Experiments in Computational Metaphysics: Gödel's Proof of God

Christoph Benzmüller[1], FU Berlin & Stanford U

jww: **B. Woltzenlogel Paleo**, ANU Canberra

AISSQ 2015, IIT Kharagpur

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>
>
> Beweise-mit-Leo2 Notwendigerweise-existiert-Gott.p

Leo-II tries to prove
=====================

  Goedel's Theorem T3: "Necessarily, God exists"
  thf(thmT3,conjecture,
      ( v
      @ ( mbox
        @ ( mexists_ind
          @ ^ [X: mu] :
              ( g @ X ) ) ) )).

  Assumptions: D1, C, T2, D3, A5

 . searching for proof ..

 ********************
 *   Proof found    *
 ********************
 % SZS status Theorem for Notwendigerweise-existiert-Gott.p

 . generating proof object
```

[1] Supported by DFG Heisenberg Fellowship BE 2501/9-1/2

Since 2013: Huge Media Attention

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Nachrichten > Wissenschaft > Mensch > Mathematik > Formel von Kurt Gödel: Mathematiker bestätigen Gottesbeweis

Formel von Kurt Gödel: Mathematiker bestätigen Gottesbeweis

Von Tobias Hürter

picture-alliance/ Imagno/ Wiener Stadt- und Landesbibliothek

Kurt Gödel (um das Jahr 1935): Der Mathematiker hielt seinen Gottesbeweis jahrzehntelang geheim

Ein Wesen existiert, das alle positiven Eigenschaften in sich vereint. Das bewies der legendäre Mathematiker Kurt Gödel mit einem komplizierten Formelgebilde. Zwei Wissenschaftler haben diesen Gottesbeweis nun überprüft - und für gültig befunden.

Montag, 09.09.2013 – 12:03 Uhr

Drucken | Versenden | Merken

Jetzt sind die letzten Zweifel ausgeräumt: Gott existiert tatsächlich. Ein Computer hat es mit kalter Logik bewiesen - das MacBook des Computerwissenschaftlers Christoph Benzmüller von der Freien Universität Berlin.


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English Site > Germany > Science > Scientists Use Computer to Mathematically Prove Gödel God Theorem

Holy Logic: Computer Scientists 'Prove' God Exists

By David Knight

picture-alliance/ Imagno/ Wiener Stadt- und Landesbibliothek

Austrian mathematician Kurt Gödel kept his proof of God's existence a secret for decades. Now two scientists say they have proven it mathematically using a computer.

Two scientists have formalized a theorem regarding the existence of God penned by mathematician Kurt Gödel. But the God angle is somewhat of a red herring -- the real step forward is the example it sets of how computers can make scientific progress simpler.

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- Spiegel Online
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DIE WELT

FREITAG, 18. OKTOBER 2013

Zippert zappt

Teuer erkaufte Harmonie

Mindestlohn lockt SPD in dritte Ehe mit der Union

Mittwoch soll über eine große Koalition verhandelt werden. Votum des SPD-Konvents steht aus

THEMEN

Sport
Bundestrainer Löw unterschreibt beim DFB bis 2016

Wirtschaft
Der Jumbojet wird bei Boeing zum Ladenhüter

Karriere
Stress im Job: So legen Sie sich einen dicken Panzer zu

Aus aller Welt
Mietprozess in Paris: Dürfen 15 Quadratmeter 330 Euro kosten?

Dax
Im Minus

Göttliche Mathematik

Haben Forscher mit einem Computerprogramm die Existenz Gottes bewiesen?

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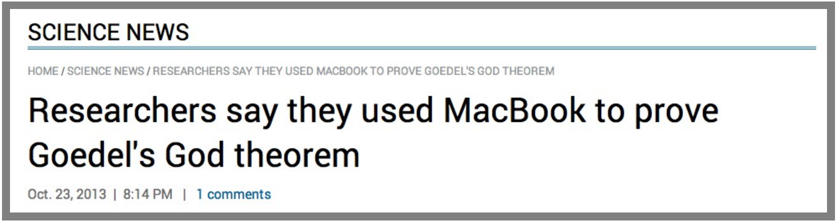

SCIENCE NEWS

HOME / SCIENCE NEWS / RESEARCHERS SAY THEY USED MACBOOK TO PROVE GOEDEL'S GOD THEOREM

Researchers say they used MacBook to prove Goedel's God theorem

Oct. 23, 2013 | 8:14 PM | 1 comments

See more serious and funny news links at
`https://github.com/FormalTheology/GoedelGod/tree/master/Press`

Overall Motivation: Leibniz (1646–1716) — *Calculemus!*

If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, to sit down to their slates, and to say to each other . . . : Let us calculate.
(Translation by Russell)

Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo . . . dicere: calculemus. (Leibniz, 1684)

Required:
characteristica universalis and **calculus ratiocinator**

Ontological argument for the existence of God

- Long tradition in (western) philosophy
- Focus on Gödel's modern version in higher-order modal logic
- Experiments with theorem provers
  (theorem provers = computer programs that try to prove theorems)

Different interests in ontological arguments

- Philosophical: Boundaries of metaphysics & epistemology
- Theistic: Successful argument could convince atheists?
- Ours: Computational metaphysics (Leibniz' vision)

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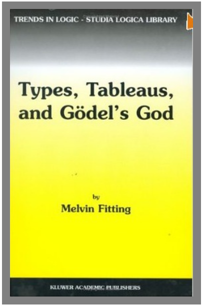
Presentation to Kurt Gödel Society in Vienna in October 2012

Got introduced after the talk to Bruno Woltzenlogel Paleo

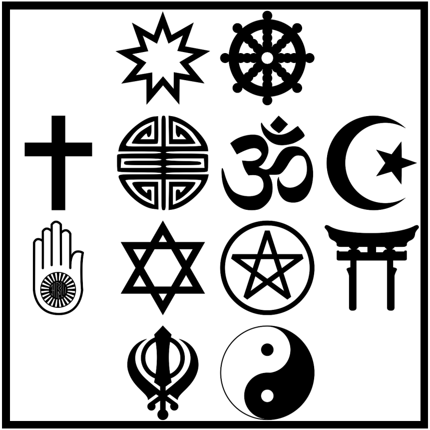


$$\dfrac{\dfrac{\text{Axiom 3}}{P(G)} \qquad \dfrac{\dfrac{\text{Theorem 1}}{\forall\varphi.[P(\varphi)\rightarrow\Diamond\exists x.\varphi(x)]}}{P(G)\rightarrow\Diamond\exists x.G(x)}\,\forall_E}{\Diamond\exists x.G(x)}\,\rightarrow_E$$

A gift to **Priest Edvaldo** and his church in Piracicaba, Brazil

Talk Outline

1. Ontological argument for the existence of God
2. Gödel's modern variant of the argument — two versions
3. Automation on the computer — how?
