Progress in Higher-Order
Automated Ontology Reasoning

Christoph Benzmüller and Adam Pease

Articulate Software, Angwin, CA, USA

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Ontology Reasoning
— SUMO and Sigma —
SUMO and Sigma

- **SUMO — Suggested Upper Merged Ontology**  
  (NilesPease, FOIS, 2001)
  - open source, formal ontology: www.ontologyportal.org
  - has been extended for a number of domain specific ontologies
  - altogether approx. 20,000 terms and 70,000 axioms
  - employs the SUO-KIF representation language, a simplification of Genesereth’s original Knowledge Interchange Format (KIF)

- **Sigma**  
  (Pease, CEUR-71, 2003)
  - browsing and inference system for ontology development
  - integrates KIF-Vampire and SystemOnTPTP

SUMO (and similarly Cyc) contains higher-order representations, but there is only very limited automation support so far

⇒ better automation support is goal of DFG project
SUMO and Sigma

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Higher-Order Aspects in SUO-KIF and SUMO: Examples

- Embedded formulas

\[
\text{term ::= variable|word|string|funterm|number|sentence}
\]

(holdsDuring (YearFn 2009) (likes Mary Bill))

- ...often in combination with modal operators such as holdsDuring, knows, believes, etc.
- Predicate variables, function variables, propositional variables
- Lambda-Abstraction with KappaFN
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\[
\begin{align*}
\text{funterm} & ::= (\text{funword} \ \text{arg}+) \quad \text{relsent} ::= (\text{relword} \ \text{arg}+)
\end{align*}
\]

\[
\begin{align*}
\text{funword}, \text{relword} & ::= \text{initialchar wordchar}^* \mid \text{variable}
\end{align*}
\]

- Lambda-Abstraction with KappaFN
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Current FO translation 'tricks'

First-order reasoning on a large ontology
(PeaseSutcliffe, CEUR 257, 2007)

- Quoting of embedded formulas

\[ A: (\text{holdsDuring} (\text{YearFn} 2009) (\text{likes Mary Bill})) \]
\[ Q: (\text{holdsDuring} (\text{YearFn} \ ?Y) (\text{likes} \ ?X \ Bill)) \]

Current project focus:
embedded formulas and modal operators
Current FO translation 'tricks'

First-order reasoning on a large ontology

(Pease Sutcliffe, CEUR 257, 2007)

- Quoting of embedded formulas

\( A: \) (holdsDuring (YearFn 2009) ' (likes Mary Bill) )
\( Q: \) (holdsDuring (YearFn ?Y) ' (likes ?X Bill) )

Answer with FO-ATPs (?Y ← 2009, ?X ← Mary)

Current project focus:

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First-order reasoning on a large ontology
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- Quoting of embedded formulas

A: (holdsDuring (YearFn 2009) ’(and (likes Mary Bill) (likes Sue Bill)))
Q: (holdsDuring (YearFn ?Y) ’(likes ?X Bill))

Failure with FO-ATP

Current project focus:
embedded formulas and modal operators
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Why not trying higher-order automated theorem proving directly?

Current project focus:
embedded formulas and modal operators
The SUO-KIF to TPTP THF0 Translation
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- **THF0**: new TPTP format for simple type theory  
  *(SutcliffeBenzmüller, J. Formalized Reasoning, 2010)*

- **THF0 ATPs**: LEO-II, TPS, IsabelleP, Satallax
- **THF0 (counter-)model finders**: IsabelleM, IsabelleN, Satallax

- achieved:

  \[ \text{SUO-KIF} \rightarrow \text{TPTP THF0} \]

  translation mechanism for SUMO as part of Sigma

- so far only exploits base type $\iota$ and $\sigma$ in THF0 ($\rightarrow$ improvable)

- generally applicable to SUO-KIF representations

- translation example (for SUMO) available at:

  http://www.ags.uni-sb.de/~chris/papers/SUMO.thf
The SUO-KIF to TPTP THF0 Translation

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Main challenge: find consistent typing for untyped SUO-KIF

(instance instance BinaryPredicate)

→ ...and so on ...
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→ ...and so on ...
Higher-Order Automated Theorem Proving in Ontology Reasoning
Example (1: Embedded Formulas)

During 2009 Mary liked Bill and Sue liked Bill. Who liked Bill in 2009?

