

# Implementation of Carmo and Jones Dyadic Deontic Logic in Isabelle/HOL

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**Background literature:** Normative notions such as obligation and permission are the subject of deontic logics [5], and conditional obligations are addressed in so called dyadic deontic logic. A particular dyadic deontic logic has been proposed by Carmo and Jones (CJ) [3]. This dyadic deontic logic comes with a neighborhood semantics and a weakly complete axiomatization over the class of finite models. Their framework is immune to some well known contrary-to-duty issues which can still be found in many other, related approaches. As Carmo and Jones logic is very complicated logic the automation of it is problematic. Isabelle/HOL is a useful tool for studying automate reasoning and computational aspects of logics. For this purpose we should present an “*implementation*” of our logic in Isabelle/HOL. As mentioned in Handbook of Deontic Logic and Normative Systems [4] deontic logic is not studied with computational tools very well, so the implementation of CJ logic in HOL is not clear. In this paper, our implementation utilizes the shallow semantical embedding approach that has been put forward by Benz Müller as a pragmatistical solution towards universal logic reasoning (see [2, 1]).

**Research question:** We investigate two important question in this paper: How to implement CJ logic in Isabelle/HOL and how can we study and improve CJ logic with this implementation? How can we use computational tools (Isabelle/HOL) for other deontic logics more generally?

**Methodology:** We present an “*implementation*” of the Carmo-Jones-Logic (CJL) in Isabelle/HOL. This approach [1] uses classical higher-order logic as (universal) meta-logic to specify, in a shallow way, the syntax and semantics of various object logics, in our case CJL. CJL provides and combines modal and conditional operators and it comes with higher-order relational semantics, which makes it a non-trivial object logic to implement in the shallow semantical embedding approach. The proof theory of CJL is not on our target.

**Technical problem:** Our main technical problem is translating Carmo and Jones logic to Isabelle/HOL and showing faithfulness of this embedding. (For two logics  $L_1$  and  $L_2$  an embedding  $f : L_1 \rightarrow L_2$  is faithful if we have:  $L_1 \vdash f^{-1}(\varphi)$  iff  $f(L_1) \vdash \varphi$ .)

There are other problems related to our research questions although they are not actually technical problems. We employ our implementation to study some meta-logical properties of CJL in Isabelle/HOL. This also includes questions about the relationship of dyadic deontic fragment of CJL to other deontic logics, for example, Input/Output logic [6]. For this, we analysis a list of normative inference patterns that have been suggested by Parent and van der Torre [7]. A particular focus of our experiments is on nested dyadic obligations and we show that nested dyadic obligations in CJL can be eliminated. Moreover, we investigate the relationship of this dyadic obligation operator to standard conditional operators. Finally, we illustrate how our implementation supports the reasoning at object-level. More precisely, we show how classical deontic reasoning examples from the literature can now be represented in Isabelle/HOL and we examine how our implementation performs when being applied to these examples. (Also, this part provides a way for using Isabelle/HOL for other deontic logics.)

## References

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