4. Results — theorem provers contributed relevant knowledge
5. Recent studies — theorem provers settled a dispute
6. Related work, discussion and conclusion



1. Ontological argument for the existence of God

Def: Ontological Argument

- deductive argument
- for the existence of God
- starting from premises, which are justified by pure reasoning
- i.e. premises do not depend on observation of the world
- "a priori" argument (versus "a posteriori" argument)

Ontological Argument: A long history

proponents and opponents

… Anselm v. C. Gaunilo … Th. Aquinas …… Descartes Spinoza Leibniz … Hume Kant … Hegel … Frege … Hartshorne Malcolm Lewis Plantinga Gödel …

Anselm's notion of God (Proslogion, 1078):

"God is that, than which nothing greater can be conceived."

Gödel's notion of God:

"A God-like being possesses all 'positive' properties."

To show by logical, deductive reasoning:

"God exists."

$$\exists x G(x)$$

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"Necessarily, God exists."

$\Box \exists x G(x)$

Modal Logic

$\Box\varphi$	—	Necessarily, φ holds
$\Diamond\varphi$	—	Possibly, φ holds

Classical Logic

$\neg\,\varphi$	—	not φ
$\varphi \vee \psi$	—	φ or ψ
$\varphi \wedge \psi$	—	φ and ψ
$\varphi \rightarrow \psi$	—	φ implies ψ
$\varphi \leftrightarrow \psi$	—	φ is equivalent to ψ
$\forall x\, \varphi$	—	For all x we have φ
$\exists x\, \varphi$	—	There exists x such that φ

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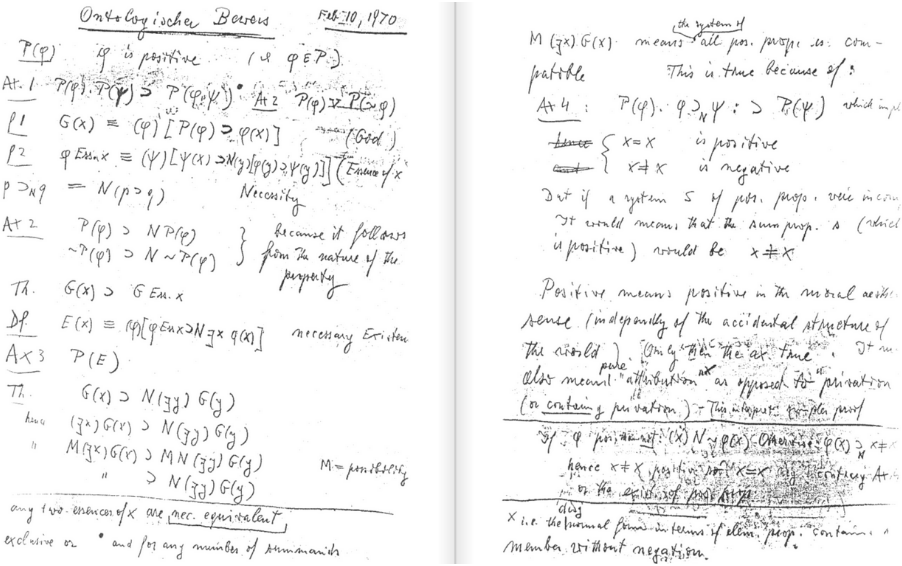
2. Gödel's modern variant of the argument — two versions

Gödel's Manuscript: 1930's, 1941, 1946-1955, 1970



Ontologischer Beweis   Feb 10, 1970

$P(\varphi)$ φ is positive (or $\varphi \in P$)

At. 1 $P(\varphi).P(\psi) \supset P(\varphi.\psi)$ • At 2 $P(\varphi) \vee P(\sim\varphi)$

P1 $G(x) \equiv (\varphi)[P(\varphi) \supset \varphi(x)]$ (God)

P2 $\varphi \, Ess \, x \equiv (\psi)[\psi(x) \supset N(y)[\varphi(y) \supset \psi(y)]]$ (Essence of x)

$p \supset_N q = N(p \supset q)$ Necessity

At 2 $P(\varphi) \supset NP(\varphi)$, $\sim P(\varphi) \supset N \sim P(\varphi)$ } because it follows from the nature of the property

Th. $G(x) \supset G \, Ess. \, x$

Df. $E(x) \equiv (\varphi)[\varphi \, Ess \, x \supset N \exists x \, \varphi(x)]$ necessary Existence

Ax 3 $P(E)$

Th. $G(x) \supset N(\exists y) G(y)$

hence $(\exists x) G(x) \supset N(\exists y) G(y)$

" $M(\exists x) G(x) \supset MN(\exists y) G(y)$ M = possibility

" $\supset N(\exists y) G(y)$

any two essences of x are nec. equivalent

exclusive or • and for any number of summands

$M(\exists x) G(x)$ means the system of all pos. prop. is compatible   This is true because of:

At 4: $P(\varphi). \varphi \supset_N \psi : \supset P(\psi)$ which impl

$x = x$ is positive

$x \neq x$ is negative

But if a system S of pos. prop. were incon

It would mean that the sum prop. s (which is positive) would be $x \neq x$

Positive means positive in the moral aesth. sense (independently of the accidental structure of the world). Only then the ax. true. It m also meant pure "attribution" as opposed to "privation" (or containing privation). This interpret. simpler proof

If φ pos. not: $(x) N \sim\varphi(x)$. Otherwise: $\varphi(x) \supset_N x \neq x$ hence $x \neq x$ positive not $x = x$ nor the contrary At or the exist. of pos. prop.

X i.e. the disj. normal form in terms of elem. prop. contains a member without negation.

Scott's Version of Gödel's Axioms, Definitions and Theorems

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2 A property necessarily implied by a positive property is positive:
$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

Thm. T1 Positive properties are possibly exemplified: $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

Def. D1 A *God-like* being possesses all positive properties: $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$

Axiom A3 The property of being God-like is positive: $P(G)$

Cor. C Possibly, God exists: $\Diamond\exists xG(x)$

Axiom A4 Positive properties are necessarily positive: $\forall\phi[P(\phi) \rightarrow \Box P(\phi)]$

Def. D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess.\ x \leftrightarrow \phi(x) \wedge \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

Thm. T2 Being God-like is an essence of any God-like being: $\forall x[G(x) \rightarrow G\ ess.\ x]$

Def. D3 *Necessary existence* of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$

Axiom A5 Necessary existence is a positive property: $P(NE)$

Thm. T3 Necessarily, God exists: $\Box\exists xG(x)$

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Modal operators are used

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second-order quantifiers

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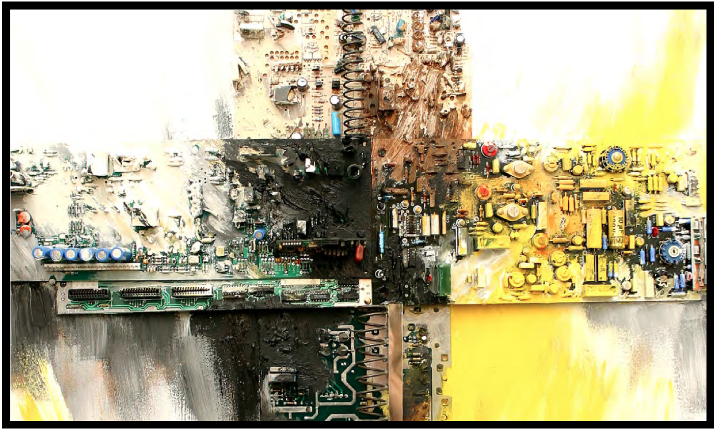
Difference to Gödel (who omits this conjunct)



3. Automation on the computer — how?

Challenge: No provers for *Higher-order Quantified Modal Logic* (QML)

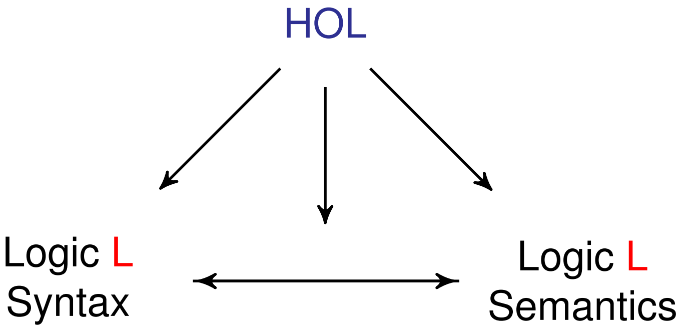
Our solution: Embedding in *Higher-order Classical Logic* (HOL)
Then use existing HOL theorem provers for reasoning in QML

