Automating Gödel's Ontological Proof of God's Existence with Higher-order Automated Theorem Provers

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## Vision of Leibniz (1646–1716): Calculemus!



If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, to sit down to their slates, and to say to each other ...: Let us calculate.

(Translation by Russell)

Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo ... dicere: calculemus. (Leibniz, 1684)



Required: characteristica universalis and calculus ratiocinator

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Ontological argument for the existence of God

We focused on Gödel's modern version in higher-order modal logic

Automation with provers for higher-order classical logic (HOL)

- confirmation of known results
- detection of some novel results
- systematic variation of the logic settings
- exploited HOL as a universal metalogic (characteristica universalis)

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Anselm's notion of God (Proslogion, 1078):

"God is that, than which nothing greater can be conceived."

Gödel's notion of God:

"A God-like being possesses all 'positive' properties."

To show by logical reasoning:

"God exists."

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 $\exists x G(x)$ 



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"God is that, than which nothing greater can be conceived."

# Gödel's notion of God:

"A God-like being possesses all 'positive' properties."

To show by logical reasoning:

# "Necessarily God exists."

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# The Ontological Proof Today









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Gödel's Manuscript: 1930's, 1941, 1946-1955, 1970

Onto Coy ischer Basers Feb 10, 1970 P(q) is positive (if qEP.) At. 1 Prof. Prof. S Propy S Proget Prof. Prof. Prof. Prof.  $G(x) = (\varphi) [P(\varphi) \supset \varphi(x)]$  (God) PI  $\varphi E_{Max} \equiv (\gamma) [\gamma(x) \rightarrow M(y) [\varphi(y) \rightarrow \gamma(y)]] (Entries of x)$ P2. p > Ng = N(p>g) Neconstry At 2 P(q) > NP(q) } become it follows ~P(q) > N~P(q) } from The mature of the The General Form Th. G(x) > GEM. X  $E(x) \equiv i q \left[ q E x X M \right] x q(x) \right]$  mecessary Eristen Df.  $A \times 3 P(E)$ Th G(x) > N(32) G(1) Hence (3x) G(x) > N(32) G(3) M (JX) G(r) > MN (JJ) G(J) "> N (37) F(y) M= pontereity any two enences of x are mer. equivalent exclusive on " and for any mumber of Aummanich.

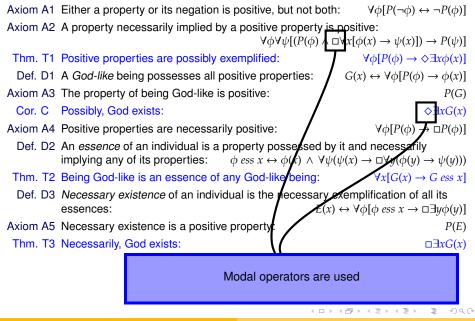
M (zx) F(x) - means all pos. prope is compatible This is the because of : At 4: P(q), q ), y: > P(y) which inpl the { x=x is positive x=x is negative Dut if a yetem 5 of pets, peopo, veic incom It would mean, that the Aun prop. A (which " prositive) would be x + x Positive means positive in the moral acide sense ( indepartily of the accidental structure of The avoild who Only The the at time . It we also means "attenduction at an opposed to privation (or crutain y privation ) - This interprets for pla proof 3/ q pontac at (X) N ~ por) - Onesting - q(X) > x+ hance x + X positive port X=X and theraping Ar in or the existing post Att 74 .... X i.e. the promoel former in terms if ellow phops . Contains a member without negation. < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □

Axiom A1 Either a property or its negation is positive, but not both:  $\forall \phi[P(\neg \phi) \leftrightarrow \neg P(\phi)]$ Axiom A2 A property necessarily implied by a positive property is positive:  $\forall \phi \forall \psi [(P(\phi) \land \Box \forall x [\phi(x) \to \psi(x)]) \to P(\psi)]$ Thm. T1 Positive properties are possibly exemplified:  $\forall \phi [P(\phi) \rightarrow \Diamond \exists x \phi(x)]$ Def. D1 A God-like being possesses all positive properties:  $G(x) \leftrightarrow \forall \phi[P(\phi) \rightarrow \phi(x)]$ Axiom A3 The property of being God-like is positive: P(G)Cor. C Possibly, God exists:  $\diamond \exists x G(x)$ Axiom A4 Positive properties are necessarily positive:  $\forall \phi[P(\phi) \to \Box P(\phi)]$ Def. D2 An essence of an individual is a property possessed by it and necessarily implying any of its properties:  $\phi ess \ x \leftrightarrow \phi(x) \land \forall \psi(\psi(x) \rightarrow \Box \forall \psi(\phi(y) \rightarrow \psi(y)))$  $\forall x[G(x) \rightarrow G \ ess \ x]$ Thm. T2 Being God-like is an essence of any God-like being: Def. D3 Necessary existence of an individual is the necessary exemplification of all its  $E(x) \leftrightarrow \forall \phi [\phi ess \ x \rightarrow \Box \exists y \phi(y)]$ essences: Axiom A5 Necessary existence is a positive property: P(E)Thm. T3 Necessarily, God exists:  $\Box \exists x G(x)$ 

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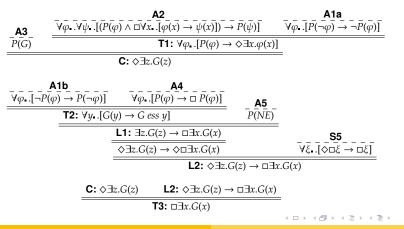
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#### **Proof Overview**

**D1:**  $G(x) \equiv \forall \varphi . [P(\varphi) \rightarrow \varphi(x)]$ 

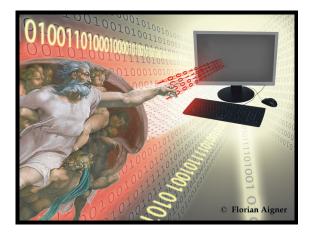
**D2:** 
$$\varphi ess \ x \equiv \varphi(x) \land \forall \psi . (\psi(x) \to \Box \forall x . (\varphi(x) \to \psi(x)))$$

**D3:**  $NE(x) \equiv \forall \varphi_{\bullet} . [\varphi \ ess \ x \to \Box \exists y. \varphi(y)]$ 



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#### How to automate Higher-Order Modal Logic?

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 Challenge:
 No provers for Higher-order Modal Logic (HOML)

 Our solution:
 Embedding in Higher-order Classical Logic (HOL)

 Then use existing HOL theorem provers for reasoning in HOML

[BenzmüllerPaulson, Logica Universalis, 2013]

Previous empirical findings:

Embedding of First-order Modal Logic in HOL works well

[BenzmüllerOttenRaths, ECAI, 2012]

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[Benzmüller, LPAR, 2013]

 $\mathsf{HOML} \quad \varphi, \psi \quad ::= \quad \dots \mid \neg \varphi \mid \varphi \land \psi \mid \varphi \to \psi \mid \Box \varphi \mid \Diamond \varphi \mid \forall x \varphi \mid \exists x \varphi \mid \forall P \varphi$ 

• Kripke style semantics (possible world semantics)

HOL  $s, t ::= C |x| \lambda xs |st| \neg s |s \lor t | \forall x t$ 

- Church's simple type theory [Church, 1940], [Henkin, 1950]
- various theorem provers exist

interactive: Isabelle/HOL, HOL4, Hol Light, Coq/HOL, PVS, ... automated: TPS, LEO-II, Satallax, Nitpick, Isabelle/HOL, ...

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# HOML $\varphi, \psi$ ::= $\ldots \mid \neg \varphi \mid \varphi \land \psi \mid \varphi \rightarrow \psi \mid \Box \varphi \mid \Diamond \varphi \mid \forall x \varphi \mid \exists x \varphi \mid \forall P \varphi$ HOLs, t::= $C \mid x \mid \lambda xs \mid st \mid \neg s \mid s \lor t \mid \forall x t$

HOML in HOL: HOML formulas  $\varphi$  are mapped to HOL predicates  $\varphi_{\mu \rightarrow o}$ 

The equations in Ax are given as axioms to the HOL provers!

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HOML in HOL: HOML formulas  $\varphi$  are mapped to HOL predicates  $\varphi_{\mu \rightarrow o}$ 

$$\neg = \lambda \varphi_{\mu \to o} \lambda w_{\mu} \neg \varphi w$$

$$\land = \lambda \varphi_{\mu \to o} \lambda \psi_{\mu \to o} \lambda w_{\mu} (\varphi w \land \psi w)$$

$$\rightarrow = \lambda \varphi_{\mu \to o} \lambda \psi_{\mu \to o} \lambda w_{\mu} (\neg \varphi w \lor \psi w)$$

$$\forall = \lambda h_{\gamma \to (\mu \to o)} \lambda w_{\mu} \forall d_{\gamma} h dw$$

$$\exists = \lambda h_{\gamma \to (\mu \to o)} \lambda w_{\mu} \exists d_{\gamma} h dw$$

$$\Box = \lambda \varphi_{\mu \to o} \lambda w_{\mu} \forall u_{\mu} (\neg r w u \lor \varphi u)$$

$$\diamond = \lambda \varphi_{\mu \to o} \lambda w_{\mu} \exists u_{\mu} (r w u \land \varphi u)$$

$$valid = \lambda \varphi_{\mu \to o} \forall w_{\mu \bullet} \varphi w$$

The equations in Ax are given as axioms to the HOL provers!

Ax

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#### **HOML** formula

HOML formula in HOL expansion,  $\beta\eta$ -conversion expansion,  $\beta\eta$ -conversion expansion,  $\beta\eta$ -conversion

## $\Diamond \exists x G(x)$

 $\begin{array}{l} \text{valid} (\diamond \exists x G(x))_{\mu \to o} \\ \forall w_{\mu}(\diamond \exists x G(x))_{\mu \to o} w \\ \forall w_{\mu} \exists u_{\mu}(rwu \land (\exists x G(x))_{\mu \to o} u) \\ \forall w_{\mu} \exists u_{\mu}(rwu \land \exists x Gxu) \end{array}$ 

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Expansion:

user or prover may flexibly choose expansion depth

#### What are we doing?

In order to prove that  $\varphi$  is valid in HOML, -> we instead prove that valid  $\varphi_{\mu \to o}$  can be derived from Ax in HOL.

This can be done with interactive or automated HOL theorem provers.

#### HOML formula HOML formula in HOL

expansion,  $\beta\eta$ -conversion expansion,  $\beta\eta$ -conversion expansion,  $\beta\eta$ -conversion

# $\begin{array}{c} \diamond \exists x G(x) \\ \text{valid} (\diamond \exists x G(x))_{\mu \to o} \\ \forall w_{\mu} (\diamond \exists x G(x))_{\mu \to o} w \\ \forall w_{\mu} \exists u_{\mu} (rwu \land (\exists x G(x))_{\mu \to o} u) \\ \forall w_{\mu} \exists u_{\mu} (rwu \land \exists x Gxu) \end{array}$

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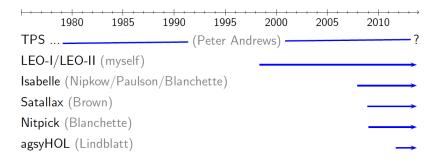
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In order to prove that  $\varphi$  is valid in HOML, -> we instead prove that valid  $\varphi_{\mu \to 0}$  can be derived from Ax in HOL.

This can be done with interactive or automated HOL theorem provers.

