LEO

A Higher Order Theorem Prover

This work was supported by the Deutsche Forschungsgemeinschaft in Grant HOTEL and by the Studienstiftung des Deutschen Volkes

Extensional Higher Order Resolution

The Calculus

Higher Order Resolution

$$\frac{[\mathbf{N}]^\alpha \vee C \quad [\mathbf{M}]^\beta \vee D \quad \alpha \neq \beta}{C \vee D \vee [\mathbf{N} = \mathbf{M}]^F}\ Res \qquad \frac{[\mathbf{N}]^\alpha \vee [\mathbf{M}]^\alpha \vee C \quad \alpha \in \{T, F\}}{[\mathbf{N}]^\alpha \vee C \vee [\mathbf{N} = \mathbf{M}]^F}\ Fac$$

$$\frac{[Q_\gamma \overline{\mathbf{U}^k}]^\alpha \vee C \quad \mathbf{P} \in \mathcal{G}_\gamma^{\{\neg,\vee\} \cup \{\Pi^\beta \mid \beta \in \mathcal{T}\}}}{[Q_\gamma \overline{\mathbf{U}^k}]^\alpha \vee C \vee [Q = \mathbf{P}]^F}\ Prim$$

Higher Order Pre-Unification

$$\frac{C \vee [\mathbf{A} = \mathbf{B}]^F \quad \mathbf{A}, \mathbf{B}\ \text{functional} \quad s_\alpha\ \text{Skolem term}}{C \vee [\mathbf{A}s = \mathbf{B}s]^F}\ \alpha$$

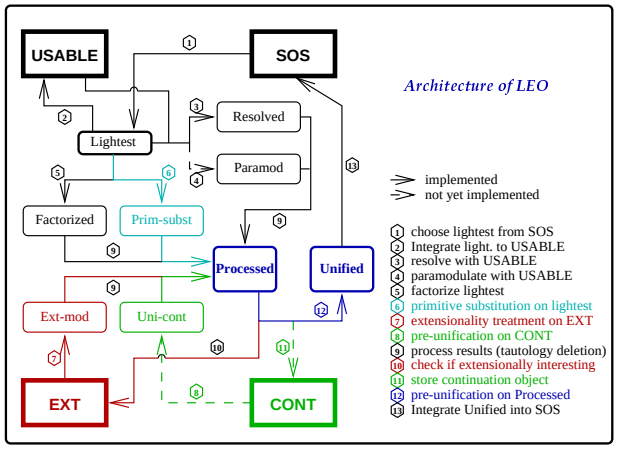
$$\frac{C \vee [h\overline{\mathbf{U}^n} = h\overline{\mathbf{V}^n}]^P}{C \vee [\mathbf{U}^1 = \mathbf{V}^1]^P \vee \ldots \vee [\mathbf{U}^n = \mathbf{V}^n]^P}\ Dec \qquad \frac{C \vee [\mathbf{A} = \mathbf{A}]^F}{C}\ Triv$$

$$\frac{C \vee E \quad E\ \text{solved for}\ C}{\mathrm{CNF}(\mathrm{subst}_E(C))}\ Subst \qquad \frac{C \vee [F_\gamma \overline{\mathbf{U}^n} = h\overline{\mathbf{V}}]^P \quad \mathbf{G} \in \mathcal{G}_\gamma^h}{C \vee [F = \mathbf{G}]^P \vee [F\overline{\mathbf{U}} = h\overline{\mathbf{V}}]^P}\ flex/rigid$$

Extensionality (Interleave Resolution an Unification)

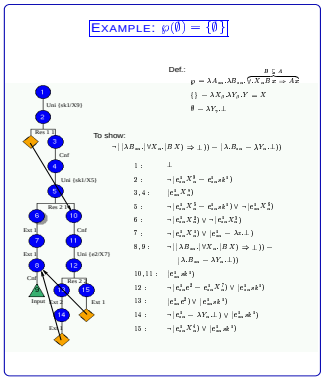
$$\frac{C \vee [\mathbf{M}_o = \mathbf{N}_o]^F}{\mathrm{CNF}(C \vee [\mathbf{M}_o \equiv \mathbf{N}_o]^F)}\ Equiv \qquad \frac{C \vee [\mathbf{M}_\alpha = \mathbf{N}_\alpha]^F \quad \alpha \in \{o, \iota\}}{\mathrm{CNF}(C \vee [\forall P_{\alpha \to o} PM \Rightarrow PN]^F)}\ Leib$$

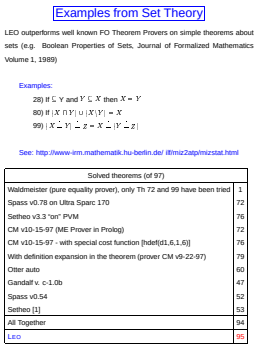
Extended Set Of Support Architecture



Proving simple theorems about sets with LEO

Boolean and Basic Properties of Sets, Journal of Formalized Mathematics Volume 1, 1989





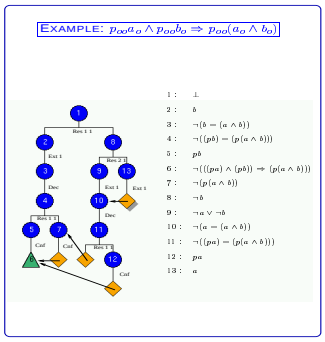
Technical Aspects of LEO

- Implemented in Allegro Common Lisp
- Based on the KEIM-Toolbox
- Automatic Mode
- Interactive Mode
- Graphical User Interface (LEO3)
- Integrated in OMEGA (LEO3)

Difficulties

- efficient Higher Order Subsumption
- Higher Order Termindexing is not compatible with Extensional Resolution
- Leibnizequality or Primitive Equality

A very simple example with embedded Propositions



- not provable by other Systems

Conclusion and Further Work

- **LEO:** Henkin-Complete Extensional Higher Order Resolution
  No Extensionality Axioms required
  Interleaving of Resolution and Unification
  Well suited for simple theorems from Set Theory
- **Further work:** More efficient implementation
  Integration in OMEGA
  Cooperation with other Reasoning Systems
- **Availability:**
  http://www.ags.uni-sb.de/projects/deduktion/projects/hot/leo/

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