A Faithful Semantic Embedding of the Dyadic Deontic Logic E in HOL
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Isabelle/HOL: Modal Operators

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Shallow Semantical Embedding

A semantic embedding of a target logical system defines the syntactic elements of the target language in a background logic (HOL) [2].

Comprehension axiom:
\[ \neg \varphi = \{ x \mid \neg \varphi(x) \} = \lambda x. \neg \varphi(x) \]
\[ M, s \models \neg \varphi \text{ if and only if } M, s \not\models \varphi \text{ (that is, not } M, s \models \varphi) \]

System E: Syntax

Axiom defined dyadic deontic logic system E [1] by the following axioms and rules: (S5-schemata for necessity) and (-(--)) (for conditional obligation)

\[ \Box(\phi \rightarrow \psi) \rightarrow (\Box \phi \rightarrow \Box \psi) \]
\[ \Box(\psi) \rightarrow \Box(\psi) \]
\[ \Diamond(\phi \rightarrow \psi) \rightarrow (\Diamond \phi \lor \Diamond \psi) \]
\[ \Box(\psi) \rightarrow \Box(\psi) \]
\[ \Box(\psi) \land \Box(\psi) \rightarrow \Box(\psi) \lor \Box(\psi) \]

Soundness and Completeness System E is (strongly) sound and complete with respect to the class of all preference models [1].

Contrary-To-Duties

Chisholm’s CTD-paradox [4] (a) It ought to be that a certain man go to help his neighbours.
(b) It ought to be that if he goes then he is coming.
(c) If he does not go, he ought not to tell them he is coming.
(d) He does not go.

For example actual world \( s \) satisfies:
\[ \Box(\psi) \rightarrow \Box(\psi) \]
\[ \Diamond(\psi) \rightarrow \Diamond(\psi) \]
\[ \Box(\psi) \land \Box(\psi) \rightarrow \Box(\psi) \lor \Box(\psi) \]

Formulas E as Certain HOL Terms

We assume a set of basic types \( B T = \{ o, i \} \). The mapping \( [\cdot] \) translates E formulas \( s \) into HOL terms \( t \) of type \( o \rightarrow o \). Type \( o \rightarrow o \) is abbreviated as \( r \) in the remainder.

\[ |\phi| = \begin{cases} \text{true} & \text{if } \phi, \\ \text{false} & \text{otherwise} \end{cases} \]
\[ |s \land t| = |s| \land |t| \]
\[ |s \lor t| = |s| \lor |t| \]
\[ |\neg s| = |\neg| \]

\[ \neg \neg s = \lambda x. \neg \lambda x = \lambda x \]
\[ \lambda \neg \neg s = \lambda x = \lambda x \]

Corresponding Preference Model \( M_t \) for Henkin Model \( H \)

For every Henkin model \( H = \langle (D_t)_{t \in T}, I \rangle \) there exists a corresponding preference model \( M_t \). Corresponding means that for all \( E \) formulas \( \delta \) and for all assignment \( g \) and worlds \( s, ||\delta||_{M_t} = T \) if and only if \( M_t(s) \models \delta \). We construct the corresponding preference model \( M_t \) as follows:

\[ S = D_t \]
\[ s \models \varphi \text{ for } s, u \in S \text{ iff } \varphi \text{ at } u \]
\[ s \models \psi \text{ if } \exists \tau \in S \text{ such that } \tau \models \psi \text{ at } s \]

Result: Soundness and Completeness of the Embedding

Given \( \varphi \), \( \models_{\text{HOL}} \varphi \) if and only if \( \models_{E} \varphi \)

Isabelle/HOL: Propositional Connectives

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