## Automated Theorem Provers and Model Finders for HOL



- all accept TPTP THF Syntax [SutcliffeBenzmüller, J.Form.Reas, 2009]
  - can be called remotely via SystemOnTPTP at Miami
  - $\bullet$  they significantly gained in strength over the last years
    - they can be bundled into a combined prover HOL-P

Exploit HOL with Henkin semantics as metalogic Automate other logics (& combinations) via semantic embeddings — HOL-P becomes a Universal Reasoner —

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#### Proof Automation and Consistency Checking with HOL-P

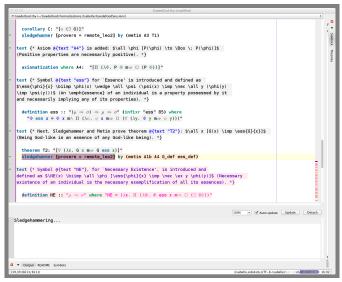
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Ferminal — bash — 125×32
MacBook-Chris %
MacBook-Chris %
MacBook-Chris % ./call_tptp.sh T3.p
Asking various HOL-ATPs in Miami remotely (thanks to Geoff Sutcliffe)
MacBook-Chris % aasyHOL---1.0 : T3.p ++++++ RESULT: SOT_7L4x_Y - aasyHOL---1.0 says Unknown - CPU = 0.00 WC = 0.02
LEO-II---1.6.0 : T3.p ++++++ RESULT: S0T_E4SCha - LEO-II---1.6.0 says Theorem - CPU = 0.03 WC = 0.09
Satallax---2.7 : T3.p ++++++ RESULT: S0T_kVZ1cB - Satallax---2.7 says Theorem - CPU = 0.00 WC = 0.14
Isabelle---2013 : T3.p ++++++ RESULT: S0T_xa0aEp - Isabelle---2013 says Theorem - CPU = 14.06 WC = 17.73 SolvedBy = auto
TPS---3.12060151b : T3.p ++++++ RESULT: SOT ROEasa - TPS---3.12060151b says Unknown - CPU = 33.56 WC = 41.57
Nitrox---2013 : T3.p ++++++ RESULT: SOT WGY1Tx - Nitrox---2013 says Unknown - CPU = 75.55 WC = 49.24
MacBook-Chris %
MacBook-Chris % ./call tptp.sh Consistency.p
Asking various HOL-ATPs in Miami remotely (thanks to Geoff Sutcliffe)
MacBook-Chris % agsyH0L---1.0 : Consistency.p ++++++ RESULT: SOT_ZtY_70 - agsyH0L---1.0 says Unknown - CPU = 0.00 WC = 0.00
Nitrox---2013 : Consistency.p ++++++ RESULT: SOT_HUZ10C - Nitrox---2013 says Satisfiable - CPU = 6.56 WC = 8.50
TPS---3.120601S1b : Consistency.p ++++++ RESULT: S0T_fpJxTM - TPS---3.120601S1b says Unknown - CPU = 43.00 WC = 49.42
Isabelle---2013 : Consistency, p ++++++ RESULT: SOT_6Tpp9i - Isabelle---2013 says Unknown - CPU = 69.96 WC = 72.62
LEO-II---1.6.0 : Consistency, p ++++++ RESULT: SOT_dY10sj - LEO-II---1.6.0 says Timeout - CPU = 90 WC = 89.86
Satallax---2.7 : Consistency.p ++++++ RESULT: SOT_09WSLf - Satallax---2.7 says Timeout - CPU = 90 WC = 90.50
MacBook-Chris % 🗌
```

Provers are called remotely in Miami - no local installation needed!

Download our experiments from https://github.com/ FormalTheology/GoedelGod/tree/master/Formalizations/THF

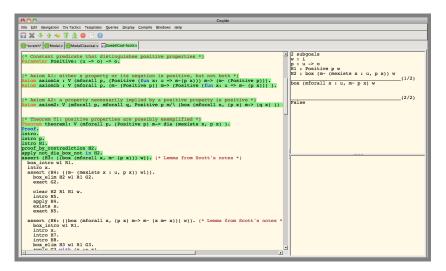
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#### Interaction and Automation in Proof Assistant Isabelle/HOL



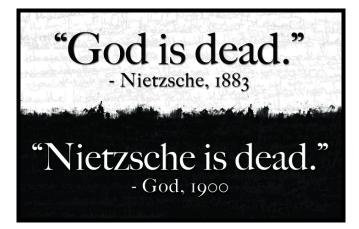
See verifiable Isabelle/HOL journal article at: http://afp.sourceforge.net/entries/GoedelGod.shtml

#### Interaction in Proof Assistant Coo



See verifiable Coq document at: https://github.com/ FormalTheology/GoedelGod/tree/master/Formalizations/Coq

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#### **Main Findings**

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	HOL encoding	dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall}\phi_{\mu\to\sigma}, p_{(\mu\to\sigma)\to\sigma}(\lambda X_{\mu}, \dot{\neg}(\phi X)) \doteq \dot{\neg}(p\phi)]$						
A2	$[\dot{\forall}\phi_{\mu\to\sigma^{\bullet}}\dot{\forall}\psi_{\mu\to\sigma^{\bullet}}(p_{(\mu\to\sigma)\to\sigma}\phi\dot{\wedge}\dot{\Box}\dot{\forall}X_{\mu^{\bullet}}(\phi X$	$(\neg \psi X)) \supset p\psi$					
T1	$[\dot{\forall}\phi_{\mu\to\sigma^*}p_{(\mu\to\sigma)\to\sigma}\phi \supset \dot{\Diamond}\dot{\exists}X_{\mu^*}\phi X]$	$A1(\supset), A2$	K	THM	0.1/0.1	0.0/0.0	_/_
	2 /µ = 1 µ = 0 / 0 / 0 / 0 / 0 / 0 / 0 / 0 / 0 / 0	A1, A2	K	THM	0.1/0.1	0.0/5.2	
D1	$g_{\mu \to \sigma} = \lambda X_{\mu} \cdot \dot{\forall} \phi_{\mu \to \sigma} \cdot p_{(\mu \to \sigma) \to \sigma} \phi \supset \phi X$						
A3	$[p_{(\mu \to \sigma) \to \sigma} g_{\mu \to \sigma}]$						
С	$[\diamond \exists X_{\mu} g_{\mu \to \sigma} X]$	T1.D1.A3	K	THM	0.0/0.0	0.0/0.0	_/_
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	
A4	$[\dot{\forall}\phi_{\mu\to\sigma^*}p_{(\mu\to\sigma)\to\sigma}\phi \stackrel{.}{\supset} \dot{\Box}p\phi]$						-
D2	$\operatorname{ess}_{(\mu \to \sigma) \to \mu \to \sigma} = \lambda \phi_{\mu \to \sigma^*} \lambda X_{\mu^*} \phi X \dot{\wedge} \dot{\forall} \psi_{\mu \to \sigma}$	$(\psi X \supset \Box \forall Y_{\dots} (\phi Y \supset \psi Y))$					
T2	$[\dot{\forall} X_{\mu}, g_{\mu \to \sigma} X \stackrel{i}{\supset} (ess_{(\mu \to \sigma) \to \mu \to \sigma} gX)]$	A1, D1, A4, D2	К	THM	19.1/18.3	0.0/0.0	_/
	L···μ·δμ=σ··· = (····(μ=σ)=μ=σδδ···/3	A1, A2, D1, A3, A4, D2	ĸ	THM	12.9/14.0	0.0/0.0	_/
D3	$NE_{\mu\to\sigma} = \lambda X_{\mu} \cdot \dot{\forall} \phi_{\mu\to\sigma} \cdot (\operatorname{ess} \phi X \dot{\supset}  \dot{\Box} \dot{\exists} Y_{\mu} \cdot \phi$						
A5	$[p_{(\mu \to \sigma) \to \sigma} \mathbf{N} \mathbf{E}_{\mu \to \sigma}]$	- ,					
Т3	$[\dot{\Box}\dot{\exists}X_{\mu}, g_{\mu\to\sigma}X]$	D1, C, T2, D3, A5	К	CSA	_/	_/	3.8/6.2
	$l =\mu \cdot s \mu \Rightarrow 0 = -3$	A1, A2, D1, A3, A4, D2, D3, A5	ĸ	CSA	_/	_/	8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	_/_
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	— <i>i</i> —	—/—	—/—
MC	$[s_{\sigma} \supset \Box s_{\sigma}]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
	2-0	A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	_/_	_/_	_/_
FG	$[\dot{\forall}\phi_{\mu\to\sigma^*}\dot{\forall}X_{\mu^*}(g_{\mu\to\sigma}X\dot{\supset}(\dot{\neg}(p_{(\mu\to\sigma)\to\sigma}\phi)\dot{\supset}$		KB	THM	16.5/—	0.0/0.0	_/
	$\mathbf{L} \cdot \mathbf{\tau} \mu \rightarrow 0 \cdot \cdot \cdot \mathbf{\tau} \mu \cdot (\mathbf{B} \mu \rightarrow 0 \cdot \mathbf{\tau} = \mathbf{U} \cdot (\mathbf{T} (\mu \rightarrow 0) \rightarrow 0 \mathbf{\tau}) =$	A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	12.8/15.1	0.0/5.4	_/
MT	$[\dot{\forall} X_{\mu^*} \dot{\forall} Y_{\mu^*} (g_{\mu \to \sigma} X \supset (g_{\mu \to \sigma} Y \supset X \doteq Y))]$	D1.FG	KB	THM	_/	0.0/3.3	_/
	$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{i$	A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	_/	_/	_/
		A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	_/	_/	7.3/7.4
CO	V(no goal, check for consistency)						
	$\emptyset$ (no goal, check for consistency)						
CO D2' CO'	(no goal, check for consistency) $ess_{(\mu \to \sigma) \to \mu \to \sigma} = \lambda \phi_{\mu \to \sigma} \cdot \lambda X_{\mu} \cdot \dot{\nabla} \psi_{\mu \to \sigma} \cdot (\psi X = \emptyset \text{ (no goal, check for consistency)})$		KB	UNS	7.5/7.8	_/	_/

# Main Findings

	HOL encoding	dependencies	logic	status	LEO-II	Satallax	Nitpick
4.1					const/varv	const/varv	const/varv
A1 A2	$\begin{bmatrix} \dot{\mathbf{V}} \boldsymbol{\phi}_{\mu \to \sigma} & p_{(\mu \to \sigma) \to \sigma} (\lambda X_{\mu} \ \dot{\neg} (\boldsymbol{\phi} X)) \equiv \dot{\neg} (\boldsymbol{p} \boldsymbol{\phi}) \end{bmatrix}$						
T1	$\begin{bmatrix} \dot{\forall} \phi_{\mu \to \sigma^*} \dot{\forall} \psi_{\mu \to \sigma^*} (p_{(\mu \to \sigma) \to \sigma} \phi \land \dot{\Box} \dot{\forall} X_{\mu^*} (\phi X) \\ \begin{bmatrix} \dot{\forall} \phi_{\mu \to \sigma^*} p_{(\mu \to \sigma) \to \sigma} \phi \supset \dot{\Diamond} \dot{\exists} X_{\mu^*} \phi X \end{bmatrix}$	$A1(\supset), A2$	к	THM	0.1/0.1	0.0/0.0	,
11	$[\forall \varphi_{\mu \to \sigma}, p_{(\mu \to \sigma) \to \sigma} \varphi \cup \bigtriangledown \exists A_{\mu}, \varphi A]$	A1, A2	ĸ	THM	0.1/0.1	0.0/5.2	_/_ _/_
D1	$g_{\mu \to \sigma} = \lambda X_{\mu} \cdot \dot{\forall} \phi_{\mu \to \sigma} \cdot p_{(\mu \to \sigma) \to \sigma} \phi \stackrel{.}{\Rightarrow} \phi X$	11,12	n.	11101	0.1/0.1	0.075.2	_/
A3	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} g_{\mu \to \sigma} \end{bmatrix}$						
C	$[\diamond \exists X_{\mu}, g_{\mu \to \sigma} X]$	T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	_/_
		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	_/_
A4	$[ \dot{\forall} \phi_{\mu \to \sigma^*} p_{(\mu \to \sigma) \to \sigma} \phi \supset \dot{\Box} p \phi ]$						
D2	$\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda \phi_{\mu\to\sigma} \cdot \lambda X_{\mu} \cdot \phi X \land \forall \psi_{\mu\to\sigma}$						
T2	$[ \dot{\forall} X_{\mu} \cdot g_{\mu \to \sigma} X  \dot{\supset}  ( \operatorname{ess}_{(\mu \to \sigma) \to \mu \to \sigma} g X ) ]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
