Term Indexing for the LEO-II Prover

Frank Theiss and Christoph Benzmüller

FR Informatik
Universität des Saarlandes
Saarbrücken, Germany

Computer Laboratory
The University of Cambridge
Cambridge, UK

Overview

- Motivation

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- Some introductory conventions and remarks

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- Closer look at some key features of the approach

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- Conclusion

LEO

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$$\forall B_{\alpha\to o}, C_{\alpha\to o}, D_{\alpha\to o}\text{.} B \cup (C \cap D) = (B \cup C) \cap (B \cup D)$$

Negation and definition expansion with

$$\cup = \lambda A_{\alpha\to o}, B_{\alpha\to o}, X_{\alpha}\text{.}(A\ X) \lor (B\ X) \qquad \cap = \lambda A_{\alpha\to o}, B_{\alpha\to o}, X_{\alpha}\text{.}(A\ X) \land (B\ X)$$

leads to:

$$C_1 : [\lambda X_{\alpha}\text{.}(b\ X) \lor ((c\ X) \land (d\ X)) \neq^? \lambda X_{\alpha}\text{.}((b\ X) \lor (c\ X)) \land ((b\ X) \lor (d\ X))]$$

Goal directed functional and Boolean extensionality treatment:

$$C_2 : [(b\ x) \lor ((c\ x) \land (d\ x)) \Leftrightarrow ((b\ x) \lor (c\ x)) \land ((b\ x) \lor (d\ x))]^F$$

Clause normalization results then in a pure propositional, i.e. decidable, set of clauses. Only these clauses are still in the search space of $\mathcal{LEO}$(in total there are 33 clauses generated and $\mathcal{LEO}$ finds the proof on a 2,5GHz PC in 820ms).

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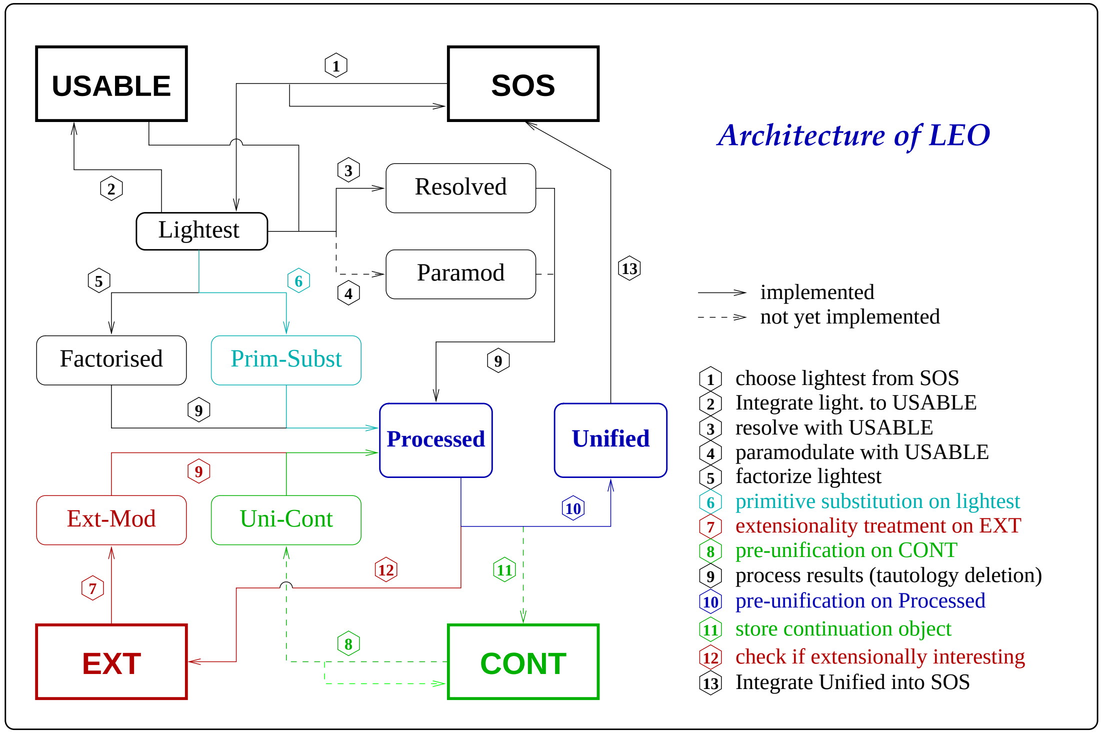

