Progress in Higher-Order Automated Ontology Reasoning

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Ontology Reasoning
— SUMO and Sigma —

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- SUMO — Suggested Upper Merged Ontology (NilesPease, FOIS, 2001)
  - open source, formal ontology: `www.ontologyportal.org`
  - has been extended for a number of domain specific ontologies
  - altogether approx. 20,000 terms and 70,000 axioms
  - employs the SUO-KIF representation language, a simplification of Genesereth's original Knowledge Interchange Format (KIF)
- Sigma (Pease, CEUR-71, 2003)
  - browsing and inference system for ontology development
  - integrates KIF-Vampire and SystemOnTPTP

SUMO (and similarly Cyc) contains higher-order representations, but there is only very limited automation support so far

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Higher-Order Aspects in SUO-KIF and SUMO: Examples

- Embedded formulas

  $$\textit{term} ::= \textit{variable} | \textit{word} | \textit{string} | \textit{funterm} | \textit{number} | \textit{sentence}$$

  `(holdsDuring (YearFn 2009) (likes Mary Bill))`

- ...often in combination with modal operators such as `holdsDuring`, `knows`, `believes`, etc.
- Predicate variables, function variables, propositional variables
- Lambda-Abstraction with KappaFN

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funterm ::= (funword arg+)   relsent ::= (relword arg+)

funword, relword ::= initialchar wordchar* | variable

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Current FO translation 'tricks'

First-order reasoning on a large ontology

(PeaseSutcliffe, CEUR 257, 2007)

- Quoting of embedded formulas

  A: `(holdsDuring (YearFn 2009) (likes Mary Bill))`

  Q: `(holdsDuring (YearFn ?Y) (likes ?X Bill))`

Current project focus:

embedded formulas and modal operators

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A: `(holdsDuring (YearFn 2009) '(likes Mary Bill))`
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Answer with FO-ATPs (?Y $\longleftarrow$ 2009, ?X $\longleftarrow$ Mary)

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- Quoting of embedded formulas

A: `(holdsDuring (YearFn 2009)`
`'(and (likes Mary Bill) (likes Sue Bill)))`
Q: `(holdsDuring (YearFn ?Y) '(likes ?X Bill))`

Failure with FO-ATP

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embedded formulas and modal operators

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Why not trying higher-order automated theorem proving directly?

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- THF0: new TPTP format for simple type theory
  (SutcliffeBenzmüller, J.Formalized Reasoning, 2010)
- THF0 ATPs: LEO-II, TPS, IsabelleP, Satallax
  THF0 (counter-)model finders: IsabelleM, IsabelleN, Satallax
- achieved:

  SUO-KIF $\longrightarrow$ TPTP THF0

  translation mechanism for SUMO as part of Sigma
- so far only exploits base type ι and o in THF0 ($\rightarrow$ improvable)
- generally applicable to SUO-KIF representations
- translation example (for SUMO) available at:

  `http://www.ags.uni-sb.de/~chris/papers/SUMO.thf`

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The SUO-KIF to TPTP THF Translation

Main challenge: find consistent typing for untyped SUO-KIF

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(instance instance BinaryPredicate)
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⟶ . . . and so on . . .

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Higher-Order Automated Theorem Proving in Ontology Reasoning

Embedded Formulas — An Easy Task for HO-ATP

Example (1: Embedded Formulas)

During 2009 Mary liked Bill and Sue liked Bill. Who liked Bill in 2009?

A: `(holdsDuring (YearFn 2009)`
`    (and (likes Mary Bill) (likes Sue Bill)))`

Q: `(holdsDuring (YearFn 2009) (likes ?X Bill))`

Proof by LEO-II(+E) in 0.19s

Embedded Formulas — An Easy Task for HO-ATP

Example (2: Embedded Formulas (1 modified))

During 2009 Mary liked Bill and Sue liked Bill. Who liked Bill in 2009?

A:
```
(holdsDuring (YearFn 2009)
   (not (or (not (likes Mary Bill))
            (not (likes Sue Bill)))))
```
Q:
```
(holdsDuring (YearFn 2009) (likes ?X Bill))
```

Proof by LEO-II(+E) in 0.19s

Embedded Formulas — An Easy Task for HO-ATP

Example (3: Embedded Formulas)

At all times Mary likes Bill. During 2009 Sue liked whomever Mary liked. Is there a year in which Sue has liked somebody?

A: `(holdsDuring ?Y (likes Mary Bill))`

B: `(holdsDuring (YearFn 2009)`
`    (forall (?X) (=> (likes Mary ?X) (likes Sue ?X))))`

Q: `(holdsDuring (YearFn ?Y) (likes Sue ?X))`

Proof by LEO-II(+E) in 0.13s

Example (4/5: Embedded Formulas (3 modified))

What holds that holds at all times. Mary likes Bill. During 2009 Sue liked whoever Mary liked. Is there a year in which Sue has liked somebody?