		A1, A2, D1, A3, A4, D2	Κ	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathbf{NE}_{\mu\to\sigma} = \lambda X_{\mu^*} \dot{\forall} \phi_{\mu\to\sigma^*} (\mathbf{ess}  \phi X  \dot{\supset}  \dot{\Box} \dot{\exists} Y_{\mu^*} \phi$	Y)					
A5	$[p_{(\mu \to \sigma) \to \sigma} \mathbf{N} \mathbf{E}_{\mu \to \sigma}]$	D1 G T2 D2 45	17	00.4	,	,	2.9/6.9
T3	$[\dot{\Box}\dot{\exists}X_{\mu}, g_{\mu ightarrow\sigma}X]$	D1, C, T2, D3, A5 A1, A2, D1, A3, A4, D2, D3, A5	K K	CSA CSA	_/ /	_/ _/	3.8/6.2 8.2/7.5
		D1, C, T2, D3, A5	KB	THM	0.0/0.1	0.1/5.3	
		A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	_/	_/	_/
					,	,	
MC	$[s_{\sigma} \supset \Box s_{\sigma}]$	D2, T2, T3	KB	THM	17.9/—	3.3/3.2	—/—
-	nin view of writing of	A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	_/_	_/	—/—
FG	$[\dot{\forall}\phi_{\mu\to\sigma^*}\dot{\forall}X_{\mu^*}(g_{\mu\to\sigma}X\dot{\supset}(\dot{\neg}(p_{(\mu\to\sigma)\to\sigma}\phi)\dot{\supset}$	$\dot{\neg}(\phi X)))$ ] A1, D1	KB	THM	16.5/-	0.0/0.0	_/
MT	$[\dot{\forall} X_{\mu^*} \dot{\forall} Y_{\mu^*} (g_{\mu \to \sigma} X \supset (g_{\mu \to \sigma} Y \supset X \doteq Y))]$	A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	12.8/15.1	0.0/5.4	_/
MI	$[\forall \mathbf{A}_{\mu^*} \forall \mathbf{Y}_{\mu^*} (\mathbf{g}_{\mu \to \sigma} \mathbf{X} \supset (\mathbf{g}_{\mu \to \sigma} \mathbf{Y} \supset \mathbf{X} = \mathbf{Y}))]$	D1, FG A1, A2, D1, A3, A4, D2, D3, A5	KB	THM	_/_ _/_	0.0/3.3	_/_ _/_
		A1, A2, D1, A3, A4, D2, D3, A3	кD	11101			
CO	Ø (no goal, check for consistency)	A1, A2, D1, A3, A4, D2, D3, A5	KB	SAT	_/_	_/_	7.3/7.4
D2'	$\operatorname{ess}_{(\mu \to \sigma) \to \mu \to \sigma} = \lambda \phi_{\mu \to \sigma} \cdot \lambda X_{\mu} \cdot \dot{\forall} \psi_{\mu \to \sigma} \cdot (\psi X)$						
CO'	$\emptyset$ (no goal, check for consistency)	A1(⊃), A2, D2', D3, A5	KB	UNS	7.5/7.8	_/	—/—
		A1, A2, D1, A3, A4, D2', D3, A5	KB	UNS	_/	_/	—/—
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# Main Findings

	HOL encoding		dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall} \phi_{\mu \to \sigma}, p_{(\mu \to \sigma) \to \sigma}(\lambda X_{\mu}, \dot{\neg})]$	-(φX)) ≟ ∹(pφ)]				consy vary	consevery	consy varj
42			$K \rightarrow \psi(K)) \rightarrow p\psi[$					
T1	$[\dot{\forall} \phi_{\mu  o \sigma^*} p_{(\mu  o \sigma)  o \sigma} \phi  \supset  \dot{\Diamond}  \dot{\exists}$	$[X_{\mu}, \phi X]$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
			A1, A2	K	THM	0.1/0.1	0.0/5.2	—/—
A3	$g_{\mu \to \sigma} - \lambda \Lambda_{\mu} \cdot \nabla \psi_{\mu \to \sigma} \cdot p_{(\mu \to \sigma)}$	$\rightarrow \sigma \rightarrow \sigma \psi \neg \psi \Lambda$						
C	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} g_{\mu \to \sigma} \end{bmatrix}$ $[\diamondsuit \exists X_{\mu} \cdot g_{\mu \to \sigma} X]$		T1, D1, A3	к	THM	0.0/0.0	0.0/0.0	_/_
	<b>[∨⊐</b> Λμ• <u>β</u> μ→σΛ]		A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	
A4	$[\dot{\forall}\phi_{\mu\to\sigma^*}p_{(\mu\to\sigma)\to\sigma}\phi \stackrel{.}{\supset}\dot{\Box}p$	φ]						
D2	$\operatorname{ess}_{(\mu\to\sigma)\to\mu\to\sigma}=\lambda\phi_{\mu\to\sigma}$	$\lambda X_{\mu} \cdot \phi X \land \forall \psi_{\mu \rightarrow}$	$_{\sigma^{\bullet}}(\psi X \stackrel{.}{\supset} \dot{\Box} \stackrel{.}{\forall} Y_{\mu^{\bullet}}(\phi Y \stackrel{.}{\supset} \psi Y))$					
T2	$[\dot{\forall} X_{\mu}, g_{\mu \to \sigma} X \stackrel{.}{\supset} (\operatorname{ess}_{(\mu \to \sigma)}$	$\rightarrow \mu \rightarrow \sigma gX)]$	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
			A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	_/
<b>P</b> <sup>3</sup>	$\mathbf{NE}_{\mu\to\sigma} = \lambda X_{\mu} \cdot \dot{\mathbf{V}} \boldsymbol{\phi}_{\mu\to\sigma} \cdot (\boldsymbol{\phi}_{\mu\to\sigma})$	Autor	nating Scott's pro	of scri	pt			
D3 A5 T3	$[p_{(\mu \to \sigma) \to \sigma} \mathbf{NE}_{\mu \to \sigma}]$	Auton	nating Scott's pro	oof scri	pt			
D3 A5 T3	$NE_{\mu\to\sigma} = \lambda X_{\mu} \cdot \dot{\nabla} \phi_{\mu\to\sigma^*} (\phi_{\mu\to\sigma^*}) = [p_{(\mu\to\sigma)\to\sigma} NE_{\mu\to\sigma}] \\ [\dot{\Box} \exists X_{\mu^*} g_{\mu\to\sigma} X]$	Autor			•			
D3 5 T3	$[p_{(\mu \to \sigma) \to \sigma} \mathbf{NE}_{\mu \to \sigma}]$	Autor	nating Scott's pro		•	ssibly (	exempl	ified''
D3 A5 T3	$[p_{(\mu \to \sigma) \to \sigma} \mathbf{NE}_{\mu \to \sigma}]$	T1: '	'Positive propert	ies are	po:	ssibly (	exempl	ified''
D3 5 T3	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} N E_{\mu \to \sigma} \end{bmatrix}$ $\begin{bmatrix} \dot{\Box} \exists X_{\mu^*} g_{\mu \to \sigma} X \end{bmatrix}$	T1: ' prove	Positive propert	ies are	po:	ssibly (	exempl	ified"
D3 75 Т3 МС	$[p_{(\mu \to \sigma) \to \sigma} \mathbf{NE}_{\mu \to \sigma}]$	T1: ' prove	'Positive propert	ies are	po:	ssibly (	exempl	ified"
D3 T3 MC FG	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} N E_{\mu \to \sigma} \end{bmatrix}$ $\begin{bmatrix} \dot{\Box} \exists X_{\mu^*} g_{\mu \to \sigma} X \end{bmatrix}$	T1: ' prove	Positive propert	ies are	po:	ssibly	exempl	ified''
FG	$[p_{(\mu \to \sigma) \to \sigma} N \mathbb{E}_{\mu \to \sigma}]$ $[\dot{\Box} \exists X_{\mu^*} g_{\mu \to \sigma} X]$ $[s_{\sigma} \Rightarrow \dot{\Box} s_{\sigma}]$ $[\dot{\forall} \phi_{\mu \to \sigma^*} \dot{\forall} X_{\mu^*} (s_{\mu \to \sigma} X \Rightarrow 0)]$	T1: ' prove • ir	'Positive propert d by LEO-II and S n logic: K rom axioms:	ies are	po:	ssibly	exempl	ified"
	$ \begin{matrix} [p_{(\mu \to \sigma) \to \sigma} \mathbf{N} \mathbf{E}_{\mu \to \sigma}] \\ [\dot{\Box} \exists X_{\mu^{*}} g_{\mu \to \sigma} X] \end{matrix} \\ [s_{\sigma} \exists \mathbf{N} s_{\sigma}] $	T1: ' prove • ir	'Positive propert d by LEO-II and S n logic: K	ies are	po:	ssibly (	exempl	ified''
FG	$[p_{(\mu \to \sigma) \to \sigma} N \mathbb{E}_{\mu \to \sigma}]$ $[\dot{\Box} \exists X_{\mu^*} g_{\mu \to \sigma} X]$ $[s_{\sigma} \Rightarrow \dot{\Box} s_{\sigma}]$ $[\dot{\forall} \phi_{\mu \to \sigma^*} \dot{\forall} X_{\mu^*} (s_{\mu \to \sigma} X \Rightarrow x)]$	T1: ' prove • ir	'Positive propert d by LEO-II and S n logic: K rom axioms:	ies are	po:	ssibly (	exempl	ified''
FG	$[p_{\mu \to \sigma} \rightarrow e^{\mathbf{N} \mathbf{E}_{\mu \to \sigma}}]$ $[\dot{\alpha} \exists X_{\mu^*} g_{\mu \to \sigma} X]$ $[s_{\sigma} \supset \mathbf{u}_{S_{\sigma}}]$ $[\dot{\nu} \phi_{\mu \to \sigma^*} \dot{\nu} X_{\mu^*} \underbrace{g_{\mu \to \sigma}}_{x \to x} X \supset$ $[\dot{\nu} X_{\mu^*} \dot{\nu} Y_{\mu^*} (g_{\mu \to \sigma} X \supset (g_{\mu}))$	T1: ' prove • ir	'Positive propert d by LEO-II and S n logic: K rom axioms: • A1 and A2	ies are Satallax	po:	ssibly (	exempl	ified''
FG MT	$[p_{\mu} - \sigma) \rightarrow \sigma \mathbf{N} \mathbf{E}_{\mu \rightarrow \sigma}]$ $[\dot{\Box} \exists X_{\mu} * g_{\mu \rightarrow \sigma} X]$ $[s_{\sigma} \Rightarrow \mathbf{N}_{\sigma}]$ $[\dot{\nabla} \phi_{\mu \rightarrow \sigma} * \dot{\nabla} X_{\mu} * (g_{\mu \rightarrow \sigma} X \Rightarrow (g_{\mu} + g_{\mu \rightarrow \sigma} X \Rightarrow (g_{\mu \rightarrow \sigma$	T1: ' prove • ir • fr	'Positive propert d by LEO-II and S n logic: K rom axioms: • A1 and A2 or domain conditi	ies are Satallax	po:	ssibly (	exempl	ified"
FG MT CO	$[p_{\mu \to \sigma} \rightarrow e^{\mathbf{N} \mathbf{E}_{\mu \to \sigma}}]$ $[\dot{\alpha} \exists X_{\mu^*} g_{\mu \to \sigma} X]$ $[s_{\sigma} \supset \mathbf{u}_{S_{\sigma}}]$ $[\dot{\nu} \phi_{\mu \to \sigma^*} \dot{\nu} X_{\mu^*} \underbrace{g_{\mu \to \sigma}}_{x \to x} X \supset$ $[\dot{\nu} X_{\mu^*} \dot{\nu} Y_{\mu^*} (g_{\mu \to \sigma} X \supset (g_{\mu}))$	T1: ' prove • ir • fr	'Positive propert d by LEO-II and S n logic: K rom axioms: • A1 and A2	ies are Satallax	po:	ssibly (	exempl	ified"
FG MT CO D2'	$[p_{(\mu-\sigma)\rightarrow\sigma}NE_{\mu\rightarrow\sigma}]$ $[\dot{D}\exists X_{\mu^*}g_{\mu\rightarrow\sigma}X]$ $[s_{\sigma}\supset s_{\sigma}]$ $[\dot{V}\phi_{\mu\rightarrow\sigma^*}\dot{V}X_{\mu^*}G_{\mu\rightarrow\sigma}X \supset (g_{\mu})$ $[\dot{V}X_{\mu},\dot{V}Y_{\mu^*}(g_{\mu\rightarrow\sigma}X \supset (g_{\mu})$ $\emptyset \text{ (no goal, check for con ess} (\mu-\sigma)-\mu-\sigma = \lambda\phi\mu-\sigma-\lambda$	T1: ' prove • ir • fr	'Positive propert d by LEO-II and S n logic: K rom axioms: • A1 and A2 or domain conditi	ies are Satallax	po:	ssibly (	exempl	ified"
FG MT CO D2'	$[p_{(\mu-\sigma)\rightarrow\sigma}NE_{\mu\rightarrow\sigma}]$ $[\dot{D}\exists X_{\mu^*}g_{\mu\rightarrow\sigma}X]$ $[s_{\sigma}\supset s_{\sigma}]$ $[\dot{V}\phi_{\mu\rightarrow\sigma^*}\dot{V}X_{\mu^*}G_{\mu\rightarrow\sigma}X \supset (g_{\mu})$ $[\dot{V}X_{\mu},\dot{V}Y_{\mu^*}(g_{\mu\rightarrow\sigma}X \supset (g_{\mu})$ $\emptyset \text{ (no goal, check for con ess} (\mu-\sigma)-\mu-\sigma = \lambda\phi\mu-\sigma-\lambda$	T1: ' prove • ir • fr	'Positive propert d by LEO-II and S n logic: K rom axioms: • A1 and A2 or domain conditi	ies are Satallax	po:	ssibly	exempl	ified''