A: \((\text{holdsDuring } (\text{YearFn } 2009))\)
   \(\quad\text{and } (\text{likes } \text{Mary Bill}) \text{ and } (\text{likes } \text{Sue Bill}))\)

Q: \((\text{holdsDuring } (\text{YearFn } 2009) \text{ and } (\text{likes } ?X \text{ Bill}))\)

Proof by LEO-II(\(+E\)) in 0.19s
Example (2: Embedded Formulas (1 modified))
During 2009 Mary liked Bill and Sue liked Bill. Who liked Bill in 2009?

A: (holdsDuring (YearFn 2009)
   (not (or (not (likes Mary Bill))
     (not (likes Sue Bill)))))
Q: (holdsDuring (YearFn 2009) (likes ?X Bill))

Proof by LEO-II(+E) in 0.19s
Example (3: Embedded Formulas)

At all times Mary likes Bill. During 2009 Sue liked whomever Mary liked. Is there a year in which Sue has liked somebody?

\[ A: (\text{holdsDuring } ?Y (\text{likes Mary Bill})) \]
\[ B: (\text{holdsDuring (YearFn 2009)} \]
\[ \quad (\forall ?X (\rightarrow (\text{likes Mary } ?X) (\text{likes Sue } ?X))) \]
\[ Q: (\text{holdsDuring (YearFn } ?Y) (\text{likes Sue } ?X)) \]

Proof by LEO-II(+E) in 0.13s
Example (4/5: Embedded Formulas (3 modified))

What holds that holds at all times. Mary likes Bill. During 2009 Sue liked whoever Mary liked. Is there a year in which Sue has liked somebody?

A: (=> ?P (holdsDuring ?Y ?P))
B: (likes Mary Bill)
C: (holdsDuring (YearFn 2009)
   (forall (?X) (=> (likes Mary ?X) (likes Sue ?X))))
Q: (holdsDuring (YearFn ?Y) (likes Sue ?X))

Proof by LEO-II(+E) in 0.16s
Embedded Formulas — An Easy Task for HO-ATP

Example (4/5: Embedded Formulas (3 modified))
What holds that holds at all times. Mary likes Bill. During 2009 Sue liked whoever Mary liked. Is there a year in which Sue has liked somebody?

A': (holdsDuring ?Y True)
B: (likes Mary Bill)
C: (holdsDuring (YearFn 2009)
   (forall (?X) (=> (likes Mary ?X) (likes Sue ?X))))
Q: (holdsDuring (YearFn ?Y) (likes Sue ?X))
Example (4/5: Embedded Formulas (3 modified))

What holds that holds at all times. Mary likes Bill. During 2009 Sue liked whoever Mary liked. Is there a year in which Sue has liked somebody?

A': (holdsDuring ?Y (1 + 1 = 2))
B: (likes Mary Bill)
C: (holdsDuring (YearFn 2009)
    (forall (?X) (=> (likes Mary ?X) (likes Sue ?X))))
Q: (holdsDuring (YearFn ?Y) (likes Sue ?X))
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B: (likes Mary Bill)
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Boolean extensionality: $(P \Leftrightarrow Q) \Leftrightarrow (P = Q)$
Example (4/5: Embedded Formulas (3 modified))

What holds that holds at all times. Mary likes Bill. During 2009 Sue liked whoever Mary liked. Is there a year in which Sue has liked somebody?

A': (holdsDuring ?Y True)
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Q: (holdsDuring (YearFn ?Y) (likes Sue ?X))

Proof by LEO-II(+E) in 0.08s
Example (6: Embedded Formulas and KappaFn)

The number of people John is grandparent of is less than or equal to three. How many grandchildren does John at most have?

A: (\(\iff\) (grandchild ?X ?Y)
B: (\(\iff\) (grandparent ?X ?Y)
C: (lessThanOrEqualTo
            (CardinalityFn (KappaFn ?X (grandparent John ?X)))
             3)
Q: (lessThanOrEqualTo
            (CardinalityFn (KappaFn ?X (grandchild ?X John)))
             ?Y)

Proof by LEO-II(+E) in 0.34s
Significant Improvements since Paper Submission
**Significant Improvements since Paper Submission**

**LEO-II(+E) version v1.1**

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One reviewer: ...only local versions... this is not impressive...