[BenzmüllerPaulson, Logica Universalis, 2013]

Theorem provers for HOL do exists

interactive: Isabelle/HOL, HOL4, Hol Light, Coq/HOL, PVS, ...

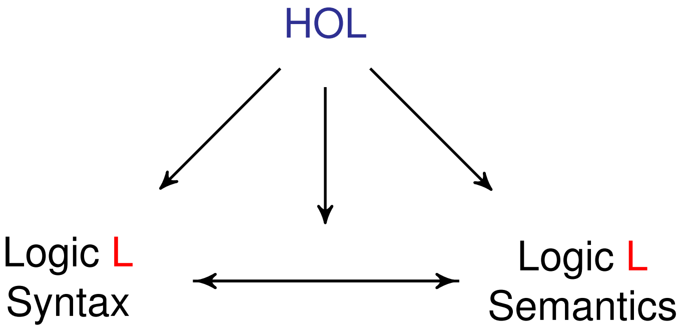
automated: TPS, **LEO-II**, Satallax, Nitpick, Isabelle/HOL, ...



Examples for L we have already studied:

Modal Logics, Conditional Logics, Intuitionistic Logics, Access Control Logics, Nominal Logics, Multivalued Logics (SIXTEEN), Logics based on Neighborhood Semantics, (Mathematical) Fuzzy Logics, Paraconsistent Logics, ...

Works also for (first-order & higher-order) quantifiers



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Embedding Approach — Idea

HOL (meta-logic) $\varphi ::=$

Your-logic (object-logic) $\psi ::=$

Embedding of in

=

=

=

=

Embedding of meta-logical notions on in

valid =

satisfiable =

... =

Pass this set of equations to a higher-order automated theorem prover

Embedding Approach — HOML in HOL

HOL $\quad s, t \quad ::= \quad C_\alpha \mid x_\alpha \mid (\lambda x_\alpha s_\beta)_{\alpha\to\beta} \mid (s_{\alpha\to\beta}\, t_\alpha)_\beta \mid \neg s_o \mid s_o \vee t_o \mid \forall x_\alpha\, t_o$

HOML $\quad \varphi, \psi \quad ::= \quad \ldots \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \to \psi \mid \Box\varphi \mid \Diamond\varphi \mid \forall x_\gamma\, \varphi \mid \exists x_\gamma\, \varphi$

HOML in HOL: HOML formulas φ are mapped to HOL predicates $\varphi_{\mu\to o}$
(explicit representation of labelled formulas)

$$
\begin{array}{lcl}
\neg & = & \lambda\varphi_{\mu\to o}\lambda w_\mu \neg\varphi w \\
\wedge & = & \lambda\varphi_{\mu\to o}\lambda\psi_{\mu\to o}\lambda w_\mu(\varphi w \wedge \psi w) \\
\to & = & \lambda\varphi_{\mu\to o}\lambda\psi_{\mu\to o}\lambda w_\mu(\neg\varphi w \vee \psi w) \\
\forall & = & \lambda h_{\gamma\to(\mu\to o)}\lambda w_\mu \forall d_\gamma\, hdw \\
\exists & = & \lambda h_{\gamma\to(\mu\to o)}\lambda w_\mu \exists d_\gamma\, hdw \\
\Box & = & \lambda\varphi_{\mu\to o}\lambda w_\mu \forall u_\mu\, (\neg rwu \vee \varphi u) \\
\Diamond & = & \lambda\varphi_{\mu\to o}\lambda w_\mu \exists u_\mu\, (rwu \wedge \varphi u) \\
\text{valid} & = & \lambda\varphi_{\mu\to o}\forall w_\mu \varphi w
\end{array}
$$

Ax (polymorphic over γ)

The equations in Ax are given as axioms to the HOL provers!

HOL $s, t \quad ::= \quad C_\alpha \mid x_\alpha \mid (\lambda x_\alpha s_\beta)_{\alpha\to\beta} \mid (s_{\alpha\to\beta}\, t_\alpha)_\beta \mid \neg s_o \mid s_o \vee t_o \mid \forall x_\alpha\, t_o$

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$\to$	$=$	$\lambda\varphi_{\mu\to o}\lambda\psi_{\mu\to o}\lambda w_\mu(\neg\varphi w \vee \psi w)$
$\forall$	$=$	$\lambda h_{\gamma\to(\mu\to o)}\lambda w_\mu \forall d_\gamma\, hdw$
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valid	$=$	$\lambda\varphi_{\mu\to o}\forall w_\mu \varphi w$

Ax (polymorphic over γ)

The equations in Ax are given as axioms to the HOL provers!

Example

$\Diamond \exists x G(x)$

valid $(\Diamond \exists x G(x))_{\mu \to o}$

$(\lambda \varphi \forall w_\mu \varphi\, w)(\Diamond \exists x G(x))_{\mu \to o}$

$\forall w_\mu ((\Diamond \exists x G(x))_{\mu \to o}\, w)$

$\forall w_\mu (((\lambda \varphi_{\mu \to o} \lambda w_\mu \exists u_\mu\, (rwu \wedge \varphi u)) \exists x G(x))_{\mu \to o}\, w)$

$\forall w_\mu \exists u_\mu (rwu \wedge (\exists x G(x))_{\mu \to o} u)$

$\forall w_\mu \exists u_\mu (rwu \wedge (\exists (\lambda x G(x)))_{\mu \to o} u)$

$\forall w_\mu \exists u_\mu (rwu \wedge ((\lambda h_{\gamma \to (\mu \to o)} \lambda w_\mu \exists d_\gamma\, hdw)(\lambda x G(x)))_{\mu \to o} u)$

$\forall w_\mu \exists u_\mu (rwu \wedge \exists x G x u)$

What are we doing?

In order to prove that φ is valid in HOML,
–> we instead prove that valid $\varphi_{\mu \to o}$ can be derived from Ax in HOL.

This can be done with interactive or automated HOL theorem provers.

Example

$\Diamond\exists x G(x)$

valid $(\Diamond\exists x G(x))_{\mu\to o}$

$(\lambda\varphi\forall w_\mu \varphi\, w)(\Diamond\exists x G(x))_{\mu\to o}$

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$\forall w_\mu \exists u_\mu (rwu \wedge \exists x Gxu)$

What are we doing?

In order to prove that φ is valid in HOML,
–> we instead prove that valid $\varphi_{\mu\to o}$ can be derived from Ax in HOL.

This can be done with interactive or automated HOL theorem provers.

Example

$\Diamond\exists x G(x)$

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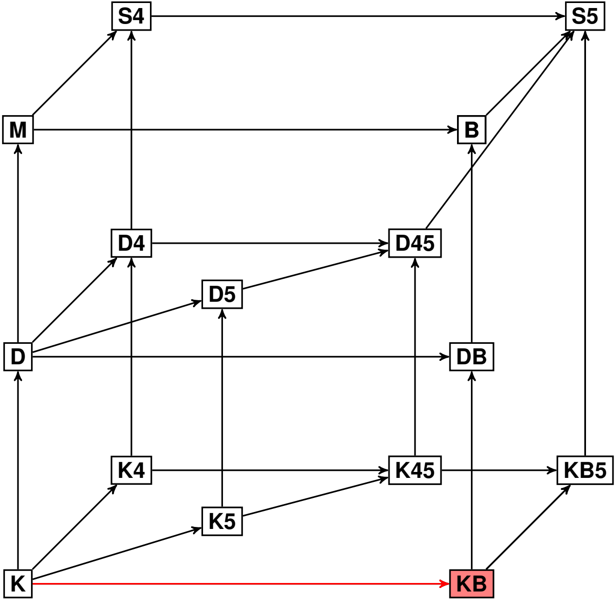
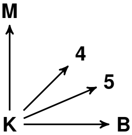
There is not just *one* modal logic: The Modal Logic Cube



B: $\varphi \rightarrow \Box\Diamond\varphi$
M: $\Box\varphi \rightarrow \varphi$
D: $\Box\varphi \rightarrow \Diamond\varphi$
4: $\Box\varphi \rightarrow \Box\Box\varphi$
5: $\Diamond\varphi \rightarrow \Box\Diamond\varphi$



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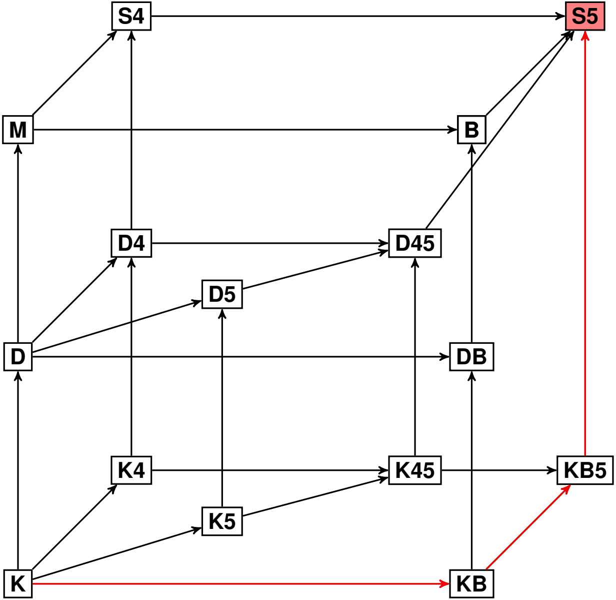

$$\Diamond\Box\varphi \rightarrow \Box\varphi$$

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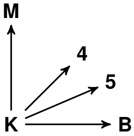


Gödel's proof of God: Automation with theorem provers for TPTP THF