A1 $[\dot{V}\phi_{\mu\to\sigma}, p_{(\mu\to\sigma)\to\sigma}(\lambda X_{\mu}, \dot{\neg}(\phi X)) \doteq \dot{\neg}(p\phi)]$ A2 $[\dot{V}\phi_{\mu\to\sigma}, \dot{V}_{\mu\to\sigma}, (p_{\mu\to\sigma}, \phi, \dot{\gamma}(\phi X)) \doteq \dot{\neg}(p\phi)]$		Satallax
A1 $[\dot{\mathbf{v}}\phi_{\mu\to\sigma}, p_{(\mu\to\sigma)\to\sigma}(\lambda X_{\mu}, \dot{\boldsymbol{\tau}}(\phi X)) \doteq \dot{\boldsymbol{\tau}}(\phi X)) \doteq \dot{\boldsymbol{\tau}}(\phi X)$ A2 $[\dot{\mathbf{v}}\phi_{\mu\to\sigma}, \dot{\mathbf{v}}\psi_{\mu\to\sigma}, (p_{\mu\to\sigma}, \phi, \dot{\boldsymbol{\tau}})\dot{\boldsymbol{v}}Y, (\phi X \dot{\boldsymbol{\tau}}, \psi Y)) \dot{\boldsymbol{\tau}}(\phi X)$		const/vary
$\Delta 2  [V_{A}  \dot{V}_{V_{a}}  (n_{a}  A \land \dot{n} \dot{V}_{V_{a}} (A V \dot{n} V \dot{N}) \dot{n} n k]$		conse vary
T1 $[\dot{\mathbf{v}}\phi_{\mu\to\sigma^*}\mathbf{p}_{(\mu\to\sigma)\to\sigma}\phi\dot{\mathbf{o}}\dot{\mathbf{o}}\dot{\mathbf{d}}\mathbf{X}_{\mu^*}\phi\mathbf{X}]$ A1( $\supset$ ), A2 K THM 0.1/0.1 0.0/	).0 —/—	0.0/0.0
A1, A2 K THM 0.1/0.1 0.0/	i.2 —/—	0.0/5.2
$D_{1} = g_{\mu \to \sigma} - \lambda A_{\mu} \cdot \Psi _{\mu \to \sigma} \cdot p_{(\mu \to \sigma) \to \sigma} \psi \to \psi \Lambda$		
A3 $[p_{(\mu \to \sigma) \to \sigma} g_{\mu \to \sigma}]$		0.0/0.0
		0.0/0.0 5.2/31.3
$\begin{bmatrix} A1, A2, D1, A3 \end{bmatrix} \times \begin{bmatrix} i \ \phi_{\mu \to \sigma}, p_{(\mu \to \sigma) \to \sigma} \phi \Rightarrow \dot{\alpha} p \phi \end{bmatrix}$	1.5 —/—	3.2/31.3
$D2 = \operatorname{ess}_{(\mu \to \sigma) \to \mu \to \sigma} = \lambda \phi_{\mu \to \sigma^*} \lambda X_{\mu^*} \phi X \dot{\lambda} \dot{\Psi}_{\mu \to \sigma^*} (\psi X \dot{\supset} \dot{\Box} \dot{\Psi} Y_{\mu^*} (\phi Y \dot{\supset} \psi Y))$		
	)0 _/_	0.0/0.0
D3 NE <sub>10</sub> $= \lambda X_{10} \dot{Y} \phi_{10} g_{10}$		
$\sum_{\substack{S \\ [P_{\mu} - \sigma] \to \sigma \in \mathbb{R}_{\mu - \sigma}] \\ T[G_{\mu} = \sigma] \\ T[G_{\mu$		
$\frac{X_{\text{IIII}}}{T_{\text{IIII}}} = \frac{Automating Scott's proof script}{T_{\text{IIII}}}$ $\frac{Automating Scott's proof script}{T_{IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII$	nplified''	exempli
$\begin{array}{c} S \\ T3 \\ [ \Box \exists X_{\mu} * g_{\mu \to \sigma} X ] \end{array} \qquad \qquad$	mplified''	exempli
$\frac{X_{\text{IIII}}}{T_{\text{IIII}}} = \frac{Automating Scott's proof script}{T_{\text{IIII}}}$ $\frac{Automating Scott's proof script}{T_{IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII$	nplified"	exempli
$\begin{array}{c} \begin{array}{c} \begin{array}{c} X \\ T \\ T \end{array} & \begin{bmatrix} p_{(\mu \circ \sigma) \rightarrow \sigma} N E_{\mu \circ \sigma} \end{bmatrix} \\ \hline T \\ \hline D \exists X_{\mu} \cdot g_{\mu \rightarrow \sigma} X \end{bmatrix} \end{array} \begin{array}{c} \begin{array}{c} \begin{array}{c} Automating Scott's proof script \\ \hline T \\ T \end{bmatrix} \\ \hline T \\ T \\$	nplified"	exempli
$\begin{array}{c} & \begin{array}{c} & Automating Scott's proof script\\ \hline T3 & [D \exists X_{\mu}, g_{\mu - \sigma} X] \\ \hline MC & [s_{\sigma} \exists x_{\nu}, g_{\mu - \sigma} X] \end{array} \end{array} \begin{array}{c} & \begin{array}{c} & \begin{array}{c} & \begin{array}{c} & \begin{array}{c} & \begin{array}{c} & \end{array} \\ \hline T1 & \\ \end{array} \end{array} \begin{array}{c} & \begin{array}{c} & \end{array} \end{array} \\ \hline T1 & \\ \hline T1 & \\ \end{array} \begin{array}{c} & \begin{array}{c} & Positive \\ proved \\ by \\ LEO-II \\ and \\ Satallax \\ \bullet \\ \end{array} \end{array} \begin{array}{c} & \begin{array}{c} & \end{array} \end{array} \\ \hline \end{array} \end{array}$	nplified"	exempli
$\begin{array}{c} \Lambda \\ \Lambda $	nplified"	exempli
$\begin{array}{c c} & & & \\ & & & \\ & & & \\ \hline D \stackrel{S}{=} X_{\mu} \cdot g_{\mu \rightarrow \sigma} \cdot X \stackrel{T}{=} \\ \hline T \stackrel{T}{=} \begin{array}{c} & & \\ \hline D \stackrel{S}{=} X_{\mu} \cdot g_{\mu \rightarrow \sigma} \cdot X \stackrel{T}{=} \\ \hline D \stackrel{T}{=} X_{\mu} \cdot g_{\mu \rightarrow \sigma} \cdot X \stackrel{T}{=} \\ \hline \end{array} \\ & & \\ \hline MC  [s_{\sigma} \stackrel{S}{=} s_{\alpha}] \\ \hline FG  [V \stackrel{T}{=} q_{\mu \rightarrow \sigma} \cdot V_{\mu} \cdot s_{\alpha \rightarrow \sigma} X \stackrel{T}{=} ( \begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	mplified"	exempli
$\begin{array}{c} \overset{s}{\underset{[D]{} = \varphi \rangle - \varphi \wedge \tilde{X}_{\mu \to \varphi} - \varphi}{\text{T}} \\ \overset{s}{\underset{[D]{} = \tilde{X}_{\mu}, g_{\mu \to \varphi} \times \tilde{X}]}{\text{T}} \end{array} \xrightarrow{\text{Automating Scott's proof script}} \\ \begin{array}{c} \text{Automating Scott's proof script} \\ \hline \text{T1: "Positive properties are possibly exerption of the script} \\ \hline \text{T1: "Positive properties are possibly exerption of the script} \\ \hline \text{T1: "Positive properties are possibly exerption of the script} \\ \hline \text{T1: "Positive properties are possibly exerption of the script} \\ \hline \text{FG}  [\dot{v}_{\mu \to \sigma}, \dot{v}_{\mu \to \sigma}, \chi_{\Delta}] \\ \hline \text{MT}  [\dot{v}_{x_{\mu}}, \dot{v}_{y_{\mu}}(g_{\mu \to \sigma}, \chi_{\Delta})(g_{\mu}) \\ \hline \text{MT}  [\dot{v}_{x_{\mu}}, \dot{v}_{x_{\mu}}(g_{\mu \to \sigma}, \chi_{\Delta})(g_{\mu}) \\ \hline \text{MT}  [\dot{v}_{x_{\mu}}(g_{\mu \to \sigma}, \chi_{\Delta})(g_{\mu}) \\ \hline \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	mplified"	exempli

	HOL encoding		dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
<b>A</b> 1	$[\dot{\forall}\phi_{\mu\to\sigma}, p_{(\mu\to\sigma)\to\sigma}(\lambda X_{\mu}, \neg)]$						••••••	
A2				TZ.		0.1/0.1	0.0/0.0	,
T1	$[\dot{\forall}\phi_{\mu\to\sigma^*}p_{(\mu\to\sigma)\to\sigma}\phi\dot{\supset}\dot{\Diamond}\dot{\exists}X$	μ• <b>φ</b> Χ]	A1(⊃), A2 A1, A2	K K	THM THM	0.1/0.1 0.1/0.1	0.0/0.0 0.0/5.2	_/_
DI	$g_{\mu\to\sigma} = \lambda \overline{\lambda}_{\mu} \cdot \overline{\psi}_{\mu\to\sigma} \cdot p_{(\mu\to\sigma)}$		111,112	к	11100	0.1/0.1	0.0/5.2	_/_
A3	$[p_{(\mu\to\sigma)\to\sigma}g_{\mu\to\sigma}]$	,,						
C	$[\dot{\diamond} \exists X_{\mu} \ g_{\mu \to \sigma} X]$		T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/—
			A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot{\forall} \phi_{\mu  o \sigma^*} p_{(\mu  o \sigma)  o \sigma} \phi  \dot{\supset}  \dot{\Box} p \phi]$							
D2	$\operatorname{ess}_{(\mu \to \sigma) \to \mu \to \sigma} = \lambda \phi_{\mu \to \sigma} \cdot \lambda X$	<i>Ҟ<sub>μ</sub>₌φX</i> Ҡ҅ Ϋψ <sub>μ</sub>	$_{ ightarrow\sigma^{*}}(\psi X  \dot{\supset}  \dot{\Box}  \dot{\forall} Y_{\mu^{*}}(\phi Y  \dot{\supset}  \psi Y))$					
T2	$[\dot{\forall} X_{\mu}, g_{\mu \to \sigma} X \stackrel{.}{\supset} (\operatorname{ess}_{(\mu \to \sigma) \to \mu}$	$\mu \rightarrow \sigma gX$ ]	A1, D1, A4, D2	K		19.1/18.3	0.0/0.0	—/—
			A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	_/
1111	NE $= IX \cdot VA$ (e)				_			
03 A5 T3	$ \begin{split} \mathbf{N} \mathbf{E}_{\mu \to \sigma} &= \lambda X_{\mu^{\star}} \dot{\mathbf{V}} \phi_{\mu \to \sigma^{\star}} (\mathbf{e} \\ & [p_{(\mu \to \sigma) \to \sigma} \mathbf{N} \mathbf{E}_{\mu \to \sigma}] \\ & [\dot{\mathbf{D}} \dot{\exists} X_{\mu^{\star}} g_{\mu \to \sigma} X] \end{split} $	Auto	mating Scott's pro	of scri	pt			
Д3 Д5 Т3	$\begin{split} \mathbf{N} \mathbf{E}_{\mu \to \sigma} &= A X_{\mu^*} \mathbf{V} \boldsymbol{\phi}_{\mu \to \sigma^*} \mathbf{e} \\ [p_{(\mu \to \sigma) \to \sigma} \mathbf{N} \mathbf{E}_{\mu \to \sigma}] \\ [\dot{\mathbf{D}} \dot{\mathbf{X}}_{\mu^*} g_{\mu \to \sigma} X] \end{split}$	T1:	"Positive propert	ies are	e po	ssibly (	exempl	ified"
03 A5 T3	$ \begin{split} & \operatorname{NE}_{\mu \to \sigma} = \mathcal{X}_{\mu}, \forall \boldsymbol{\varphi}_{\mu \to \sigma^*} ( \boldsymbol{e} \\ & [p_{(\mu \to \sigma) \to \sigma} \operatorname{NE}_{\mu \to \sigma}] \\ & [\dot{\boldsymbol{\Box}} \exists X_{\mu^*} \boldsymbol{g}_{\mu \to \sigma} X] \end{split} $	T1:	<u> </u>	ies are	e po	ssibly	exempl	ified"
03 Т3 МС	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} \mathbf{N} \mathbf{E}_{\mu \to \sigma} \end{bmatrix}^{\top} \begin{bmatrix} \mathbf{\dot{\Box}} \exists X_{\mu}, \mathbf{g}_{\mu \to \sigma} X \end{bmatrix}$	T1: prove	"Positive propert ed by LEO-II and S	ies are	e po	ssibly	exempl	ified"