A: `(=> ?P (holdsDuring ?Y ?P))`

B: `(likes Mary Bill)`

C:
```
(holdsDuring (YearFn 2009)
    (forall (?X) (=> (likes Mary ?X) (likes Sue ?X))))
```

Q: `(holdsDuring (YearFn ?Y) (likes Sue ?X))`

Proof by LEO-II(+E) in 0.16s

Embedded Formulas — An Easy Task for HO-ATP

Example (4/5: Embedded Formulas (3 modified))

What holds that holds at all times. Mary likes Bill. During 2009 Sue liked whoever Mary liked. Is there a year in which Sue has liked somebody?

A': `(holdsDuring ?Y True)`

B: `(likes Mary Bill)`

C:
```
(holdsDuring (YearFn 2009)
    (forall (?X) (=> (likes Mary ?X) (likes Sue ?X))))
```

Q: `(holdsDuring (YearFn ?Y) (likes Sue ?X))`

Example (4/5: Embedded Formulas (3 modified))

What holds that holds at all times. Mary likes Bill. During 2009 Sue liked whoever Mary liked. Is there a year in which Sue has liked somebody?

A': (holdsDuring ?Y $(1+1=2)$)

B: (likes Mary Bill)

C: (holdsDuring (YearFn 2009)
      (forall (?X) (=> (likes Mary ?X) (likes Sue ?X))))

Q: (holdsDuring (YearFn ?Y) (likes Sue ?X))

Embedded Formulas — An Easy Task for HO-ATP

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What holds that holds at all times. Mary likes Bill. During 2009 Sue liked whoever Mary liked. Is there a year in which Sue has liked somebody?