```
>
>
> Beweise-mit-Leo2 Notwendigerweise-existiert-Gott.p

Leo-II tries to prove
=====================

  Goedel's Theorem T3: "Necessarily, God exists"
  thf(thmT3,conjecture,
      ( v
      @ ( mbox
        @ ( mexists_ind
          @ ^ [X: mu] :
              ( g @ X ) ) ) )).

  Assumptions: D1, C, T2, D3, A5

 . searching for proof ..

 ********************
 *   Proof found    *
 ********************
 % SZS status Theorem for Notwendigerweise-existiert-Gott.p

 . generating proof object
```

GoedelGod.thy (~/Varia/Talks/2015-ReasoningWeb/DemoMaterial/DEMO-Isabelle/)

```
text {* QML formulas are translated as HOL terms of type @{typ "i ⇒ bool"}.
This type is abbreviated as @{text "σ"}. *}

  type_synonym σ = "(i ⇒ bool)"

text {* The classical connectives $\neg, \wedge, \rightarrow$, and $\forall$
(over individuals and over sets of individuals) and $\exists$ (over individuals) are
lifted to type $\sigma$. The lifted connectives are @{text "m¬"}, @{text "m∧"}, @{text "m→"},
@{text "∀"}, and @{text "∃"} (the latter two are modeled as constant symbols).
Other connectives can be introduced analogously. We exemplarily do this for @{text "m∨"} ,
@{text "m≡"}, and @{text "mL="} (Leibniz equality on individuals). Moreover, the modal
operators @{text "□"} and @{text "◇"}  are introduced. Definitions could be used instead of
abbreviations. *}