03 Т3 МС	$\begin{split} NE_{\mu,\sigma\sigma} &= \mathcal{AI}_{\mu} \cdot \forall \varphi_{\mu-\sigma} \cdot (\mathbf{e}_{[p(\mu \to \sigma) \to \sigma} NE_{\mu \to \sigma}] \\ & [p(\mu \to \sigma) \to \sigma NE_{\mu-\sigma}] \\ & [\dot{\mathbf{D}} \exists \mathcal{X}_{\mu} \cdot g_{\mu \to \sigma} \mathcal{X}] \end{split}$	T1: prove	"Positive propert	ies are	e po	ssibly (	exempl	ified"
MC FG	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} \mathbf{N} \mathbf{E}_{\mu \to \sigma} \end{bmatrix}^{\top} \begin{bmatrix} \mathbf{\dot{\Box}} \exists X_{\mu}, \mathbf{g}_{\mu \to \sigma} X \end{bmatrix}$	T1: prove ● i	"Positive propert ed by LEO-II and S	ies are	e po	ssibly (	exempl	ified"
FG	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} \operatorname{NE}_{\mu \to \sigma} \\ [\dot{\Box} \exists X_{\mu}, g_{\mu \to \sigma} X] \end{bmatrix}$ $[s_{\sigma} \ni \mathfrak{s}_{\sigma}]$ $[\dot{\forall} \phi_{\mu \to \sigma}, \dot{\forall} X_{\mu}, \mathfrak{s}_{\mu \to \sigma} X \ni 0]$	T1: prove ● i	"Positive propert ed by LEO-II and S n logic: K rom axioms:	ies are	e po	ssibly	exempl	ified"
	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} \operatorname{NE}_{\mu \to \sigma} \end{bmatrix}$ $\begin{bmatrix} \dot{\mu} \exists X_{\mu *} g_{\mu \to \sigma} X \end{bmatrix}$ $\begin{bmatrix} s_{\sigma} \exists \forall s_{\sigma} \end{bmatrix}$	T1: prove ● i	"Positive propert ed by LEO-II and S n logic: K rom axioms: • A1 and A2	ies are	e po	ssibly (	exempl	ified"
FG	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} \operatorname{NE}_{\mu \to \sigma} \\ [\dot{\Box} \exists X_{\mu}, g_{\mu \to \sigma} X] \end{bmatrix}$ $[s_{\sigma} \ni \mathfrak{s}_{\sigma}]$ $[\dot{\forall} \phi_{\mu \to \sigma}, \dot{\forall} X_{\mu}, \mathfrak{s}_{\mu \to \sigma} X \ni 0]$	T1: prove ● i	"Positive propert ed by LEO-II and S n logic: K rom axioms:	ies are	e po	ssibly (	exempl	ified"
FG MT	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} \mathbf{N} \mathbf{E}_{\mu \to \sigma} \\ [\dot{\mathbf{D}} \exists X_{\mu^*} g_{\mu \to \sigma} X] \end{bmatrix}$ $\begin{bmatrix} \mathbf{S}_{\sigma} \ni \mathbf{S}_{\sigma} \end{bmatrix}$ $\begin{bmatrix} \dot{\mathbf{V}} \phi_{\mu \to \sigma^*} \dot{\mathbf{V}} X_{\mu^*} g_{\mu \to \sigma} X \ni \begin{bmatrix} \mathbf{V} \\ \mathbf{V}_{\mu^*} \dot{\mathbf{V}} Y_{\mu^*} (g_{\mu \to \sigma} X \ni (g_{\mu} + g_{\mu \to \sigma} X)) \end{bmatrix}$	T1: prove o i o f	"Positive propert ed by LEO-II and S n logic: K rom axioms: ● A1 and A2 ● A1(⊃) and A2	ies are atallax	e po	ssibly	exempl	ified"
FG MT CO	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} N E_{\mu \to \sigma} \\ [\dot{D} \exists X_{\mu} \cdot g_{\mu \to \sigma} X] \end{bmatrix}$ $\begin{bmatrix} s_{\sigma} \ni s_{\sigma} \end{bmatrix}$ $\begin{bmatrix} \dot{V} \phi_{\mu \to \sigma} \cdot \dot{X}_{\mu} \cdot g_{\mu \to \sigma} X \ni (g_{\mu} - \sigma X) \end{bmatrix}$ $\begin{bmatrix} \dot{V} X_{\mu} \cdot \dot{V}_{\mu} \cdot (g_{\mu \to \sigma} X) = (g_{\mu} - \sigma X) \end{bmatrix}$	T1: prove o i o f	"Positive propert ed by LEO-II and S n logic: K rom axioms: • A1 and A2	ies are atallax	e po	ssibly (	exempl	ified"
FG MT	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} N E_{\mu \to \sigma} \\ [\dot{\Box} \exists X_{\mu}, g_{\mu \to \sigma} X] \end{bmatrix}$ $\begin{bmatrix} s_{\sigma} \ni s_{\sigma} \end{bmatrix}$ $\begin{bmatrix} \dot{\forall} \phi_{\mu \to \sigma}, \dot{\forall} X_{\mu}, \dots, \chi \ni (G_{\mu} \to G_{\mu}) \\ [\dot{\forall} X_{\mu}, \dot{\forall} Y_{\mu}, (g_{\mu \to \sigma} X \ni (G_{\mu} \to G_{\mu}))] \\ 0 \text{ (no goal, check for consess} \\ ess_{(\mu \to \sigma) \to \mu \to \sigma} = \lambda \phi_{\mu \to \sigma}, A \end{bmatrix}$	T1: prove o i o f	"Positive propert ed by LEO-II and S n logic: K rom axioms: ● A1 and A2 ● A1(⊃) and A2	ies are atallax ons:	e po	ssibly (	exempl	ified"
FG MT CO D2'	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} \mathbf{N} \mathbf{E}_{\mu \to \sigma} \\ [\dot{\mathbf{D}} \exists X_{\mu} \cdot \mathbf{g}_{\mu \to \sigma} X] \end{bmatrix}$ $\begin{bmatrix} \mathbf{s}_{\sigma} \rightarrow \mathbf{s}_{\sigma} \\ [\dot{\mathbf{v}}_{\sigma} \rightarrow \mathbf{v} \cdot \dot{\mathbf{x}}_{\mu} \cdot \mathbf{g}_{\mu \to \sigma} X \rightarrow 0 \\ [\dot{\mathbf{v}}_{X_{\mu}} \cdot \dot{\mathbf{v}}_{\mu_{\mu}} (\mathbf{g}_{\mu \to \sigma} X \rightarrow 0_{\mu} \\ 0 \text{ (no goal, check for const} \end{bmatrix}$	T1: prove o i o f	"Positive propert ed by LEO-II and S n logic: K rom axioms: ● A1 and A2 ● A1(⊃) and A2 for domain conditi ● constant domai	ies are atallax ons: ns	• po:	·	exempl	ified"
FG MT CO D2'	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} N E_{\mu \to \sigma} \\ [\dot{\Box} \exists X_{\mu}, g_{\mu \to \sigma} X] \end{bmatrix}$ $\begin{bmatrix} s_{\sigma} \ni s_{\sigma} \end{bmatrix}$ $\begin{bmatrix} \dot{\forall} \phi_{\mu \to \sigma}, \dot{\forall} X_{\mu}, \dots, \chi \ni (G_{\mu} \to G_{\mu}) \\ [\dot{\forall} X_{\mu}, \dot{\forall} Y_{\mu}, (g_{\mu \to \sigma} X \ni (G_{\mu} \to G_{\mu}))] \\ 0 \text{ (no goal, check for consess} \\ ess_{(\mu \to \sigma) \to \mu \to \sigma} = \lambda \phi_{\mu \to \sigma}, A \end{bmatrix}$	T1: prove o i o f	"Positive propert ed by LEO-II and S n logic: K from axioms: ● A1 and A2 ● A1(⊃) and A2 For domain conditi	ies are atallax ons: ns	• po:	·	exempl	ified"
FG MT CO D2'	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} N E_{\mu \to \sigma} \\ [\dot{\Box} \exists X_{\mu}, g_{\mu \to \sigma} X] \end{bmatrix}$ $\begin{bmatrix} s_{\sigma} \ni s_{\sigma} \end{bmatrix}$ $\begin{bmatrix} \dot{\forall} \phi_{\mu \to \sigma}, \dot{\forall} X_{\mu}, \dots, \chi \ni (G_{\mu} \to G_{\mu}) \\ [\dot{\forall} X_{\mu}, \dot{\forall} Y_{\mu}, (g_{\mu \to \sigma} X \ni (G_{\mu} \to G_{\mu}))] \\ 0 \text{ (no goal, check for consess} \\ ess_{(\mu \to \sigma) \to \mu \to \sigma} = \lambda \phi_{\mu \to \sigma}, A \end{bmatrix}$	T1: prove o i o f	"Positive propert ed by LEO-II and S n logic: K rom axioms: ● A1 and A2 ● A1(⊃) and A2 for domain conditi ● constant domai	ies are atallax ons: ns	• po:	·	exempl	ified"

	HOL encoding		dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall}\phi_{\mu\to\sigma}, p_{(\mu\to\sigma)\to\sigma}(\lambda X_{\mu}, \dot{\neg})]$	$(\phi X)) \doteq \neg (p\phi)$						
A2	$[\dot{\forall}\phi_{\mu\to\sigma},\dot{\forall}\psi_{\mu\to\sigma},(p_{(\mu\to\sigma)\to\sigma})$							
T1	$[\dot{\forall}\phi_{\mu\to\sigma}, p_{(\mu\to\sigma)\to\sigma}\phi \supset \dot{\Diamond}\dot{\exists})$	$X_{\mu} \cdot \phi X$	A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	—/—
			A1, A2	K	THM	0.1/0.1	0.0/5.2	<i>—/—</i>
D1	$g_{\mu\to\sigma} = \lambda X_{\mu} \cdot \dot{\forall} \phi_{\mu\to\sigma} \cdot p_{(\mu\to\sigma)}$	$\sigma \to \sigma \phi \supset \phi X$						
A3				**		0.010.0	0.0/0.0	
С	$[\dot{\Diamond} \exists X_{\mu} \ g_{\mu \to \sigma} X]$		T1, D1, A3	K K	THM	0.0/0.0	0.0/0.0	_/_
			A1, A2, D1, A3	ĸ	THM	0.0/0.0	5.2/31.3	_/_
D2	$(\psi_{\mu\to\sigma}, p_{(\mu\to\sigma)\to\sigma}, \phi \to \Box p_{\phi})$		$_{\sigma^*}(\psi X \stackrel{.}{\supset} \stackrel{.}{\Box} \stackrel{.}{\forall} Y_{\mu^*}(\phi Y \stackrel{.}{\supset} \psi Y))$					
T2	$[ \dot{\forall} X_{\mu} g_{\mu \to \sigma} X \stackrel{i}{\supset} (\operatorname{ess}_{(\mu \to \sigma) \to}$		A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	_/
12	$[ \bullet I \mu \bullet g \mu \rightarrow \sigma I \mu \rightarrow \sigma I \mu \rightarrow \sigma ] \rightarrow$	$\mu \rightarrow \sigma S^{(1)}$	A1, A2, D1, A3, A4, D2	ĸ	THM	12.9/14.0	0.0/0.0	
D3	$NE_{\mu\to\sigma} = \lambda X_{\mu} \cdot \dot{\forall} \phi_{\mu\to\sigma} \cdot (e$	A						
A5	$[p_{(\mu\to\sigma)\to\sigma}NE_{\mu\to\sigma}]$	Autor	nating Scott's pro	of scri	ρτ			
Т3	$[\Box \exists X_{\mu}, g_{\mu \to \sigma} X]$							
	$[\Box \neg A \mu \delta \mu \rightarrow \sigma A]$							
$\mathbf{N}$	[⊔⊐ <b>∧</b> μ• <b>σ</b> μ→σ <b>∧</b> ]	C. "D	ossibly God ovist	c"				
$\backslash$	[⊔⊐Aµ*Sµ→σA]		ossibly, God exist					
			ossibly, God exist d by LEO-II and S					
мс		prove	d by LEO-II and S					
мс		prove	• •					
MC FG		prove • ir	ed by LEO-II and S n logic: K	atallax				
FG	$[s_{\sigma} \dot{\Box} s_{\sigma}]$ $[\dot{V}\phi_{\mu+\sigma}, \dot{V} \lambda_{\mu}, a_{\mu+\sigma} X \dot{\Box}]$	prove • ir	ed by LEO-II and S n logic: K rom assumptions:	atallax				
		prove • ir	ed by LEO-II and S n logic: K rom assumptions: • T1, D1, A3	atallax				