**LEO-II(+E) version v1.2.1 (with relevance filtering)**

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Problem for SUO-KIF Semantics:
Boolean Extensionality versus Modal Operators
Problem: Boolean Extensionality versus Modal Operators

Example (5: Embedded Formulas – Temporal Contexts)

A': (holdsDuring ?Y True)
B: (likes Mary Bill)
C: (holdsDuring (YearFn 2009)
    (forall (?X) (=> (likes Mary ?X) (likes Sue ?X))))
Q: (holdsDuring (YearFn 2009) (likes Sue Bill))

Proof by LEO-II(+E) in < 0.16s

Boolean extensionality is in conflict with (epistemic) modalities!
(Has Boolean extensionality ever been questioned for KIF?)

Problem relevant not only for HO-ATPs!
Problem: Boolean Extensionality versus Modal Operators

Example (8: Embedded Formulas – Epistemic Contexts)

A': (knows ?Y True)
B: (likes Mary Bill)
C': (knows Chris
     (forall (?X) (=> (likes Mary ?X) (likes Sue ?X)))
Q': (knows Chris (likes Sue Bill))

Proof by LEO-II(+E) in 0.04s

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Problem: Boolean Extensionality versus Modal Operators

Example (8: Embedded Formulas – Epistemic Contexts)

A'': (\textit{knows} ?Y (forall (?P) (=> ?P ?P)))
B: (\textit{likes} Mary Bill)
C': (\textit{knows} Chris
    (forall (?X) (=> (\textit{likes} Mary ?X) (\textit{likes} Sue ?X)))
Q': (\textit{knows} Chris (\textit{likes} Sue Bill))

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Example (8: Embedded Formulas – Epistemic Contexts)

\[ A'': (\text{knows } ?Y \text{ True}) \]
\[ B: (\text{likes Mary Bill}) \]
\[ C': (\text{knows Chris} \]
\[ \quad (\text{forall } (?X) (\rightarrow (\text{likes Mary } ?X) (\text{likes Sue } ?X))) \]
\[ Q': (\text{knows Chris (likes Sue Bill)}) \]

Proof by LEO-II(+E) in 0.04s

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(Has Boolean extensionality ever been questioned for KIF?)

Problem relevant not only for HO-ATPs!
Example (8: Embedded Formulas – Epistemic Contexts)

\(A'': (\textsf{knows} \ ?Y \ True)\)

\(B: (\textsf{likes} \ Mary \ Bill)\)

\(C': (\textsf{knows} \ Chris\)
\quad (\text{forall} \ (?X) \ (\Rightarrow (\textsf{likes} \ Mary \ ?X) \ (\textsf{likes} \ Sue \ ?X)))\)

\(Q': (\textsf{knows} \ Chris \ (\textsf{likes} \ Sue \ Bill))\)

Proof by LEO-II(+E) in 0.04s

Boolean extensionality is in conflict with (epistemic) modalities!
(Has Boolean extensionality ever been questioned for KIF?)

Problem relevant not only for HO-ATPs!
SUMO $\rightarrow$ Quantified Multimodal Logic (QML) $\rightarrow$ TPTP THF

(QML is fragment of HOL (BenzmüllerPaulson, SR-2009-02, 2009))

- T-Box like information in SUMO:

\[
(\text{instance holdsDuring AsymmetricRelation}) \rightarrow \\
\forall W_i. (\text{instance holdsDuring AsymmetricRelation})_{i \rightarrow o} W
\]

- A-Box like information as in query problem: current world $cw_i$

\[(\text{likes Mary Bill}) \rightarrow (\text{likes Mary Bill})_{i \rightarrow o} cw\]

\[(\text{knows Chris (likes Sue Bill)}) \rightarrow (\Box_{\text{Chris (likes Sue Bill)})_{i \rightarrow o} cw\]

\[\Box_{\text{Chris (likes Sue Bill)}},cw\]
Proposed Solution: Possible World Semantics for SUMO

SUMO $\longrightarrow$ Quantified Multimodal Logic (QML) $\longrightarrow$ TPTP THF
(QML is fragment of HOL (BenzmüllerPaulson, SR-2009-02, 2009))

- T-Box like information in SUMO:

  \[(\text{instance holdsDuring AsymmetricRelation}) \rightarrow \forall W.(\text{instance holdsDuring AsymmetricRelation})_{i \rightarrow o} W\]

