Implementation of Dyadic Deontic Logic E in Isabelle/HOL

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Abstract

We have devised a shallow semantical embedding of a dyadic deontic logic (by B.Hansson and Åqvist) in classical higher-order logic. This embedding has been encoded in Isabelle/HOL, which turns this system into a proof assistant for deontic logic reasoning. The experiments with this environment provide evidence that this logic *implementation* fruitfully enables interactive and automated reasoning at the meta-level and the object-level.

1 Introduction

Normative notions such as obligation and permission are the subject of deontic logics [9] and conditional obligations are addressed in so-called dyadic deontic logic. A landmark and historically important dyadic deontic logic have been proposed by B.Hansson [10] and Åqvist [1]. This dyadic deontic logic comes with a preference models semantics [12]. Their framework is immune to some well known contrary-to-duty (CTD) obligation sentences such as Chisholm's paradox (cf. [6]) and defeasible conditional obligation. CTD sentences tell us what comes into force when some other obligations are violated. An appropriate semantics for CTD sentences calls for a preference on possible worlds (for more details see [7]). System **E** is deductively the weakest system that is sound and complete for the class of all preference models [12].

When applied as a meta-logical tool, simple type theory [8], aka classical higher-order logic (HOL), can help to better understand semantical issues (of embedded object logics). The syntax and semantics of HOL are well understood [3] and there exist automated proof tools for it; examples include Isabelle/HOL [11] and Leo-II. As mentioned in the Handbook of Deontic Logic and Normative Systems [7] deontic logic is not studied with computational tools very well, so the implementation of system **E** in proof assistants and theorem provers is not obvious.

We have devised an *embedding* of dyadic deontic logic system **E** [12] in HOL. This embedding utilizes the *shallow semantical embedding* approach that has been put forward by Benzmüller (cf. [2] and references therein) as a pragmatical solution towards universal logic reasoning. This approach uses classical higher-order logic as (universal) meta-logic to specify, in a shallow way, the syntax and semantics of various object logic, in our case system **E**. System **E** provides and combines modal and conditional operators and it comes with higher-order relational semantics. The truth condition for the dyadic deontic obligation is formulated in terms of maximality under preference relation. This is the main difference with semantics studied in [5]. We have shown that the presented embedding is sound and complete.

Our embedding has been encoded in Isabelle/HOL to enable experiment in deontic reasoning. We have shown how classical deontic reasoning examples from the literature can now be represented in Isabelle/HOL and we have examined how our implementation performs when being applied to these examples.

The work reported here and in [4] provide the theoretical foundation for the implementation and automation of dyadic deontic logic within existing theorem provers and proof assistants for HOL. We do not provide a new logic. Instead, we provide an empirical infrastructure for assessing practical aspects of an ambitious, state-of-the-art deontic logic; this has not been done before. An interesting aspect of the approach is that (based on the ideas of Benzmüller and Paulson [5]) quantified extensions of system **E** and [4] can easily be implemented and studied in the framework and experimentally assessed. There is much room for future work. For example, experiments could investigate whether the provided implementation already supports non-trivial applications in practical normative reasoning, or whether further improvements are required. Moreover, we could employ our implementation to systematically study some meta-logical properties of dyadic deontic logic system **E** within Isabelle/HOL.

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