FG	$[s_{\sigma} \dot{\Box} s_{\sigma}]$ $[\dot{V}\phi_{\mu+\sigma}, \dot{V} \lambda_{\mu}, a_{\mu+\sigma} X \dot{\Box}]$	prove • ir	ed by LEO-II and S n logic: K rom assumptions:	atallax				
FG MT	$[s_{\sigma} \dot{\Box} s_{\sigma}]$ $[\dot{\forall} \phi_{\mu \to \sigma}, \dot{\forall} X_{\mu}, \phi_{\mu \to \sigma} X \ni (g_{\mu})$ $[\dot{\forall} X_{\mu}, \dot{\forall} Y_{\mu}, (g_{\mu \to \sigma} X \ni (g_{\mu}))$	prove ● ir ● fi	ed by LEO-II and S n logic: K rom assumptions: • T1, D1, A3 • A1, A2, D1, A3	atallax				
FG MT CO	$[s_{\sigma} \stackrel{()}{\rightarrow} s_{\sigma}]$ $[\dot{\mathbf{v}} \phi_{\mu \rightarrow \sigma}, \dot{\mathbf{v}} X_{\mu}, \dot{\mathbf{g}}_{\mu \rightarrow \sigma} X \stackrel{()}{\rightarrow} (g_{\mu}$ $[\dot{\mathbf{v}} X_{\mu}, \dot{\mathbf{v}} Y_{\mu}, (g_{\mu \rightarrow \sigma} X \stackrel{()}{\rightarrow} (g_{\mu}$ $\emptyset (\text{no goal, check for cons})$	prove ● ir ● fi	ed by LEO-II and S n logic: K rom assumptions: • T1, D1, A3	atallax				
FG MT CO D2'	$[s_{\sigma}  \dot{\Box} s_{\sigma}]$ $[\dot{\forall} \phi_{\mu \rightarrow \sigma}, \dot{\forall} X_{\mu}, \phi_{\mu \rightarrow \sigma}, X \supset ($ $[\dot{\forall} X_{\mu}, \dot{\forall} Y_{\mu}, (g_{\mu \rightarrow \sigma}, X \supset (g_{\mu}$ $\emptyset \text{ (no goal, check for consess}_{(\mu \rightarrow \sigma), \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma}, \lambda$	prove ● ir ● fi	ed by LEO-II and S n logic: K rom assumptions: • T1, D1, A3 • A1, A2, D1, A3	atallax : ons:				
FG MT CO	$[s_{\sigma} \stackrel{()}{\rightarrow} s_{\sigma}]$ $[\dot{\mathbf{v}} \phi_{\mu \rightarrow \sigma}, \dot{\mathbf{v}} X_{\mu}, \dot{\mathbf{g}}_{\mu \rightarrow \sigma} X \stackrel{()}{\rightarrow} (g_{\mu}$ $[\dot{\mathbf{v}} X_{\mu}, \dot{\mathbf{v}} Y_{\mu}, (g_{\mu \rightarrow \sigma} X \stackrel{()}{\rightarrow} (g_{\mu}$ $\emptyset (\text{no goal, check for cons})$	prove ● ir ● fi	ed by LEO-II and S n logic: K rom assumptions: • T1, D1, A3 • A1, A2, D1, A3 or domain conditio • constant domain	atallax : ons: 15		-1		
FG MT CO D2'	$[s_{\sigma}  \dot{\Box} s_{\sigma}]$ $[\dot{\forall} \phi_{\mu \rightarrow \sigma}, \dot{\forall} X_{\mu}, \phi_{\mu \rightarrow \sigma}, X \supset ($ $[\dot{\forall} X_{\mu}, \dot{\forall} Y_{\mu}, (g_{\mu \rightarrow \sigma}, X \supset (g_{\mu}$ $\emptyset \text{ (no goal, check for consess}_{(\mu \rightarrow \sigma), \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma}, \lambda$	prove ● ir ● fi	ed by LEO-II and S n logic: K rom assumptions: • T1, D1, A3 • A1, A2, D1, A3 or domain condition	atallax : ons: 15		5)		
FG MT CO D2'	$[s_{\sigma}  \dot{\Box} s_{\sigma}]$ $[\dot{\forall} \phi_{\mu \rightarrow \sigma}, \dot{\forall} X_{\mu}, \phi_{\mu \rightarrow \sigma}, X \supset ($ $[\dot{\forall} X_{\mu}, \dot{\forall} Y_{\mu}, (g_{\mu \rightarrow \sigma}, X \supset (g_{\mu}$ $\emptyset \text{ (no goal, check for consess}_{(\mu \rightarrow \sigma), \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma}, \lambda$	prove ● ir ● fi	ed by LEO-II and S n logic: K rom assumptions: • T1, D1, A3 • A1, A2, D1, A3 or domain conditio • constant domain	atallax : ons: 15		;)		

	HOL encoding		dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\forall \phi_{\mu \to \sigma} \ p_{(\mu \to \sigma) \to \sigma}(\lambda X_{\mu} \ \neg)$	$(\phi X)) \doteq \neg (p\phi)$	]					
A2	$[\dot{\forall}\phi_{\mu\to\sigma},\dot{\forall}\psi_{\mu\to\sigma},(p_{(\mu\to\sigma)\to\sigma}))$	,φ Å ĠΫX <sub>μ</sub> .(φ	$X \supset \psi X$ )) $\supset p\psi$ ]					
T1	$[\dot{\forall}\phi_{\mu\to\sigma^*}p_{(\mu\to\sigma)\to\sigma}\phi \dot{\ominus}\dot{\forall}X_{\mu^*}\phi X]$		A1(⊃), A2 A1, A2	K K	THM THM	0.1/0.1 0.1/0.1	0.0/0.0 0.0/5.2	_/ /
D1 A3	$g_{\mu\to\sigma} = \lambda X_{\mu} \cdot \dot{V} \phi_{\mu\to\sigma} \cdot p_{(\mu\to\mu)}$ $[p_{(\mu\to\sigma)\to\sigma} g_{\mu\to\sigma}]$	$\sigma \to \sigma \phi \supset \phi X$	,			,	,	,
С	$[\diamond \exists X_{\mu}, g_{\mu \to \sigma} X]$		T1, D1, A3 A1, A2, D1, A3	K K	THM THM	0.0/0.0 0.0/0.0	0.0/0.0 5.2/31.3	_/
A4	$[\dot{\forall}\phi_{\mu\to\sigma}, p_{(\mu\to\sigma)\to\sigma}\phi \stackrel{!}{\supset} \dot{\Box}p\phi]$	5] V 4V 1 1/4	(LV VV (4VLV))	n	11101	0.07 0.0	5.2, 51.5	7
T2	$[\dot{\mathbf{V}}X_{\mu}, g_{\mu\to\sigma}X \stackrel{\scriptstyle{\rightarrow}}{\supset} (\mathrm{ess}_{(\mu\to\sigma)\to}$	$\mu \to \sigma gX$	A1, D1, A4, D2 A1, A2, D1, A3, A4, D2	K K	THM THM	19.1/18.3 12.9/14.0	0.0/0.0 0.0/0.0	—/— —/—
D3 A5 T3	$\mathbf{NE}_{\mu \to \sigma} = \mathbf{A} \mathbf{X}_{\mu} \cdot \mathbf{V} \boldsymbol{\varphi}_{\mu \to \sigma^*} (\mathbf{e}$ $[\mathbf{p}_{(\mu \to \sigma) \to \sigma} \mathbf{NE}_{\mu \to \sigma}]$ $[\mathbf{\dot{\Box}} \mathbf{\dot{\Xi}} \mathbf{X}_{\mu^*} \mathbf{g}_{\mu \to \sigma} \mathbf{X}]$	Auto	mating Scott's pro	of scri	pt			
	$[\Box \neg A\mu \cdot B\mu \rightarrow \sigma A]$							
	[u⊐xµ*gµ→σx]		Being God-like is			ny Goo	l-like be	eing"
мс	[تعمين من	prove	ed by LEO-II and S			iny Goo	I-like be	eing"
MC FG		prove • i	-	atallax		iny Goo	l-like be	eing"
		prove • i	ed by LEO-II and S n logic: K	atallax :		ny Goo	I-like be	eing"
FG	[νόφ <sub>μ→σ</sub> , Υ <u>τη</u> (ε <sub>μ→σ</sub> Χ ⊃ (	prove ● i ● f	ed by LEO-II and S n logic: K rom assumptions • A1, D1, A4, D2	atallax : A4, D2		iny Goo	l-like be	eing"

HOL encoding $\dot{\nabla} \phi_{\mu \to \sigma} \cdot p_{(\mu \to \sigma) \to \sigma} (\lambda X_{\mu} \cdot \neg (\phi X)$ $\dot{\nabla} \phi_{\mu \to \sigma} \cdot \nabla \psi_{\mu \to \sigma} \cdot (p_{(\mu \to \sigma) \to \sigma} \phi)$ $\dot{\nabla} \phi_{\mu \to \sigma} \cdot p_{(\mu \to \sigma) \to \sigma} \phi \supset \dot{\Diamond} \exists X_{\mu} \cdot c$	dependencies )) $\doteq \dot{\neg}(p\phi)]$ $\dot{\Box}\dot{\Psi}X_{\mu^*}(\phi X \dot{\supset} \psi X)) \dot{\supset} p\psi]$	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/var
$ \dot{\forall} \phi_{\mu \to \sigma^*} p_{(\mu \to \sigma) \to \sigma} (\lambda X_{\mu^*} \dot{\neg} (\phi X)_{\mu \to \sigma^*} \dot{\forall} \psi_{\mu \to \sigma^*} (p_{(\mu \to \sigma) \to \sigma} \phi)_{\mu \to \sigma^*} \dot{\forall} \phi_{\mu \to \sigma^*} p_{(\mu \to \sigma) \to \sigma} \phi \dot{\neg} \dot{\Diamond} \dot{\exists} X_{\mu^*} $	)) ≐ ¬( <i>p</i> φ)] \ ἀ∀ <i>X</i> <sub>μ</sub> • ( <i>φX</i> ⊃ <i>ψX</i> )) ⊃ <i>pψ</i> ]				tonoy (my	eonoy .m
$ \begin{array}{l} (\dot{\psi}\phi_{\mu\to\sigma},\dot{\psi}\phi_{\mu\to\phi},\dot{\psi}\phi_{\mu\phi},\dot{\psi}\phi,\psi$	$(\phi X) \stackrel{-}{\rightarrow} (\phi X) \stackrel{-}{\rightarrow} \psi X)) \stackrel{-}{\rightarrow} p \psi]$					
$[\dot{\mathbf{v}}\phi_{\mu\to\sigma^*}\mathbf{p}_{(\mu\to\sigma)\to\sigma}\phi\dot{\mathbf{o}}\dot{\mathbf{o}}\dot{\mathbf{d}}X_{\mu^*}]$	$(\Box ( \Box \mu) ( \varphi \Box \Box \varphi \Box ) ) = P \varphi ]$					
$(\psi \mu \rightarrow \sigma \cdot P(\mu \rightarrow \sigma) \rightarrow \sigma \psi ) \rightarrow \sigma \psi$	$\delta \mathbf{X} = \Delta 1(\neg) \Delta 2$	к	THM	0.1/0.1	0.0/0.0	_/
	A1. A2	ĸ	THM	0.1/0.1	0.0/5.2	_/_ _/_
$g_{\mu\to\sigma} = \lambda X_{\mu} \cdot \dot{\forall} \phi_{\mu\to\sigma} \cdot p_{(\mu\to\sigma)\to}$		IX.	111111	0.1/0.1	0.0/5.2	_/
$[p_{(\mu \to \sigma) \to \sigma} g_{\mu \to \sigma}]$	$\sigma \psi \supset \psi X$					
$[\dot{\phi} \exists X_{\mu} \ g_{\mu \to \sigma} X]$	T1, D1, A3	к	THM	0.0/0.0	0.0/0.0	_/_
$[\bullet \exists m \mu \cdot g \mu \rightarrow \sigma m]$		ĸ				_/
<b>ダ</b> め n				010/010	012/0110	'
	$dX \downarrow \dot{V}_{H} = (\mu X \neg \dot{\Pi} \dot{V} = (dX \neg \mu V))$					
		к	THM	10 1/18 3	0.0/0.0	_/
$( I \mu B \mu \rightarrow \sigma T D ( C S (\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma ) $	A1 A2 D1 A3 A4 D2	ĸ	THM			
$NE_{u\to \sigma} = \lambda X_u, \dot{\nabla} \phi_{u\to \sigma} (e$	Automoting Coatt's pro	of oori	<b></b> +			
$[\mathbf{p}_{(m,m)}]$ $\mathbf{NE}_{m,m}$	Automating Scott's pro	or scri	ρι			
$[\dot{\Box} \exists X_{\mu}, g_{\mu \to \sigma} X]$						
, , , , , , , , , , , , , , , , , , ,	To: "Nessearchity Code					
	15: Necessarily, God e	exists				
	proved by LEO-II and S	atallax				
	• •					
$s_{\sigma} \supset \Box s_{\sigma}$	• In logic: KB					
	a from accumptions					
$\psi \varphi_{\mu \to \sigma} = (g_{\mu \to \sigma} A )$		•				
$\dot{\mathbf{W}} \mathbf{X} \dot{\mathbf{W}} \mathbf{V} (\mathbf{a} \mathbf{X} \dot{\neg} (\mathbf{a})$	D1, C, T2, D3, A	5				
$(\mathbf{Y}_{\mu}, \mathbf{Y}_{\mu}, (\mathbf{g}_{\mu \to \sigma}, \mathbf{X} \cup (\mathbf{g}_{\mu})))$	· · · · · · · · · · · · · · · · · · ·					
	for domain conditient	ons:				
0 (no goal, check for cons	a constant domai	20				