  abbreviation mnot :: "σ ⇒ σ" ("m¬") where "m¬ φ ≡ (λw. ¬ φ w)"
  abbreviation mand :: "σ ⇒ σ ⇒ σ" (infixr "m∧" 51) where "φ m∧ ψ ≡ (λw. φ w ∧ ψ w)"
  abbreviation mor :: "σ ⇒ σ ⇒ σ" (infixr "m∨" 50) where "φ m∨ ψ ≡ (λw. φ w ∨ ψ w)"
  abbreviation mimplies :: "σ ⇒ σ ⇒ σ" (infixr "m→" 49) where "φ m→ ψ ≡ (λw. φ w ⟶ ψ w)"
  abbreviation mequiv:: "σ ⇒ σ ⇒ σ" (infixr "m≡" 48) where "φ m≡ ψ ≡ (λw. φ w ⟷ ψ w)"
  abbreviation mforall :: "('a ⇒ σ) ⇒ σ" ("∀") where "∀ Φ ≡ (λw. ∀x. Φ x w)"
  abbreviation mexists :: "('a ⇒ σ) ⇒ σ" ("∃") where "∃ Φ ≡ (λw. ∃x. Φ x w)"
  abbreviation mLeibeq :: "μ ⇒ μ ⇒ σ" (infixr "mL=" 52) where "x mL= y ≡ ∀(λφ. (φ x m→ φ y))"
  abbreviation mbox :: "σ ⇒ σ" ("□") where "□ φ ≡ (λw. ∀v.  w r v ⟶ φ v)"
  abbreviation mdia :: "σ ⇒ σ" ("◇") where "◇ φ ≡ (λw. ∃v. w r v ∧ φ v)"

text {* For grounding lifted formulas, the meta-predicate @{text "valid"} is introduced. *}

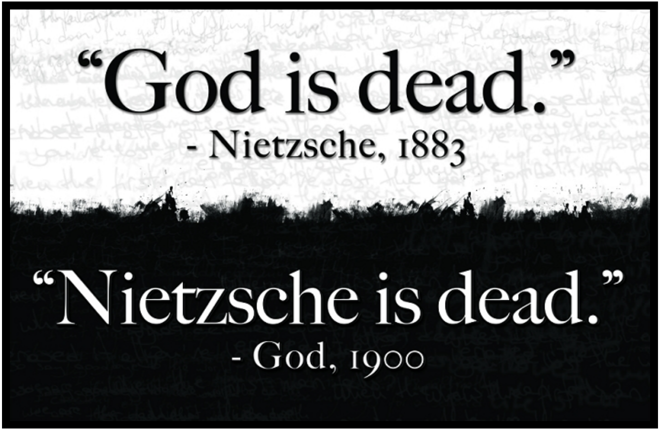
  (*<*) no_syntax "_list" :: "args ⇒ 'a list" ("[(_)]") (*>*)
  abbreviation valid :: "σ ⇒ bool" ("[_]") where "[p] ≡ ∀w. p w"
```

100% Auto update Update Detach

Output README Symbols

41,7 (1708/9125) (isabelle,sidekick,UTF-8-Isabelle)Nmro UG 68/124MB 6:28 PM

See verifiable Isabelle/HOL document (Archive of Formal Proofs) at:
`http://afp.sourceforge.net/entries/GoedelGod.shtml`



4. Results — Theorem provers contributed relevant (and even some new) knowledge

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2 A property necessarily implied by a positive property is positive:
$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

Thm. T1 Positive properties are possibly exemplified: $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

Def. D1 A *God-like* being possesses all positive properties: $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$

Axiom A3 The property of being God-like is positive: $P(G)$

Cor. C Possibly, God exists: $\Diamond\exists x G(x)$

Axiom A4 Positive properties are necessarily positive: $\forall\phi[P(\phi) \rightarrow \Box P(\phi)]$

Def. D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess.\ x \leftrightarrow \phi(x) \wedge \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

Thm. T2 Being God-like is an essence of any God-like being: $\forall x[G(x) \rightarrow G\ ess.\ x]$

Def. D3 *Necessary existence* of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$

Axiom A5 Necessary existence is a positive property: $P(NE)$

Thm. T3 Necessarily, God exists: $\Box\exists x G(x)$

Main Findings — Scott's version [ECAI, 2014]

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

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$\boxed{\textbf{Thm. T1}\ \text{Positive properties are possibly exemplified:}\ \forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]}$

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Def. D2 An *essence* o
implying any o

Thm. T2 Being God-like

Def. D3 *Necessary ex*
essences:

Axiom A5 Necessary ex

Thm. T3 Necessarily, G

Automating Scott's proof script

- **Provers: LEO-II and Satallax**
- **Show: T1 follows from A1($\rightarrow$) and A2**
- **Time: few milliseconds**
- **Logic: K is sufficent (S5 not needed)**
- **(Quantifiers: actualist and possibilist)**

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

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$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \to \psi(x)]) \to P(\psi)]$$

Thm. T1 Positive properties are possibly exemplified: $\forall\phi[P(\phi) \to \Diamond\exists x\phi(x)]$

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Axiom A5 Necessary ex

Thm. T3 Necessarily, G

Automating Scott's proof script

- **Provers: LEO-II and Satallax**
- **Show: C follows from T1, D1, A3**
- **Time: few milliseconds**
- **Logic: K is sufficent (S5 not needed)**
- **(Quantifiers: actualist and possibilist)**

Main Findings — Scott's version [ECAI, 2014]

Automating Scott's proof script

- **Provers: LEO-II and Satallax**
- **Show: T2 follows from A1, D1, A4, D2**
- **Time: few milliseconds**
- **Logic: K is sufficent (S5 not needed)**
- **(Quantifiers: actualist and possibilist)**

Axiom A1 Either a prope

Axiom A2 A property ne

Thm. T1 Positive prope

Def. D1 A *God-like* bei

Axiom A3 The property

Cor. C Possibly, God

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Thm. T3 Necessarily, God exists: $\Box\exists x G(x)$

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Axiom A2 A property ne

Thm. T1 Positive prope

Def. D1 A *God-like* bei

Axiom A3 The property

Cor. C Possibly, God

Automating Scott's proof script

- **Provers: LEO-II and Satallax — fail**
- **Modelfinder: Nitpick — countermodel**
- **Hence: T3 does *not* follow in logic K**

Axiom A4 Positive properties are necessarily positive: $\forall \phi[P(\phi) \rightarrow \Box P(\phi)]$

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Thm. T3 Necessarily, God exists: $\Box \exists x G(x)$

Automating Scott's proof script

- **Provers: LEO-II and Satallax**
- **Show: T3 follows from D1, C, T2, D3, A5**
- **Time: few milliseconds**
- **Logic: KB is sufficent (S5 not needed)**
- **(Quantifiers: actualist and possibilist)**

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Thm. T3 Necessarily, God exists: $\Box\exists x G(x)$

Automating Scott's proof script

- **Important question: Assumptions consistent?**
- **Modelfinder: Nitpick — presents simple model**
- **Time: few seconds**
- **Hence: consistency shown**

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Thm. T3 Necessarily, G

Automating Scott's proof script — Summary

- Axioms/definitions are consistent — verified
- T1, C, T2 and T3 indeed follow — verified
- Logic KB is sufficient — logic S5 not needed
- (Quantifiers: actualist and possibilist)
- Exact dependencies determined experimentally
- Provers found alternative proofs to humans:
  e.g. self-identity $\lambda x(x = x)$ is not needed
- Excellent match between argumentation granularity and strengths of the provers

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2 A property necessarily implied by a positive property is positive:
$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

Thm. T1 Positive properties are possibly exemplified: $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$

Def. D1 A *God-like* being possesses all positive properties: $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$

Axiom A3 The property of being God-like is positive: $P(G)$

Cor. C Possibly, God exists: $\Diamond\exists x G(x)$

Axiom A4 Positive properties are necessarily positive: $\forall\phi[P(\phi) \rightarrow \Box P(\phi)]$

Def. D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess.\ x \leftrightarrow \phi(x) \wedge \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

Thm. T2 Being God-like is an essence of any God-like being: $\forall x[G(x) \rightarrow G\ ess.\ x]$

Def. D3 *Necessary existence* of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$

Axiom A5 Necessary existence is a positive property: $P(NE)$

Thm. T3 Necessarily, God exists: $\Box\exists x G(x)$

difference in the definition of "essential properties"

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

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Thm. T3 Necessarily, God exists: $\Box\exists x G(x)$

Automating Gödel's proof script

- **Important question: Assumptions consistent?**
- **Prover LEO-II: Assumptions are inconsistent**
- **Time: some seconds**
- **Hence: Gödel's original script fails**
- **New philosophical result?!**

Axiom A1 Either a prope

Axiom A2 A property ne

Thm. T1 Positive prope

Def. D1 A *God-like* bei

Axiom A3 The property

Cor. C Possibly, God

Axiom A4 Positive prope

Def. D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess.\ x \leftrightarrow \boxed{\cancel{\phi(x)} \wedge} \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

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Axiom A5 Necessary existence is a positive property: $P(NE)$

Thm. T3 Necessarily, God exists: $\Box\exists x G(x)$

Inconsistency (Gödel): Proof by LEO-II in KB

DemoMaterial — bash — 166×52