- A-Box like information as in query problem: current world $cw_i$

  \[(\text{likes Mary Bill}) \rightarrow (\text{likes Mary Bill})_{i \rightarrow o} cw\]

  \[(\text{knows Chris (likes Sue Bill)}) \rightarrow (\square_{\text{Chris (likes Sue Bill)})_{i \rightarrow o} cw\]
**Challenge: Embedded Formulas — Epistemic Context**

**Example (8: Embedded Formulas – Epistemic Contexts)**

A’’: \( \forall Y \rightarrow \rightarrow_{o} (\square Y \top) \) cw

B: \( (\text{likes Mary Bill}) \) cw

C’: \( (\square_{\text{Chris}} (\forall^{i} X_{\mu}.((\text{likes Mary X}) \supset (\text{likes Sue X})))) \) cw

Q’: \( (\square_{\text{Chris}} (\text{likes Sue Bill})) \) cw

Axioms for \( \square_{\text{Chris}} \) can be added:

M: \( \forall W_{l}. (\forall^{p} \phi_{l} \rightarrow_{o} \square_{\text{Chris}} \phi \supset \phi) \) \( W \)

4: \( \forall W_{l}. (\forall^{p} \phi_{l} \rightarrow_{o} \square_{\text{Chris}} \phi \supset \square_{\text{Chris}} \square_{\text{Chris}} \phi) \) \( W \)

5: \( \forall W_{l}. (\forall^{p} \phi_{l} \rightarrow_{o} \square_{\text{Chris}} \neg \phi \supset \square_{\text{Chris}} \neg \square_{\text{Chris}} \phi) \) \( W \)
Example (8: Embedded Formulas – Epistemic Contexts)

A": \( \forall Y_{l \to o} \cdot (\Box_Y \top) \) cw
B: (likes Mary Bill) cw
C': (\( \Box_{Chris} (\forall^i X_{l \cdot} \cdot ((likes Mary X) \supset (likes Sue X))) \)) cw
Q': (\( \Box_{Chris} (likes Sue Bill) \)) cw

Axioms for \( \Box_{Chris} \) can be added:

M: \( \forall W_{l \cdot} \cdot (\forall^p \phi_{l \to o} \cdot \Box_{Chris} \phi \supset \phi) \) W
4: \( \forall W_{l \cdot} \cdot (\forall^p \phi_{l \to o} \cdot \Box_{Chris} \phi \supset \Box_{Chris} \Box_{Chris} \phi) \) W
5: \( \forall W_{l \cdot} \cdot (\forall^p \phi_{l \to o} \cdot \Box_{Chris} \neg \phi \supset \Box_{Chris} \neg \Box_{Chris} \phi) \) W
Example (8: Embedded Formulas – Epistemic Contexts)

A'': \( \forall Y_{t \rightarrow t \rightarrow o} (\Box_Y \top) \) \(cw\)
B: \((\text{likes Mary Bill})\) \(cw\)
C': \((\Box_{\text{Chris}} (\forall i X_{i \rightarrow o} ((\text{likes Mary X}) \supset (\text{likes Sue X}))))\) \(cw\)
Q': \((\Box_{\text{Chris}} (\text{likes Sue Bill}))\) \(cw\)

Axioms for \(\Box_{\text{Chris}}\) can be added:

M: \(\forall W_{t} (\forall p \phi_{t \rightarrow o} \Box_{\text{Chris}} \phi \supset \phi) W\)
4: \(\forall W_{t} (\forall p \phi_{t \rightarrow o} \Box_{\text{Chris}} \phi \supset \Box_{\text{Chris}} \Box_{\text{Chris}} \phi) W\)
5: \(\forall W_{t} (\forall p \phi_{t \rightarrow o} \Box_{\text{Chris}} \neg \phi \supset \Box_{\text{Chris}} \neg \Box_{\text{Chris}} \phi) W\)

LEO-II(+E) cannot solve this problem anymore!
Example (8: Embedded Formulas – Epistemic Contexts)

A’: \( \forall Y_{l \rightarrow l \rightarrow o} (\Box_Y \top) \) cw

B: \( (\Box_{Chris} (likes \ Mary \ Bill)) \) cw

C’: \( (\Box_{Chris} (\forall^i X_{\mu} . ((likes \ Mary \ X) \supset (likes \ Sue \ X)))) \) cw

