Ωmega: From Proof Planning towards Mathematical Knowledge Management*

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1 Introduction

The vision of computer-based mathematical assistance systems providing integrated support for all work phases of a mathematician (see Figure 1 from [12]) has fascinated researchers in artificial intelligence, particularly the deduction systems area, and in mathematics for a long time. The dream of mechanizing mathematical reasoning dates back to Gottfried Wilhelm Leibniz in the 18th century. In the beginning of the 20th century modern mathematical logic was born and an important milestones in the formalization of mathematics are Hilbert's program and the 20th century Bourbakism.

After the enthusiasm of the 50s and the 60s the deduction systems area increasingly fragmented into several subareas which all developed their specific approaches and systems similar to the Artificial Intelligence area in general. It is only very recently that this trend is reversed, with the Calculemus and Mkm communities as driving forces of this movement. In Calculemus the viewpoint is bottom-up, starting from existing techniques and tools developed in the community. Mkm approaches the goal of revolutionizing computer-based mathematics in the new millennium by a complementary top-down approach starting from existing, mainly pen and paper based mathematical practice down to system support.

The Ωmega project of Jörg Siekmann at Saarland University is an innovative force in this field since the early 90s. At the heart of this project is the Ωmega system [10; 33], which today integrates several modules and subsystems addressing various of the aspects illustrated in Figure 1.

In this paper we first provide a compact overview and the main references on subsequent developments in the Ωmega project w.r.t. the long-term goal of building a powerful mathematical assistant (MA) and we then point to current research directions and some novel ideas.

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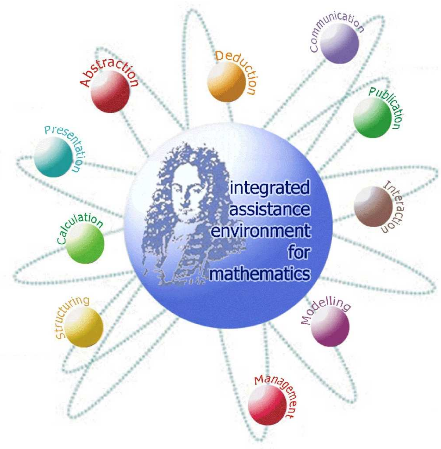

Figure 1: Calculemus illustration of different challenges of a mathematical assistance system

2 Mathematical Assistant In-the-small

Knowledge-based Proof Planning Ωmega has been born in the early 90s as a result of the paradigm shift in Jörg Siekmanns research group from classical automated theorem proving (ATP) in first-order logic (FOL) to knowledge-based proof planning (PP) [29] in classical higher-order logic (HOL) [16]: after many years of experience in building classical ATPs the cumulative conviction was that that this approach alone is insufficient w.r.t. the ambitious goal of powerful MAs. PP in Ωmega is on the one hand inspired by the work of Alan Bundy [14], on the other hand it contributed some novel aspects: it provides declarative meta-level control structures (control rules and strategies; see [27]), it is based on an expressive HOL framework, it supports underspecified pre-conditions in the proof operators and sound and non-sound proof plans can be explicitly represented, and it guarantees soundness of proof plans via plan operator expansion to and verification at its base calculus (OMEGA-ND) [9], which is a HOL variant of Gentzen's natural deduction calculus.

Interaction The Ωmega group is convinced that a symbiosis of interaction and automation is required in

MAs. However, initially there was no tight integration of both paradigms in the project and the system simply offered in addition to PP a tactic based approach for interactive proof construction (IP) in HOL; see [35]. The differences to the Edinburgh LCF approach [20] include: potentially non-sound tactics are supported (cf. planning operators above), tactic-level proofs can be explicitly represented and their soundness is guaranteed only if a non-failing expansion to OMEGA-ND is possible. Recently the group has investigated a decalarative style of interactive proof based on the idea of island planning [35]. In this approach the user provides a network of proof islands and the gaps between these island are then ideally automatically refined by the system to OMEGA-ND.

Proof Data Structure ΩMEGA's proof expansion approach is supported by its hierarchical proof data structure (PDS) developed since the mid 90s [15]. It allows to maintain proof developments (sound or non-sound, see above) at different albeit connected levels of granularity.

