:

- (1) From previous observations we know that A or B .
- (2) The former implies D by Lemma X .
- (3) Similarly, from the latter follows C .

Alternative user utterances with underspecification:

- (a) From **this** follows D since C implies D by Lemma Y .
— **this** may refer to (1)+(2)+(3), to (3), or even (2) with wrong justification
- (b) It holds D since C implies D by Lemma Y .
— no underspecified anaphoric reference but **ambiguity at domain reasoning level**

- Ambiguities may arise at linguistic **and** domain reasoning level.
- Ambiguities are resolved by a **combination of linguistic processing and proof step evaluation**.
- Remaining ambiguous readings are explicitly represented and **ranked**.
- The use of **underspecification techniques** (CHORUS) will be explored.

Further ambiguity examples

Example	Where does ambiguity arise?	Ambiguity resolution means
(1) $x \in B$ und somit $x \subseteq K(B)$ und $x \subseteq K(A)$ wegen Voraussetzung	linguistic meaning level;	linguistic means;
(2) A enthaelt B	attachment, coordination	type checking in (2)
(3) $P((A \cup C) \cap (B \cup C)) = PC \cup (A \cap B)$	linguistic meaning level;	type checking for (3);
(4) $K((A \cup C) \cap (B \cup C)) = KC \cup (A \cap B)$	informal character of discourse	mathematical domain reasoning for (4)
(5) T1: Bitte zeigen Sie: $K((A \cup B) \cap (C \cup D)) = (K(A) \cap K(B)) \cup (K(C) \cap K(D))$! S1: nach deMorgan-Regel-2 ist $K((A \cup B) \cap (C \cup D)) = (K(A \cup B) \cup K(C \cup D))$	underspecified proof step	mathematical domain reasoning