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- Often generates lots of first order or propositional clauses
- Has been successfully combined with first order ATPs

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- Shall cooperate with first order automated theorem provers
- Shall be way more efficient than LEO (which was developed rather as an academic demonstrator than a prototype)

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- One example: Brigitte Pientka's work [ICLP-03]
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- Our approach
  - based on Stickel's coordinate and path indexing [SRI-Report-89]
  - low level operations (e.g., operations on hashtables)

HOL-Syntax: Simple Types

Simple Types $\mathcal{T}$:

	o	(truth values)
	ι	(individuals)
	$(\alpha \rightarrow \beta)$	(functions from α to β)

HOL-Syntax: Simply Typed λ-Terms

Typed Terms:

X_α	Variables ($\mathcal{V}$)
c_α	Constants & Parameters (Σ)
$(\mathbf{F}_{\alpha\to\beta}\,\mathbf{B}_\alpha)_\beta$	Application
$(\lambda Y_\alpha\,\mathbf{A}_\beta)_{\alpha\to\beta}$	λ-abstraction

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Equality of Terms:

α-conversion	Changing bound variables	
β-reduction	$((\lambda Y_\beta\, \mathbf{A}_\alpha)\, \mathbf{B}_\beta) \xrightarrow{\beta} [\mathbf{B}/Y]\mathbf{A}$	
η-reduction	$(\lambda Y_\alpha\, (\mathbf{F}_{\alpha\rightarrow\beta}\, Y)) \xrightarrow{\eta} \mathbf{F}$	$(Y_\beta \notin \mathbf{Free}(\mathbf{F}))$

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Equality of Terms:

Every term has a unique $\beta\eta$-normal form (up to α-conversion).

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- Different occurrences of the same de Bruijn index may refer to different λ-binders:

  x_0 relates to bound variable b and c

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- Types
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  - no further impact on the indexing mechanism
- Due to Currying all our applications have just one argument

Key Features of Our Approach

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3. Indexing of subterm occurrences
4. Indexing of bound variable occurrences
   (support for explicit substitutions)

Shared Representation of Terms

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  - symbol $s \in \Sigma$
    $\longrightarrow$ a term node $n : \texttt{symbol}(s)$ is created
  - bound variable x_d (d is de Bruijn index)
    $\longrightarrow$ a term node $m : \texttt{bound}(type, d)$ is created
  - application $(s\ t)$ (s, t already represented by nodes i, j)
    $\longrightarrow$ a term node $l : \texttt{application}(i, j)$ is created
  - abstraction λt (t is already represented by i)
    $\longrightarrow$ a term node $k : \texttt{abstraction}(type, i)$ is created

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- Example:

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- Example: Original problem terms
  - def-emptyset: $\emptyset := \lambda x_\iota.\bot$
  - def-element: $\in := \lambda y_\iota.\lambda s_{\iota\to o}.(s\ y)$
  - theorem1: $\neg(A_\iota \in \emptyset)$
  - theorem1alt: $(A_\iota \in \emptyset) \Rightarrow \bot$

Shared Representation of Terms

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- Example: HOTPTP encoding

```
thf(emptyset,definition,
        (emptyset :=
                (^ [Z : $i] : $false))).

thf(element,definition,
        (element :=
                (^ [Y:$i, S:($i > $o)] : (S @ Y)))).

thf(theorem1,conjecture,
        (~ ((element @ A) @ emptyset))).

thf(theorem1alt,conjecture,
        (((element @ A) @ emptyset) => $false)).
```