Proof Verbalization In the early 90s the proof verbalization tool PROVERB [21] has been developed; the successor of PROVERB is P.REX [17; 18]. These systems lift any proof in the PDS to the assertion level and then create — after macro-planning the text structure and micro-planning the sentence structure and linguistic realization — a natural language representation of it. PROVERB and P.REX assume that respective domain specific linguistic information is provided in the knowledge base.

User Interface The graphical user interface LOUI [34] developed since the mid 90s provides different views on proofs maintained in ΩMEGA's PDS — including linearized ND, proof tree, and natural language. LOUI furthermore supports the different hierarchical layers in the PDS.

Mathematical Knowledge Hierarchically structured mathematical knowledge (an ontology of mathematical theories providing among others axioms, theorems, and lemmas, i.e assertions) has initially been stored in ΩMEGAs hardwired mathematical knowledge base (MKB). This MKB was later (end of the 90s) outsourced which fostered the development of the MBASE MKB [19]. ΩMEGA nowadays assumes that an MKB ideally also supports maintenance of its domain specific control rules, strategies, and linguistic knowledge.

External Reasoners Despite the ΩMEGA project's initial shift from classical FOL-ATP — which in the authors' view sweeps towards a local maximum w.r.t. the goal of powerful MAs — to HOL-PP and HOL-IP the project from the very beginning fostered the integration of FOL-ATPs as one species of external specialist reasoners (SR) into MAs. Early versions of ΩMEGA already support the transformation of HOL subproofs (proof goal together with its local and global assumptions) by employing a HOL-2-FOL translation mechanism [23] into pure FOL representations; thereby relevant information on the translation mappings are memorized. The resulting FOL proof problems can be tackled, for instance, by OTTER. White-box integration is supported by TRAMP [28], which is capable of retranslating machine-oriented FOL proof objects into assertion level proof representations in ΩMEGA using the memorized translation mappings above. This way proof verbalization and independent proof checking becomes available for SRs called within ΩMEGA. Today ΩMEGA has access to more than twenty different SRs (and to many of them in white-box style). This includes computer algebra systems like MAPLE or MATHEMATICA exploiting the CAS-ΩMEGA-translator SAPPER [36], the HOL-ATP TPS [1] exploiting a tactic based proof translator [6], model generators, and the constraint-solver COSIE [30].

Modularization In the mid of the 90s ΩMEGAs initial monolithic architecture got subsequently replaced by a modular concept for MAs. This move started with the outsourcing of the previously hardwired external reasoners. It has resulted in the MATHWEB-SB software bus, which in addition to the various SRs offered by MATHWEB-SB connects ΩMEGA with the outsourced systems LOUI and MBASE.

Agent-based Theorem Proving The symbiosis of IP, ATP, and SRs is supported in ΩMEGA by the agent-based suggestion and reasoning mechanism ΩANTS [8; 37]. The initial motivation for ΩANTS was to turn the thitherto passive ΩMEGA system into a pro-active counter-player of the user which — in cooperation with and competition to the user — autonomously exploits available resources to reason on possible directions for continuing the proof under construction.

The ΩANTS solution provides societies of pro-active agents in a hierarchical blackboard architecture that dynamically and concurrently generate suggestions on applicable proof operators. These ΩANTS agents may also call SRs [31] or perform search in MKBs [7]. The approach has furthermore been applied to realize agent-based ATP [13] and interactive PP [32].

3 Mathematical Assistant In-the-large

Current and future research of the ΩMEGA project is concerned with widening the frontiers of the system such that it integrates more smoothly into the spectrum of usual tasks of a mathematician. In addition to the above to-date streams of research this comprises the following new aspects:

- Mathematical knowledge management: there is an increasing interest in (i) MA independent representation of mathematical knowledge such as theory definitions and domain specific proof search strategies, and (ii) improved support for the distribution and exchange of mathematical knowledge.

- Proof development in-the-large: we aim at (i) lifting the argumentative level of proof construction in MAs in order to support more natural proof styles in combination with possibly underspecified proof steps, (ii) the combination of different proof search paradigms, and (iii) the integration of various kinds of available structured mathematical knowledge into the assisted proof construction process.
- Other mathematical activities: we want to support additional activities such as (i) writing mathematical publication and (ii) tutoring for mathematics students.