```
@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: $false))@SV3)=$true))),inference(prim_subst,[status(thm)],[66:[bind(SV11,$thf(^[SV23:mu,SV24:$i]: $false))]])).
 thf(84,plain,(![SV22:(mu>($i>$o)),SV3:$i,SV8:(mu>($i>$o))]: ((((SV8@(((sK2_SY33@SV3)@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV8))@(((sK1_SY31@(^[SX0:mu,SX1:$i]: (
~ ((SV22@SX0)@SX1))))@SV8)@SV3))=$true) | (((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV3)=$true))),inference(prim_subst,[status(thm)],[66
:[bind(SV11,$thf(^[SV20:mu,SV21:$i]: (~ ((SV22@SV20)@SV21))))]])).
 thf(85,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false) | ((((p@SV9)@SV4) = ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)
@SY28))))@SV4))=$false))),inference(fac_restr,[status(thm)],[56])).
 thf(86,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true) | ((((p@SV9)@SV4) = ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@
SY30))))@SV4))=$false))),inference(fac_restr,[status(thm)],[57])).
 thf(87,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((~ (((p@SV9)@SV4) | ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4))) | (~ ((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY27:mu,
SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)))))=$false) | (((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_equal_neg,[status(thm)],[85])).
 thf(89,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((~ (((p@SV9)@SV4) | ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4))) | (~ ((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY29:mu,
SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)))))=$false) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true))),inference(extcnf_equal_neg,[status(thm)],[86])).
 thf(92,plain,(![SV4:$i,SV9:(mu>($i>$o))]: (((~ ((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4))))=$false) | (((p@(^[SY27:mu,SY28:$i]: (
~ ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_or_neg,[status(thm)],[87])).
 thf(93,plain,(![SV4:$i,SV9:(mu>($i>$o))]: (((~ (((p@SV9)@SV4) | ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)))=$false) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@
SY29)@SY30))))@SV4)=$true))),inference(extcnf_or_neg,[status(thm)],[89])).
 thf(96,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((~ ((p@SV9)@SV4)) | (~ ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)))=$true) | (((p@(^[SY27:mu,SY28:$i]: (~ ((S
V9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_not_neg,[status(thm)],[92])).
 thf(97,plain,(![SV4:$i,SV9:(mu>($i>$o))]: (((((p@SV9)@SV4) | ((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4))=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)
@SY30))))@SV4)=$true))),inference(extcnf_not_neg,[status(thm)],[93])).
 thf(100,plain,(![SV4:$i,SV9:(mu>($i>$o))]: (((~ ((p@SV9)@SV4))=$true) | ((~ ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4))=$true) | (((p@(^[SY27:mu,SY28:$i]:
 (~ ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_or_pos,[status(thm)],[96])).
 thf(101,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@SV9)@SV4)=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV
9@SY29)@SY30))))@SV4)=$true))),inference(extcnf_or_pos,[status(thm)],[97])).
 thf(103,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@SV9)@SV4)=$false) | ((~ ((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4))=$true) | (((p@(^[SY27:mu,SY28:$i]: (~
 ((SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_not_pos,[status(thm)],[100])).
 thf(105,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false) | (((p@SV9)@SV4)=$false) | (((p@(^[SY27:mu,SY28:$i]: (~ ((
SV9@SY27)@SY28))))@SV4)=$false))),inference(extcnf_not_pos,[status(thm)],[103])).
 thf(107,plain,(![SV8:(mu>($i>$o)),SV3:$i,SV22:(mu>($i>$o))]: ((((SV22@(((sK2_SY33@SV3)@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV8))@(((sK1_SY31@(^[SX0:mu,SX1:$i]:
 (~ ((SV22@SX0)@SX1))))@SV8)@SV3))=$true) | (((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: (~ ((SV22@SX0)@SX1))))@SV3)=$true))),inference(extcnf_not_neg,[status(thm
)],[78])).
 thf(108,plain,(![SV11:(mu>($i>$o)),SV3:$i,SV15:(mu>($i>$o))]: ((((SV15@(((sK2_SY33@SV3)@SV11)@(^[SX0:mu,SX1:$i]: (~ ((SV15@SX0)@SX1)))))@(((sK1_SY31@SV11)@(^[SX0:mu,
SX1:$i]: (~ ((SV15@SX0)@SX1))))@SV3))=$false) | (((p@(^[SX0:mu,SX1:$i]: (~ ((SV15@SX0)@SX1))))@SV3)=$false) | (((p@SV11)@SV3)=$true))),inference(extcnf_not_pos,[statu
s(thm)],[81])).
 thf(109,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@(^[SY27:mu,SY28:$i]: (~ ((SV9@SY27)@SY28))))@SV4)=$false) | (((p@SV9)@SV4)=$false))),inference(sim,[status(thm)],[10
5])).
 thf(110,plain,(![SV4:$i,SV9:(mu>($i>$o))]: ((((p@SV9)@SV4)=$true) | (((p@(^[SY29:mu,SY30:$i]: (~ ((SV9@SY29)@SY30))))@SV4)=$true))),inference(sim,[status(thm)],[101]
)).
 thf(111,plain,(![SV3:$i,SV8:(mu>($i>$o))]: ((((p@SV8)@SV3)=$false) | (((p@(^[SX0:mu,SX1:$i]: $true))@SV3)=$true))),inference(sim,[status(thm)],[76])).
 thf(112,plain,(![SV11:(mu>($i>$o)),SV3:$i]: ((((p@(^[SX0:mu,SX1:$i]: $false))@SV3)=$false) | (((p@SV11)@SV3)=$true))),inference(sim,[status(thm)],[80])).
 thf(113,plain,((($false)=$true)),inference(fo_atp_e,[status(thm)],[25,112,111,110,109,108,107,84,83,82,75,74,73,72,71,70,69,68,67,66,65,62,57,56,51,42,29])).
 thf(114,plain,($false),inference(solved_all_splits,[solved_all_splits(join,[])],[113])).
% SZS output end CNFRefutation