Q’: \( (\Box_{Chris} (likes \ Sue \ Bill)) \) cw

Axioms for \( \Box_{Chris} \) can be added:

M: \( \forall W_{l} . (\forall^p \phi_{l \rightarrow o} \Box_{Chris} \phi \supset \phi) \) W

4: \( \forall W_{l} . (\forall^p \phi_{l \rightarrow o} \Box_{Chris} \phi \supset \Box_{Chris} \Box_{Chris} \phi) \) W

5: \( \forall W_{l} . (\forall^p \phi_{l \rightarrow o} \Box_{Chris} \neg \phi \supset \Box_{Chris} \neg \Box_{Chris} \phi) \) W

But LEO-II(+E) can solve this problem in 0.15s!
Example (8: Embedded Formulas – Epistemic Contexts)

A”**: $\forall Y_{i\rightarrow o} \left( \Box_y T \right) \text{cw}$

B: $(\Box_{fool} \left( \text{likes Mary Bill} \right)) \text{cw}$

C’: $(\Box_{Chris} \left( \forall^i X_{\mu} \left( (\text{likes Mary X}) \supset (\text{likes Sue X}) \right) \right)) \text{cw}$

Q’: $(\Box_{Chris} \left( \text{likes Sue Bill} \right)) \text{cw}$

Axioms for $\Box_{Chris}$ can be added:

M: $\forall W_{i\cdot} \left( \forall p \phi_{i\rightarrow o} \Box_{Chris} \phi \supset \phi \right) W$

4: $\forall W_{i\cdot} \left( \forall p \phi_{i\rightarrow o} \Box_{Chris} \phi \supset \Box_{Chris} \Box_{Chris} \phi \right) W$

5: $\forall W_{i\cdot} \left( \forall p \phi_{i\rightarrow o} \Box_{Chris} \neg \phi \supset \Box_{Chris} \neg \Box_{Chris} \phi \right) W$

Axioms for $\Box_{fool}$ can be added ...

$\forall W_{i\cdot} \left( \forall p \phi_{i\rightarrow o} \Box_{fool} \phi \supset \Box_{Chris} \phi \right) W$

...
Example (8: Embedded Formulas – Epistemic Contexts)

A’’: \( \forall Y \rightarrow_l \rightarrow_\circ (\Box Y \top) \) \(cw\)

B: \( (\Box_{\text{fool}} (\text{likes Mary Bill})) \) \(cw\)

C’: \( (\Box_{\text{Chris}} (\forall X_\mu ((\text{likes Mary X}) \supset (\text{likes Sue X})))) \) \(cw\)

Q’: \( (\Box_{\text{Chris}} (\text{likes Sue Bill})) \) \(cw\)

Axioms for \( \Box_{\text{Chris}} \) can be added:

M: \( \forall W \bullet (\forall \phi \rightarrow_l \rightarrow_\circ \Box_{\text{Chris}} \phi \supset \phi) \) \(W\)

4: \( \forall W \bullet (\forall \phi \rightarrow_l \rightarrow_\circ \Box_{\text{Chris}} \phi \supset \Box_{\text{Chris}} \Box_{\text{Chris}} \phi) \) \(W\)

5: \( \forall W \bullet (\forall \phi \rightarrow_l \rightarrow_\circ \Box_{\text{Chris}} \neg \phi \supset \Box_{\text{Chris}} \neg \Box_{\text{Chris}} \phi) \) \(W\)

More information: (BenzmüllerPease, ECAI-ARCOE-10, 2010)
SUMO (similarly Cyc) employs higher-order representations

support with first-order ATPs good but not perfect

additional support with higher-order ATPs seems feasible

- translation SUO-KIF $\longrightarrow$ THF0
- example problems solved effectively (in large theory context!) by LEO-II(+E)
- simple relevance filtering mechanism implemented for LEO-II(+E)

various problems in SUMO detected, including:

- Boolean extensionality versus modal operators

solution (BenzmüllerPease, ECAI-ARCOE-10, 2010)

- possible world semantics for SUO-KIF resp. SUMO
- exploitation of embedding of quantified multimodal logic in THF for automation with higher-order ATPs
- supports combinations with further logic embeddings in THF0