Mathematical knowledge management. Mathematical knowledge in the envisioned mathematical assistant consists not only of structured formal mathematical theories, but also of domain specific proof knowledge such as tactics and proof operators. This spurred the development of the OmDoc-language [25] for the representation of mathematical theories. Furthermore, mathematical activities are distributed over different physical locations such that there is a need for remote access of mathematical knowledge and provide knowledge to third parties on the other hand. Last not least the mathematical activity is an evolutionary process which requires a sophisticated management of change combined with origin tracking and version control. This spurred the development of the MBase-system [19] designed for distributed mathematical knowledge, which is currently extended to manage domain specific proof knowledge and incorporate techniques and tools like Maya [5] for management of change and version control developed in the context of formal software development.

Proof development in-the-large. A challenge is to enlarge the size of the individual proof steps that are directly supported by the proof engine. Taking up the notion of *assertion level proof steps* coined in the area of proof presentation we envision to support direct application of assertions. The CoRe-system [2], whose calculus directly supports the determination and application of available assertions to sub-formulas, is currently integrated as the uniform basis [22] in Ωmega for proof construction. CoRe shall also support the integration and combination of the different proof construction paradigms [4], which is the second aspect of in-the-large proof development. Indeed, the experience in the Ωmega-system showed that each kind of proof search paradigm, namely ATP, IP, and PP have complementary strengths. Thus, rather than being tailored to one type of proof knowledge, we envision their collaboration on the common basis provided by CoRe. Finally, work is devoted to linking more closely the structured mathematical theories with the proof construction process. For instance [39] presents a technique based on CoRe employing the Ωants idea for concurrently searching for applicable assertions in a MKB. On a more global scale the MathWeb-SB is currently redesigned to accommodate existing standards of multi-agent system design, to support more high-level problem descriptions and incorporate limited automated problem solving activities via automated coordination of the SRs provided in MathWeb-SB. This also shall allow for a better integration of SRs into a proof construction process.

Support for specific mathematical activities. Proof construction is usually only part of a much wider range of mathematical activities an ideal MA should support; see also Figure 1. Therefore the Ωmega system is currently extended to directly support additional aspects in a mathematicians usual task spectrum. The focus of our current research is on writing mathematical publications and advising students during proof construction.

With respect to the former we envision that a mathematician writes a new paper in some specific mathematical domain using a LaTeX-like environment. The definitions, lemmas, theorems and especially their proofs give rise to extensions of the original theory and the writing of some proof goes along with an interactive proof construction in Ωmega. As a result this allows the development of mathematical documents in a publishable style which in addition are formally validated by Ωmega, hence obtaining *certified mathematical documents*. A first step in that direction is currently under development by linking the WYSIWYG mathematical editor TeXmacs [38] with the Ωmega proof assistant.

As a second mathematical activity we consider the tutoring of students, which consists of advising a student to develop a proof. Thereby the interaction with the student should be conducted via a textual dialog. This scenario is currently under investigation in the Dialog-project [11] and, aside from all linguistic analysis problems, gives rise to the problem of *underspecification* in proofs. Although this problem already occurs in the writing of mathematical documents, it is much more distinctive in this scenario. We expect that this will spur a lot research, on which we initially report in [3].

4 Lessons Learned

Instead of a conclusion we briefly discuss a few lessons learned aspects:

- The modularization of the Ωmega system was an important move fostering a mutual scientific stimulation and a strongly increasing join of resources, e.g. within the Calculemus Network, in the build-up of MAs. This addresses not only the tool development level but also the actual research level and, for instance, comprises the joint development of the MKB MBase, the joint employment of SRs within MathWeb-SB, and the current joint development of an increasingly platform independent user interface. These joint developments in turn depend on and at the same time foster common communication standards such as the OmDoc language.

- It is a waste of time to fight over proof search or proof construction paradigms. Concerning the goal of powerful MAs it is instead useful to develop frameworks in which different of these paradigms may coexist and ideally even mutually benefit from each others strengths as well as share and exploit common components (e.g. user interfaces).
- The lack of a long-term employed software engineer and the imposed suboptimal application and monitoring of high quality software engineering principles is one of the ΩMEGA projects biggest problems. The current funding structure of the ΩMEGA group is due to the given funding and employment principles of the German academic system based only on short-term research projects and contracts which impede such a position. This unfortunately imposes a big challenge for a sustainable software development and also for organizing optimal transfer of knowledge from one generation of ΩMEGA researcher to the next one.
- Due to the heterogeneity of research directions in the ΩMEGA project and the beforehand mentioned problem the ΩMEGA group is strongly depending on but also benefitting from its teamwork spirit.

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