%***** End of derivation protocol ****
%***** no. of clauses in derivation: 97 ****
%***** clause counter: 113 ****

% SZS status Unsatisfiable for ConsistencyWithoutFirstConjunctinD2.p : (rf:0,axioms:6,ps:3,u:6,ude:false,rLeibEQ:true,rAndEQ:true,use_choice:true,use_extuni:true,use_
extcnf_combined:true,expand_extuni:false,foatp:e,atp_timeout:25,atp_calls_frequency:10,ordering:none,proof_output:1,clause_count:113,loop_count:0,foatp_calls:2,transl
ation:fof_full)
ontoleo:DemoMaterial cbenzmueller$
```

Axiom A1	Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$
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Thm. T1	Positive properties are possibly exemplified: $\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$
Def. D1	A *God-like* b
Axiom A3	The propert
Cor. C	Possibly, Go
Axiom A4	Positive pro
Def. D2	An *essence* implying any
Thm. T2	Being God-l
Def. D3	*Necessary e* essences:
Axiom A5	Necessary e
Thm. T3	Necessarily,

LEO-II's inconsistency proof

- **Problem: technical, machine-oriented proof output**
- **Challenge: extraction of a human-intuitive argument**
- **For a long time I failed to "understand" my prover,**
- **but . . . recently, I succeeded**
- **Once understood, the inconsistency argument is simple!**
- **Clue: Self-difference becomes an essential property of every entity.**

Axiom A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

Axiom A2 A property necessarily implied by a positive property is positive:
$$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$$

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Thm. T1 Positive prope

Def. D1 A *God-like* bei

Axiom A3 The property

Cor. C Possibly, God

Axiom A4 Positive prope

Further Results

- **Monotheism holds**
- **God is flawless**

Def. D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess.\ x \leftrightarrow \phi(x) \wedge \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

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Axiom A5 Necessary existence is a positive property: $P(NE)$

Thm. T3 Necessarily, God exists: $\Box\exists x G(x)$

$$\forall x\forall y\,(G(x) \rightarrow (G(y) \rightarrow x = y))$$
$$\forall\phi\forall x(G(x) \rightarrow (\neg P(\phi) \rightarrow \neg\phi(x)))$$

Axiom A1 Either a prope

Axiom A2 A property ne

Thm. T1 Positive prope

Def. D1 A *God-like* bei

Axiom A3 The property

Cor. C Possibly, God

Axiom A4 Positive prope

Def. D2 An *essence* o
implying any c

Thm. T2 Being God-like

Modal Collapse

$$\forall \varphi(\varphi \rightarrow \Box \varphi)$$

- **quickly proved by LEO-II and Satallax**
- **corollary**

$$\forall \varphi(\Diamond \varphi \leftrightarrow \Box \varphi)$$

Serious problem! This expresses that ...

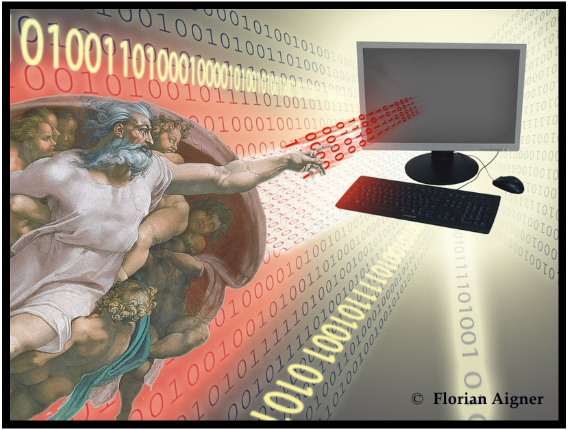
- **there are no contingent truths**
- **everything is determined / no free will**

Def. D3 *Necessary existence* of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall \phi[\phi \ ess. \ x \rightarrow \Box \exists y \phi(y)]$

Axiom A5 Necessary existence is a positive property: $P(NE)$

Thm. T3 Necessarily, God exists: $\Box \exists x G(x)$

$$\boxed{\forall \varphi(\varphi \rightarrow \Box \varphi)}$$



5. Recent work: tries to avoid the Modal Collapse
— theorem provers settled a dispute

Avoiding the Modal Collapse: Recent Variants

SOME EMENDATIONS OF GÖDEL'S ONTOLOGICAL PROOF

C. Anthony Anderson

Kurt Gödel's version of the ontological argument was shown by J. Howard Sobel to be defective, but some plausible modifications in the argument result in a version which is immune to Sobel's objection. A definition is suggested which permits the proof of some of Gödel's axioms.

Gödel's Ontological Proof Revisited *

C. Anthony Anderson and Michael Gettings

University of California, Santa Barbara
Department of Philosophy

Gödel's version of the modal ontological argument for the existence of God has been criticized by J. Howard Sobel [5] and modified by C. Anthony Anderson [1]. In the present paper we consider the extent to which Anderson's emendation is defeated by the type of objection first offered by the Monk Gaunilo to St. Anselm's original Ontological Argument. And we try to push the analysis of this Gödelian argument a bit further to bring it into closer agreement with the details of Gödel's own formulation. Finally, we indicate what seems to be the main weakness of this emendation of Gödel's attempted proof.

Magari and others on Gödel's ontological proof

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1 Introduction