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B: `(likes Mary Bill)`

C:
```
(holdsDuring (YearFn 2009)
    (forall (?X) (=> (likes Mary ?X) (likes Sue ?X))))
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C: `(holdsDuring (YearFn 2009)`
`    (forall (?X) (=> (likes Mary ?X) (likes Sue ?X))))`

Q: `(holdsDuring (YearFn ?Y) (likes Sue ?X))`

Boolean extensionality: $(P \Leftrightarrow Q) \Leftrightarrow (P = Q)$

Embedded Formulas — An Easy Task for HO-ATP

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A': `(holdsDuring ?Y True)`

B: `(likes Mary Bill)`

C: `(holdsDuring (YearFn 2009)`
`    (forall (?X) (=> (likes Mary ?X) (likes Sue ?X))))`

Q: `(holdsDuring (YearFn ?Y) (likes Sue ?X))`

Proof by LEO-II(+E) in 0.08s

Embedded Formulas — An Easy Task for HO-ATP

Example (6: Embedded Formulas and KappaFn)

The number of people John is grandparent of is less than or equal to three. How many grandchildren does John at most have?

A:
```
(<=> (grandchild ?X ?Y)
  (exists (?Z) (and (parent ?Z ?X) (parent ?Y ?Z))))
```
B:
```
(<=> (grandparent ?X ?Y)
  (exists (?Z) (and (parent ?X ?Z) (parent ?Z ?Y))))
```
C:
```
(lessThanOrEqualTo
  (CardinalityFn (KappaFn ?X (grandparent John ?X)))
  3)
```
Q:
```
(lessThanOrEqualTo
  (CardinalityFn (KappaFn ?X (grandchild ?X John)))
  ?Y)
```

Proof by LEO-II(+E) in 0.34s

Significant Improvements since Paper Submission

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LEO-II(+E) version v1.1

Ex.	1	2	3	4	5	6	7	8	9
local version	.19	.19	.13	.16	.08	.34	.18	.04	2642.55
SInE version	–	–	–	–	–	–	–	–	–
global version	–	–	–	–	–	–	–	–	–

One reviewer: ...only local versions ...this is not impressive ...

LEO-II(+E) version v1.2.1 (with relevance filtering)

Ex.	1	2	3	4	5	6	7	8	9
local version	.19	.18	.11	.08	.10	.38	.32	.14	.18
SInE version	.43	.40	.21	.54	.37	.12	.70	.06	.26
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Problem for SUO-KIF Semantics:
Boolean Extensionality versus Modal Operators

Problem: Boolean Extensionality versus Modal Operators

Example (5: Embedded Formulas – Temporal Contexts)

A': `(holdsDuring ?Y True)`

B: `(likes Mary Bill)`

C: `(holdsDuring (YearFn 2009)`
`    (forall (?X) (=> (likes Mary ?X) (likes Sue ?X))))`

Q: `(holdsDuring (YearFn 2009) (likes Sue Bill))`

Proof by LEO-II(+E) in < 0.16s

Boolean extensionality is in conflict with (epistemic) modalities!
(Has Boolean extensionality ever been questioned for KIF?)

Problem relevant not only for HO-ATPs!

Problem: Boolean Extensionality versus Modal Operators

Example (8: Embedded Formulas – Epistemic Contexts)

A'': `(knows ?Y True)`

B: `(likes Mary Bill)`

C': `(knows Chris`
`      (forall (?X) (=> (likes Mary ?X) (likes Sue ?X)))`

Q': `(knows Chris (likes Sue Bill))`

Proof by LEO-II(+E) in 0.04s

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Problem: Boolean Extensionality versus Modal Operators

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A'': `(knows ?Y (forall (?P) (=> ?P ?P)))`

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Proposed Solution: Possible World Semantics for SUMO

SUMO $\longrightarrow$ Quantified Multimodal Logic (QML) $\longrightarrow$ TPTP THF
(QML is fragment of HOL (BenzmüllerPaulson, SR-2009-02, 2009))

- T-Box like information in SUMO:

  `(instance holdsDuring AsymmetricRelation)` $\longrightarrow$
  $\forall W_\iota.(\text{instance holdsDuring AsymmetricRelation})_{\iota\to o}\ W$

- A-Box like information as in query problem: current world cw_ι

  (likes Mary Bill) $\longrightarrow$ $(\text{likes Mary Bill})_{\iota\to o}\ cw$

  (knows Chris (likes Sue Bill)) $\longrightarrow$
  $(\Box_{\text{Chris}}\,(\text{likes Sue Bill}))_{\iota\to o}\ cw$

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Example (8: Embedded Formulas – Epistemic Contexts)

A”: $\forall Y_{\iota\rightarrow\iota\rightarrow o}\blacksquare(\Box_{Y}\top)\ cw$
B: $(likes\ Mary\ Bill)\ cw$
C’: $(\Box_{Chris}(\forall^{i}X_{\mu}\blacksquare((likes\ Mary\ X)\supset(likes\ Sue\ X))))\ cw$
Q’: $(\Box_{Chris}(likes\ Sue\ Bill))\ cw$

Axioms for $\Box_{Chris}$ can be added:

M: $\forall W_{\iota}\blacksquare(\forall^{p}\phi_{\iota\rightarrow o}\blacksquare\Box_{Chris}\phi\supset\phi)\ W$
4: $\forall W_{\iota}\blacksquare(\forall^{p}\phi_{\iota\rightarrow o}\blacksquare\Box_{Chris}\phi\supset\Box_{Chris}\Box_{Chris}\phi)\ W$
5: $\forall W_{\iota}\blacksquare(\forall^{p}\phi_{\iota\rightarrow o}\blacksquare\Box_{Chris}\neg\phi\supset\Box_{Chris}\neg\Box_{Chris}\phi)\ W$

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5: $\forall W_{\iota}.(\forall^p \phi_{\iota\to o}.\Box_{Chris}\neg\phi \supset \Box_{Chris}\neg\Box_{Chris}\phi)\ W$

Challenge: Embedded Formulas — Epistemic Context

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LEO-II(+E) cannot solve this problem anymore!

Example (8: Embedded Formulas – Epistemic Contexts)

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But LEO-II(+E) can solve this problem in 0.15s!

Example (8: Embedded Formulas – Epistemic Contexts)

A”: $\forall Y_{\iota\to\iota\to o}\centerdot(\Box_Y \top)\ cw$

B: $(\Box_{fool}\ (likes\ Mary\ Bill))\ cw$

C’: $(\Box_{Chris}\ (\forall^i X_{\mu}\centerdot((likes\ Mary\ X) \supset (likes\ Sue\ X))))\ cw$

Q’: $(\Box_{Chris}\ (likes\ Sue\ Bill))\ cw$

Axioms for $\Box_{Chris}$ can be added:

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5: $\forall W_{\iota}\centerdot(\forall^p \phi_{\iota\to o}\centerdot\Box_{Chris}\ \neg\phi \supset \Box_{Chris}\ \neg\Box_{Chris}\ \phi)\ W$

Axioms for $\Box_{fool}$ can be added ...

$\forall W_{\iota}\centerdot(\forall^p \phi_{\iota\to o}\centerdot\Box_{fool}\ \phi \supset \Box_{Chris}\ \phi)\ W$

...

Example (8: Embedded Formulas – Epistemic Contexts)

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More information: (BenzmüllerPease,ECAI-ARCOE-10,2010)

Conclusion

- SUMO (similarly Cyc) employs higher-order representations
- support with first-order ATPs good but not perfect
- additional support with higher-order ATPs seems feasible
  - translation SUO-KIF $\longrightarrow$ THF0
  - example problems solved effectively (in large theory context!) by LEO-II(+E)
  - simple relevance filtering mechanism implemented for LEO-II(+E)
- various problems in SUMO detected, including:
  Boolean extensionality versus modal operators
- solution (BenzmüllerPease,ECAI-ARCOE-10,2010)
  - possible world semantics for SUO-KIF resp. SUMO
  - exploitation of embedding of quantified multimodal logic in THF for automation with higher-order ATPs
  - supports combinations with further logic embeddings in THF0