$\emptyset$ (no goal, check for cons	varying domains	s (Indivi	duals	5)		
	For logic K we got a co					
				have Mitt	niek	
	$ \begin{array}{l} \forall \dot{X}_{\mu} \cdot g_{\mu \rightarrow \sigma} X \supset (\mathrm{ess}_{(\mu \rightarrow \sigma) - \mu \rightarrow c} \\ \mathrm{ste}_{\mu \rightarrow \sigma} = \lambda X_{\mu} \cdot \dot{\forall} \phi_{\mu \rightarrow \sigma} \cdot (\mathbf{e}) \\ \mathrm{ste}_{\mu \rightarrow \sigma} \cdots \mathrm{NE}_{\mu \rightarrow \sigma} \mathbf{X} \\ \mathrm{ste}_{\mu \rightarrow \sigma} X \\ \mathrm{ste}_{\sigma} \supset \mathrm{ste}_{\sigma} \end{bmatrix} \\ \mathbf{v}_{\phi \mu \rightarrow \sigma} \mathbf{v}_{\sigma} (g_{\mu \rightarrow \sigma} X \supset (\mathbf{e})) \\ \mathrm{v}_{X\mu} \cdot \dot{\forall} Y_{\mu} \cdot (g_{\mu \rightarrow \sigma} X \supset (\mathbf{e})) \\ \mathrm{v}_{X\mu} \cdot \dot{\forall} Y_{\mu} \cdot (g_{\mu \rightarrow \sigma} X \supset (\mathbf{e})) \\ \mathrm{ste}_{\mu \rightarrow \sigma} \mathbf{x} \rightarrow (g_{\mu \rightarrow \sigma} X \supset (\mathbf{e})) \\ \mathrm{ste}_{\mu \rightarrow \sigma} \mathbf{x} \rightarrow (g_{\mu \rightarrow \sigma} X \rightarrow (\mathbf{e})) \\ \mathrm{ste}_{\mu \rightarrow \sigma} \mathbf{x} \rightarrow (g_{\mu \rightarrow \sigma} X \rightarrow (\mathbf{e})) \\ \mathrm{ste}_{\mu \rightarrow \sigma} \mathbf{x} \rightarrow (g_{\mu \rightarrow \sigma} X \rightarrow (\mathbf{e})) \\ \mathrm{ste}_{\mu \rightarrow \sigma} \mathbf{x} \rightarrow (g_{\mu \rightarrow \sigma} x \rightarrow ($	$s_{(\mu-\sigma)\rightarrow\mu-\sigma} = \lambda \phi_{\mu-\sigma} \lambda X_{\mu} \cdot \phi X \lambda \dot{\nu} \psi_{\mu-\sigma} (\psi X \exists \dot{\nu} \forall \gamma_{\mu}, (\phi Y \ni \psi Y))$ $\dot{\nu} X_{\mu} g_{\mu-\sigma} X \exists (ss_{(\mu-\sigma)\rightarrow\mu-\sigma} g X)] A1, D1, A4, D2$ $s_{\mu-\sigma} X_{\mu-\sigma} \dot{\nu} \phi_{\mu-\sigma-\sigma} g X) A1, D1, A4, D2$ $A1, D1, A4, D2$ $A1, D1, A4, D2$ $A1, D1, A4, D2$ $A1, D1, A4, D2$ $T3: "Necessarily, God e proved by LEO-II and S$ $\bullet in logic: KB$ $\bullet from assumptions$ $\bullet D1, C, T2, D3, A4$ $\bullet for domain conditi$ $\bullet constant domain$ $s_{(\mu-\sigma)} x_{-\mu-\sigma} = \lambda \phi_{\mu-\sigma} x$	$\begin{array}{c} \forall \phi_{\mu \rightarrow \sigma^*} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi \stackrel{1}{\Rightarrow} \dot{D}p \phi \\ ss_{(\mu \rightarrow \sigma) \rightarrow \mu \rightarrow \sigma} = \lambda \phi_{\mu \rightarrow \sigma^*} \lambda \chi_* \phi X \land \dot{\psi} \psi_{\mu \rightarrow \sigma^*} (\psi X \stackrel{1}{\Rightarrow} \dot{D}Y_{\mu^*} (\phi Y \stackrel{1}{\Rightarrow} \psi Y)) \\ \dot{\chi}_{\mu^*} g_{\mu \rightarrow \sigma^*} \chi \stackrel{1}{\Rightarrow} (e_{X,\mu^*} \dot{\psi}_{\mu \rightarrow \sigma^*} (e_{X,\mu^*} \dot{\psi}_{X,\mu^*} \dot{\psi}_{X,\mu^*} \dot{\psi}_{X,\mu^*} \dot{\psi}_{\mu \rightarrow \sigma^*} (e_{X,\mu^*} \dot{\psi}_{X,\mu^*} \dot{\psi}_{\mu \rightarrow \sigma^*} \dot{\chi}_{\mu^*} (g_{\mu \rightarrow \sigma^*} \chi \stackrel{1}{\Rightarrow} (e_{X,\mu^*} \dot{\psi}_{X,\mu^*} (g_{\mu \rightarrow \sigma^*} \chi \stackrel{1}{\Rightarrow} (e_{X,\mu^*} \dot{\psi}_{X,\mu^*} \dot{\psi}_{X,\mu^*} \dot{\psi}_{X,\mu^*} \dot{\psi}_{X,\mu^*} (g_{\mu \rightarrow \sigma^*} \chi \stackrel{1}{\Rightarrow} (e_{X,\mu^*} \dot{\psi}_{X,\mu^*} \psi$	$\begin{aligned} & \psi_{\phi_{\mu}\to\sigma^*} p_{(\mu=\sigma)\to\mu=\sigma} \pm \delta_{\phi_{\mu}\to\sigma^*} X_{x_{*}} \phi_{X} \hat{x} \hat{\psi}_{\psi_{\mu=\sigma}} (\phi_{X} \pm \hat{u}^{*} \hat{y}^{*}_{\mu}, (\phi_{Y} \pm \hat{u}^{*} \hat{y})) \\ & s_{\mu_{\mu}\to\sigma_{\mu=\sigma}} x_{2} (e_{s_{\mu}\to\sigma_{\mu}}, x_{\mu}, \phi_{X} \hat{x} \hat{\psi}_{\psi_{\mu=\sigma}} (\phi_{X} \pm \hat{u}^{*} \hat{y}^{*}_{\mu}, (\phi_{Y} \pm \hat{u}^{*} \hat{y})) \\ & A_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ & Alt a_{2} \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \pm 3 \text{ A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D2} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D3} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D3} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D3} & K & \text{THM} \\ \hline & a_{1} \pm 2 \text{ D1} \text{ A3 A4 D3}$	$\begin{aligned} & \psi_{\phi_{\mu}\to\sigma^{*}} p_{(\mu\to\sigma)\to\sigma}\phi \downarrow \dot{d}\rho\phi ] & In the point of the $	$\begin{aligned} & \begin{array}{lllllllllllllllllllllllllllllllllll$

	HOL encoding		dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1	$[\dot{\forall}\phi_{\mu\to\sigma},p_{(\mu\to\sigma)\to\sigma}(\lambda X_{\mu},\dot{\neg})$	$(\phi X)) \doteq \neg (p\phi)$						
A2	$[\dot{\forall}\phi_{\mu\to\sigma},\dot{\forall}\psi_{\mu\to\sigma},(p_{(\mu\to\sigma)\to\sigma})$		$(\neg \psi X)) \supset p\psi$					
T1	$[\dot{\forall}\phi_{\mu\to\sigma}, p_{(\mu\to\sigma)\to\sigma}\phi \supset \dot{\Diamond}\dot{\exists})$		A1(⊃), A2	K	THM	0.1/0.1	0.0/0.0	_/_
		<i>F</i> · · ·	A1, A2	K	THM	0.1/0.1	0.0/5.2	_/_
D1	$g_{\mu\to\sigma} = \lambda X_{\mu} \cdot \dot{\forall} \phi_{\mu\to\sigma} \cdot p_{(\mu\to\sigma)}$	$\sigma_{\sigma) \to \sigma} \phi \supset \phi X$						
A3	$[p_{(\mu \to \sigma) \to \sigma} g_{\mu \to \sigma}]$							
C	$[\diamond \exists X_{\mu} \ g_{\mu \to \sigma} X]$		T1, D1, A3	K	THM	0.0/0.0	0.0/0.0	—/— —/—
			A1, A2, D1, A3	K	THM	0.0/0.0	5.2/31.3	—/—
A4	$[\dot{\forall}\phi_{\mu ightarrow\sigma},p_{(\mu ightarrow\sigma) ightarrow\sigma}\phi \supset \dot{\Box}p\phi$							
D2			$\bullet(\psi X \mathrel{\dot{\supset}} \dot{\Box} \mathrel{\dot{\forall}} Y_{\mu} \bullet (\phi Y \mathrel{\dot{\supset}} \psi Y))$					
T2	$[\dot{\forall} X_{\mu}, g_{\mu \to \sigma} X \stackrel{.}{\supset} (\operatorname{ess}_{(\mu \to \sigma) \to \sigma})$	$(\mu \to \sigma gX)$ ]	A1, D1, A4, D2	K	THM	19.1/18.3	0.0/0.0	—/—
			A1, A2, D1, A3, A4, D2	K	THM	12.9/14.0	0.0/0.0	—/—
D3	$\mathbf{NE}_{\mu\to\sigma} = \lambda X_{\mu} \cdot \dot{\forall} \phi_{\mu\to\sigma} \cdot (\mathbf{es}$	ss $φX$ ⊃ $□∃Y_{\mu}.φ$	Y)					
A5	$[p_{(\mu \to \sigma) \to \sigma} NE_{\mu \to \sigma}]$				_			
T3	$[\dot{\Box}\dot{\exists}X_{\mu^*}g_{\mu ightarrow\sigma}X]$	Autom	nating Scott's proc	of scri	pt			
		Summ	ary					
MC	$[s_{\sigma} \stackrel{.}{\supset} \stackrel{.}{\Box} s_{\sigma}]$	e nr	oof verified and a	utoma	hated			
		• •						
FG	$[\forall \phi_{\mu \to \sigma}, \forall X_{\mu} (g_{\mu \to \sigma} X \supset ($	o K	B is sufficient (cri	tisized	h loa	ic S5 n	ot need	ed!)
								<b>.</b> .,
MT	$[\dot{\forall} X_{\mu^*} \dot{\forall} Y_{\mu^*} (g_{\mu \to \sigma} X \stackrel{.}{\supset} (g_{\mu}$	e nr	oof works for con	stant	and	varvino	ı domai	ns
		• p.				· ,	,	
co	Q (no cool shools for cone	0 e)	act dependencies	dete	rmin	ed expe	eriment	allv
D2'	$\emptyset$ (no goal, check for cons					ou onpo		<b>,</b>
CO'	$\operatorname{ess}_{(\mu \to \sigma) \to \mu \to \sigma} = \lambda \phi_{\mu \to \sigma} \lambda$ 0 (no goal, check for cons	ο Δ1	TPs have found al	ternat	ive n	roofs (	shorter	)
	v (no goal, check for cons	• 7						,

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		HOL encoding		dependencies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
	A1 A2 T1	$ [\dot{\forall} \phi_{\mu \to \sigma^*} p_{(\mu \to \sigma) \to \sigma} (\lambda X_{\mu^*} \dot{\neg} (\psi_{\mu \to \sigma^*} \psi_{\mu \to \sigma^*} (p_{(\mu \to \sigma) \to \sigma}))] $	,φ∧̀⊡҅ΫX <sub>μ</sub> .(φX	<i>⇒ψX</i> )) <i>⊃ pψ</i> ] A1(⊃), A2	V	TIN	0.1/0.1	0.0/0.0	,
	D1	$\begin{bmatrix} \dot{\forall} \phi_{\mu \to \sigma^*} p_{(\mu \to \sigma) \to \sigma} \phi \stackrel{!}{\Rightarrow} \dot{\Diamond} \stackrel{!}{\exists} \end{bmatrix}$ $g_{\mu \to \sigma} = \lambda X_{\mu^*} \dot{\forall} \phi_{\mu \to \sigma^*} p_{(\mu \to \sigma)}$		stency check: Göde	el vs			0.0/0.0	
	A3 C	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} g_{\mu \to \sigma} \end{bmatrix}$ $[\diamondsuit \exists X_{\mu}, g_{\mu \to \sigma} X]$	• Sc	cott's assumptions	are	cons	istent;		
	A4 D2	$\begin{bmatrix} \dot{\forall} \phi_{\mu \to \sigma} \cdot p_{(\mu \to \sigma) \to \sigma} \phi \supset \Box p \\ ess_{(\mu \to \sigma) \to \mu \to \sigma} = \phi_{\mu \to \sigma} \cdot \lambda \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots &$	sh	own by Nitpick					
	T2 D3	$[\dot{\nabla} X_{\mu} \cdot g_{\mu}]  X \supset (\operatorname{ess}_{(\mu \to \sigma)})$ $\operatorname{NE}_{\to \sigma} = \lambda X_{\mu} \cdot \dot{\nabla} \phi_{\mu \to \sigma^*} (e^{-\lambda})$		ödel's assumptions				· ·	