This paper is a continuation of my paper [H] and concentrates almost exclusively to mathematical properties of logical systems underlying Gödel's ontological proof [G] and its variant by Anderson [A], with special care paid to Magari's criticism [M]. Since [H] is written in German, we shall try to summarize its content in such a way that knowledge of [H] will be not obligatory for reading the present paper (even it remains advantageous). Here we describe

Der Mathematiker und die Frage der Existenz Gottes

(betreffend Gödels ontologischen Beweis)

Es ist gut, daß wir nicht wissen,
sondern glauben, daß ein Gott sei.
(Kant, Nachlaß)

1. Einführung

Gödels zu Lebzeiten unveröffentlichter Beweis für die notwendige Existenz eines Gott-ähnlichen Wesens hat sowohl philosophisches als auch mathematisches Interesse geweckt. Zweck der vorliegenden Arbeit ist es, zu einer Deutung des Gödelschen Textes beizutragen, 1. durch Kommentierung der einschlägigen Literatur und 2. durch Bereitstellung von etwas Modelltheorie. Die Arbeit enthält keinen philosophischen Beitrag. Während der letzten Jahre habe ich etliche Male über Gödels Gottesbeweis vorgetragen, insbesondere auf dem Symposium zur Feier von Professor Gert Müller (Heidelberg, Jänner 1991), doch habe ich niemals beabsichtigt, eine Veröffentlichung über das Thema zu machen. Da ich wiederholt um eine schriftliche Version gebeten wurde, entschloß ich mich, schnell eine „erweiterte Kurzfassung"[1] zu schreiben, ohne aus ihr einen

A New Small Emendation of Gödel's Ontological Proof

PETR HÁJEK

Keywords: Ontological proof, Gödel, modal logic, comprehension, positive properties.

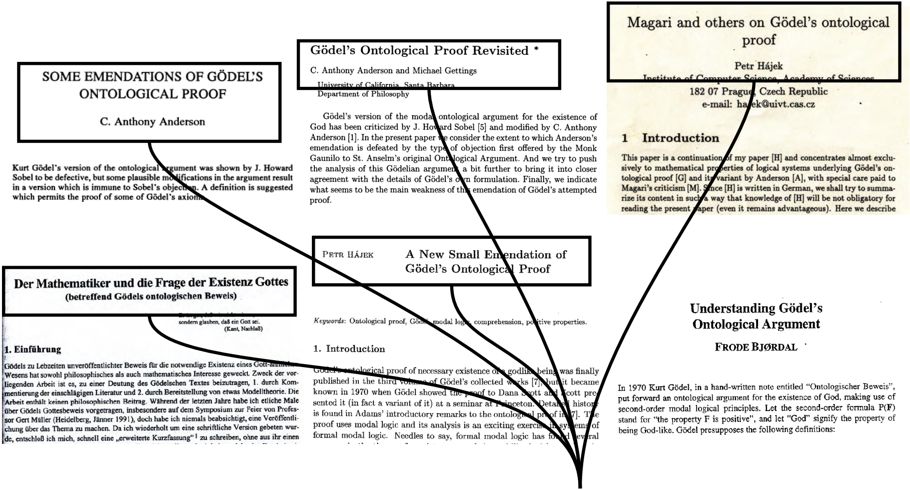
1. Introduction

Gödel's ontological proof of necessary existence of a godlike being was finally published in the third volume of Gödel's collected works [7]; but it became known in 1970 when Gödel showed the proof to Dana Scott and Scott presented it (in fact a variant of it) at a seminar at Princeton. Detailed history is found in Adams' introductory remarks to the ontological proof in [7]. The proof uses modal logic and its analysis is an exciting exercise in systems of formal modal logic. Needles to say, formal modal logic has found several

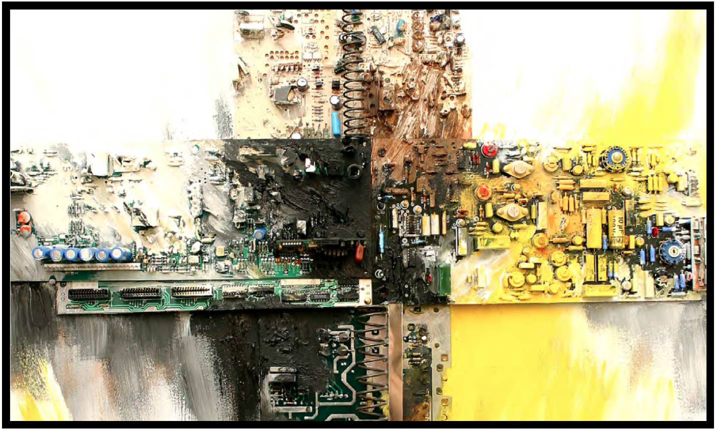
Understanding Gödel's Ontological Argument

FRODE BJØRDAL

In 1970 Kurt Gödel, in a hand-written note entitled "Ontologischer Beweis", put forward an ontological argument for the existence of God, making use of second-order modal logical principles. Let the second-order formula P(F) stand for "the property F is positive", and let "God" signify the property of being God-like. Gödel presupposes the following definitions:



Computer-supported Clarification of Controversy
1st World Congress on Logic and Religion, 2015



6. Discussion and conclusion

Overall Motivation: Leibniz (1646–1716) — *Calculemus!*

If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, to sit down to their slates, and to say to each other . . . : Let us calculate.

(Translation by Russell)

Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo . . . dicere: calculemus. (Leibniz, 1684)

Required:
characteristica universalis and **calculus ratiocinator**

Discussion

Internet bloggers reacted mostly negative to media reports!

Possible Explanations?

- Certain level of logic education required
- Few textbooks on the ontological argument for a wider audience
- Obfuscated media reports can trigger negative reactions

Conclusion

Overall Achievements

- significant contribution towards a **Computational Metaphysics**
- **novel results** contributed by **theorem provers**
- infrastructure can be adapted for **other logics and logic combinations**
- **basic technology works well**; however, improvements still needed

Relevance (wrt foundations and applications)

- Philosophy, AI, Computer Science, Computational Linguistics, Maths

Related work: only for Anselm's simpler argument

- with first-order prover PROVER9 [OppenheimerZalta, 2011]
- with interactive proof assistant PVS [Rushby, 2013]

Ongoing/Future work

- landscape of verified/falsified ontological arguments
- You may contribute: `https://github.com/FormalTheology/GoedelGod.git`

Personal Statement

(Interim) Culmination of two decades of related own research

- Theory of classical higher-order logic (HOL) (since 1995)
- Automation of HOL / own LEO provers (since 1998)
- Integration of interactive and automated theorem proving (since 1999)
- International TPTP infrastructure for HOL (since 2006)
- HOL as a universal logic via semantic embeddings (since 2008)
- jww Bruno Woltzenlogel-Paleo:
  Application in Metaphysics: Ontological Argument (since 2013)

. . . success story (despite strong criticism/opposition on the way!) . . .

Own standpoint

- I am not fully convinced (yet?) by the ontological argument.
- However, it seems to me that **"the belief in a (God-like) supreme being is not necessarily irrational/inconsistent"**.