	A5 T3	$\begin{bmatrix} \mu_{\mu\to\sigma} & -\lambda \alpha_{\mu^*} & \psi_{\mu\to\sigma^*} \\ \mu_{\mu\to\sigma} & N E_{\mu\to\sigma} \end{bmatrix}$ $\begin{bmatrix} \dot{\mu} \exists X_{\mu^*} & g_{\mu\to\sigma} X \end{bmatrix}$	Sr	lown by LEO-II <mark>(ne</mark> v	w pn	lioso	pnical	result!)	
	/			D1, C, T2, D3, A5 A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	0.0/0.1 —/—	0.1/5.3 —/—	_/ _/
Ι	MC	$[s_{\sigma} \supset \Box s_{\sigma}]$		D2, T2, T3 A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	17.9/— —/—	3.3/3.2 —/—	_/ /
	FG	$[\dot{\forall}\phi_{\mu\to\sigma},\dot{\forall}X_{\mu},(g_{\mu\to\sigma}X\dot{\supset}($		A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	16.5/— 12.8/15.1	0.0/0.0 0.0/5.4	_/ _/
	MT	$[\dot{\forall} X_{\mu^*} \dot{\forall} Y_{\mu^*} (g_{\mu \to \sigma} X \supset (g_{\mu}))]$	$_{\to\sigma}Y\supset X\doteq Y))]$	D1, FG A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	_/_ _/_	0.0/3.3 —/—	_/ /
	CO D2'	$\emptyset$ (no goal, check for cons $ess_{(\mu \to \sigma) \to \mu \to \sigma} = \lambda \phi_{\mu \to \sigma} \cdot \lambda$		A1, A2, D1, A3, A4, D2, D3, A5 $\dot{\Box}\dot{\Psi}Y_{**}(\phi Y \dot{\supset} \psi Y))$	KB	SAT	—/—	_/_	7.3/7.4
	ĊŌ'	$\emptyset$ (no goal, check for cons		$A1(\supset), A2, D2', D3, A5$ A1, A2, D1, A3, A4, D2', D3, A5	KB KB	UNS UNS	7.5/7.8 —/—	_/_ _/_	_/ /

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	HOL encoding	depende	ncies	logic	status	LEO-II const/vary	Satallax const/vary	Nitpick const/vary
A1 A2 T1	$\begin{bmatrix} \dot{\mathbf{V}} \boldsymbol{\phi}_{\mu \to \sigma^*} \boldsymbol{p}_{(\mu \to \sigma) \to \sigma} (\lambda X_{\mu^*} \neg \mathbf{i} \\ \begin{bmatrix} \dot{\mathbf{V}} \boldsymbol{\phi}_{\mu \to \sigma^*} \dot{\mathbf{V}} \boldsymbol{\psi}_{\mu \to \sigma^*} (\boldsymbol{p}_{(\mu \to \sigma) \to \sigma} \\ \begin{bmatrix} \dot{\mathbf{V}} \boldsymbol{\phi}_{\mu \to \sigma^*} \boldsymbol{p}_{(\mu \to \sigma) \to \sigma} \boldsymbol{\phi} \supset \dot{\mathbf{Q}} \end{bmatrix}$	$\phi \land \Box \forall X_{\mu} (\phi X \supset \psi X)) \supset$		K K	THM THM	0.1/0.1 0.1/0.1	0.0/0.0 0.0/5.2	_/ _/
D1 A3 C	$g_{\mu \to \sigma} = \lambda X_{\mu^{\star}} \dot{\Psi} \phi_{\mu \to \sigma^{\star}} p_{(\mu \to \sigma) \to \sigma} g_{\mu \to \sigma} ]  [\dot{\Phi} \exists X_{\mu^{\star}} g_{\mu \to \sigma} X]$	Further Res	ults					
A4 D2 T2 D3	$\begin{bmatrix} \dot{\forall} \phi_{\mu \to \sigma} \cdot p_{(\mu \to \sigma) \to \sigma} \phi \ni \Box p_{\mu} \\ ess_{(\mu \to \sigma) \to \mu \to \sigma} & \forall \phi_{\mu \to \sigma} \cdot \lambda \\ \begin{bmatrix} \dot{\forall} X_{\mu} \cdot g_{\mu \to \sigma} \\ \sigma & X \end{bmatrix} (ess_{(\mu \to \sigma)} \cdot ds) \\ E_{\mu \to \sigma} = \lambda X_{\mu} \cdot \dot{\forall} \phi_{\mu \to \sigma} \cdot (ess_{\mu \to \sigma} \cdot ds) \\ \end{bmatrix}$	God is	eism holds flawless					
A5 T3	$\begin{bmatrix} p_{(\mu \to \sigma) \to \sigma} \mathbf{N} \mathbf{E}_{\mu \to \sigma} \end{bmatrix}$ $\begin{bmatrix} \dot{\mathbf{a}} \exists X_{\mu^*} g_{\mu \to \sigma} X \end{bmatrix}$	A1, A2, D1, C, T	2, D3, A5 D1, A3, A4, D2, D3, A5 2, D3, A5 D1, A3, A4, D2, D3, A5	K K KB KB	CSA CSA THM THM	—/— —/— 0.0/0.1 —/—	—/— —/— 0.1/5.3 —/—	3.8/6.2 8.2/7.5 —/— —/—
MC	$[s_{\sigma} \supset \Box s_{\sigma}]$	D2, T2, 7	13 D1 43 44 D2 D3 45	KB KB	THM	17.9/—	3.3/3.2	_/
FG MT	$\begin{bmatrix} \dot{\forall} \phi_{\mu \to \sigma^*} \dot{\forall} X_{\mu^*} (g_{\mu \to \sigma} X \supset (g_{\mu^*}) \\ \begin{bmatrix} \dot{\forall} X_{\mu^*} \dot{\forall} Y_{\mu^*} (g_{\mu \to \sigma} X \supset (g_{\mu^*}) \end{bmatrix}$	$\begin{array}{ll} A1, A2, \\ a_{\sigma} Y \supset X \doteq Y) \end{bmatrix}  D1, FG$	A1, D1 D1, A3, A4, D2, D3, A5 D1, A3, A4, D2, D3, A5	KB KB KB KB	THM THM THM THM	16.5/— 12.8/15.1 —/— —/—	0.0/0.0 0.0/5.4 0.0/3.3 /	_/ _/
CO D2' CO'	$\emptyset$ (no goal, check for cons $\mathbf{ess}_{(\mu\to\sigma)\to\mu\to\sigma} = \lambda \phi_{\mu\to\sigma} \cdot \lambda$ $\emptyset$ (no goal, check for cons	$\begin{array}{l} X_{\mu} \cdot \dot{\mathbf{V}} \psi_{\mu \to \sigma} \cdot (\psi X \stackrel{{}_{\circ}}{\rightarrow} \dot{\mathbf{U}} \overset{{}_{\circ}}{\mathbf{Y}}_{\mu \to \sigma}, \\ \text{istency} & \text{A1}(\supset), \end{array}$	D1, A3, A4, D2, D3, A5 $\phi Y \supset \psi Y$ )) $\lambda_2, D2', D3, A5$ D1, A3, A4, D2', D3, A5	KB KB KB	SAT UNS UNS	—/— 7.5/7.8 —/—	_/_ _/	7.3/7.4 —/—

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/	A1 A2 T1 D1 A3 C A4 D2 T2 D3 A5 T3	HOL encoding $\begin{bmatrix} \dot{\forall} \phi_{\mu \rightarrow \sigma}, p_{(\mu \rightarrow \sigma) \rightarrow \sigma}(\lambda X_{\mu}, \dot{\neg} \\ \dot{\forall} \phi_{\mu \rightarrow \sigma}, \forall \phi_{\mu \rightarrow \sigma}, \phi_{(\mu \rightarrow \sigma) \rightarrow \tau}(p_{(\mu \rightarrow \sigma) \rightarrow \tau}(p_{(\mu \rightarrow \sigma) \rightarrow \sigma})) \\ \dot{\forall} \phi_{\mu \rightarrow \sigma}, p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi_{\sigma} \diamond \dot{\Rightarrow} \\ \begin{bmatrix} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} g_{\mu \rightarrow \sigma} \\ x_{\mu}, y_{\mu \rightarrow \sigma}, x_{\mu} \\ \vdots \\ \dot{\forall} x_{\mu}, g_{\mu \rightarrow \sigma}, x_{\mu} \\ \vdots \\ \dot{\forall} x_{\mu}, g_{\mu \rightarrow \sigma}, x_{\mu} \\ \vdots \\ \dot{\forall} x_{\mu}, g_{\mu \rightarrow \sigma}, x_{\mu} \\ \vdots \\ \begin{bmatrix} p_{(\mu \rightarrow \sigma) \rightarrow \sigma} \phi_{\mu \rightarrow \sigma} \\ \phi_{\mu \rightarrow \sigma} \\ \vdots \\ $	● pr ● fo Main c ● th ● ev	Collapse $\forall \varphi(\varphi)$ oved by LEO-II and r constant and vary ritique on Gödel's ere are no continger verything is determination by using modal log	Sata ving onto ent tr ined	allax doma logic ruths / no	al proc free wi	11	
Y	MC	[\$ <sub>σ</sub> ċ ċ <sub>σ</sub> ]		D2, T2, T3 A1, A2, D1, A3, A4, D2, D3, A5	KB KB	THM THM	17.9/— —/—	3.3/3.2 —/—	—/— —/—
-	РG MT	$\begin{bmatrix} \forall \varphi_{\mu \to \sigma^*} \forall X_{\mu^*} (g_{\mu \to \sigma} X \supset (\neg f) \\ [\dot{\forall} X_{\mu^*} \dot{\forall} Y_{\mu^*} (g_{\mu \to \sigma} X \supset (g_{\mu \to \sigma}) ] \end{bmatrix}$	•	A1, A2, D1, A3, A4, D2, D3, A5 D1, FG A1, A2, D1, A3, A4, D2, D3, A5	KB KB KB	THM THM THM THM	16.5/— 12.8/15.1 —/— —/—	0.0/5.4 0.0/3.3 —/—	_/_ _/_ _/_
	CO D2' CO'	$\emptyset$ (no goal, check for consistence $ess_{(\mu \to \sigma) \to \mu \to \sigma} = \lambda \phi_{\mu \to \sigma} \cdot \lambda X_{\mu} \psi$ $\emptyset$ (no goal, check for consistence for consistence $\psi$ )	$\dot{\forall}\psi_{\mu\to\sigma}$ . ( $\psi X$	A1, A2, D1, A3, A4, D2, D3, A5 $\dot{\phi}\dot{\Psi}_{\mu} \cdot (\phi Y \div \psi Y))$ A1( $\supset$ ), A2, D2', D3, A5 A1, A2, D1, A3, A4, D2', D3, A5	KB KB KB	SAT UNS UNS	—/— 7.5/7.8 —/—	_/_ _/	7.3/7.4 / /

 Avoiding the Modal Collapse: Very recent work (not yet published)

Variants of Gödel's proof that avoid the modal collapse

- [Frode Bjørdal, Understanding Gödel's Ontological Argument, 1998] (verified and automated)
- [Anthony Anderson, Some emendations of Gödel's ontological proof, 1990] (verified and automated)
- [Melvin Fitting, Types, Tableaux and Gödel's God, 2002] (ongoing)

Future work

- [André Fuhrmann, 2005]
- [Petr Hajek, 1996, 2001, 2002, 2008, 2011]
- [Szatkowski, 2011]

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Achievements

- significant contribution towards a Computational Metaphysics
- HOL very fruitfully exploited as a universal metalogic
- systematic study of a prominent philosophical argument
- even some novel results were found by HOL-ATPs
- infrastructure can be adapted for other logics and logic combinations

Relevance (wrt foundations and applications)

• Theoretical Philosophy, Artificial Intelligence, Computer Science, Maths

Little related work: only for Anselm's simpler argument

- first-order ATP PROVER9 [OppenheimerZalta, 2011]
- interactive proof assistant PVS

Future work

- continuation of systematic study of the ontological argument
- further studies in Computational Metaphysics

[Rushby, 2013]

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#### Germany

- Telepolis & Heise
- Spiegel Online
- FAZ
- Die Welt
- Berliner Morgenpost
- Hamburger Abendpost
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- Austria
- Die Presse
- Wiener Zeitung
- ORF

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#### Italy

- Repubblica
- Ilsussidario

- . . .

#### India

- DNA India
- Delhi Daily News
- India Today

- . . .

US - ABC News

- . . .

#### International

- Spiegel International
- Yahoo Finance
- United Press Intl.

- . . .

#### See links at https://github.com/FormalTheology/GoedelGod/tree/master/Press