Shared Representation of Terms

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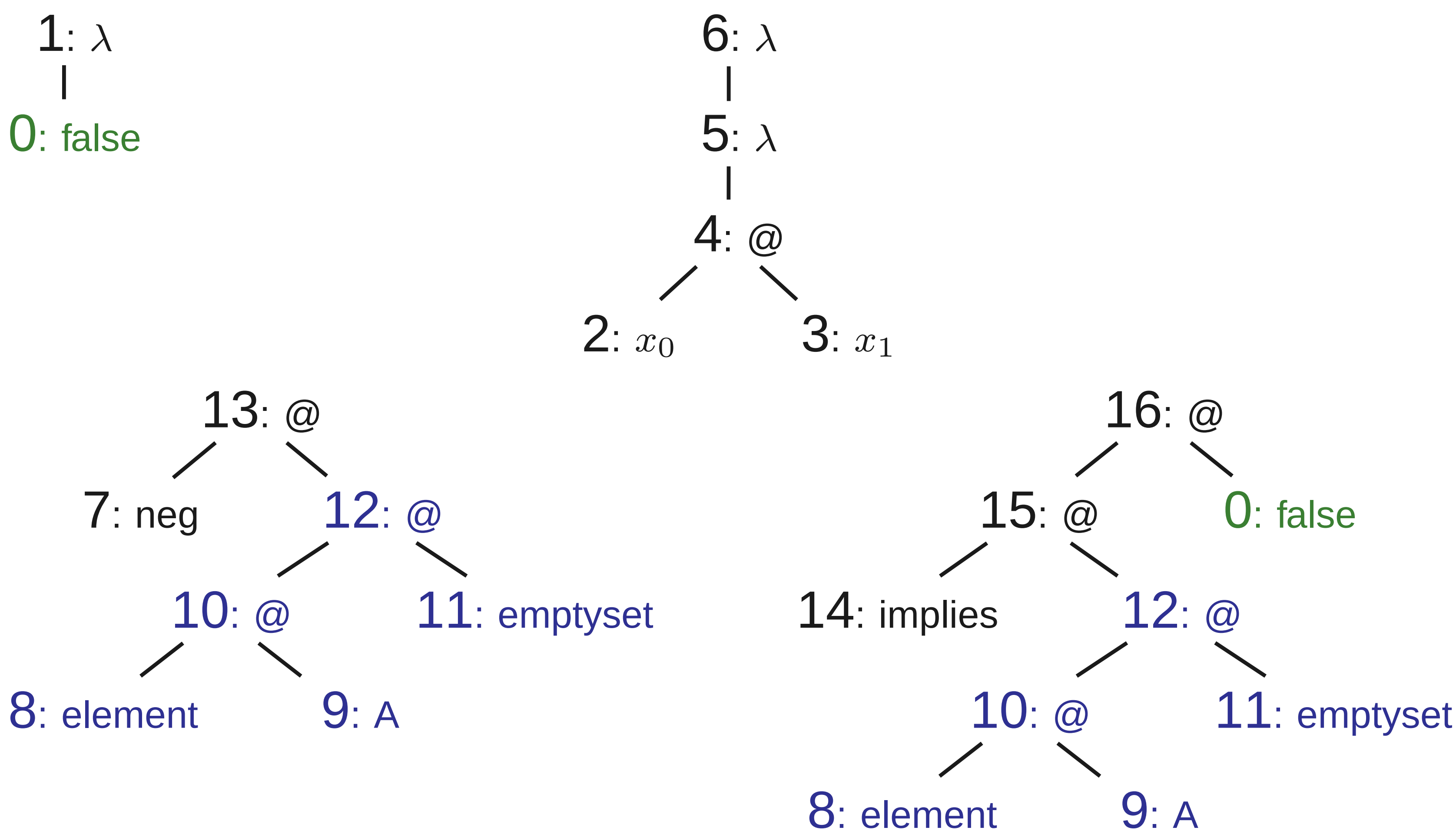
- Example: Graph representation of terms



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- Example: Representation as term sets

```
0: symbol false
1: abstr(i,0)
2: bound(i -> o,0)
3: bound(i,1)
4: appl(2,3)
5: abstr(i -> o,4)
6: abstr(i,5)
7: symbol neg
8: symbol element
9: symbol A
10: appl(8,9)
11: symbol emptyset
12: appl(10,11)
13: appl(7,12)
14: symbol implies
15: appl(14,12)
16: appl(15,0)
```

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- Terms are represented as sets of term nodes
- Example: Parsing returns pointers to this term set / index

```
emptyset: lambda [Z]: false
->index: 1

element: lambda [Y]: lambda [S]: S Y
->index: 6

theorem1: neg ((element A) emptyset)
->index: 13

theorem1alt: (implies ((element A) emptyset)) false
->index: 16
```

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- Example: Term set representation supported via hashtables

  - ht `abstr_with_scope` : $I\!N \rightarrow I\!N$:
    lookup abstractions with a given scope i
  - ht `appl_with_func` : $I\!N \rightarrow I\!N \rightarrow I\!N$:
    lookup application with a given function i and argument j
  - ht `appl_with_arg` : $I\!N \rightarrow I\!N \rightarrow I\!N$:
    lookup application with a given argument j and function i

Partial Syntax Trees (PST)

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- Allow for early detection of branches in a term's syntax tree with no occurrences of a specific symbol/subterm, since these branches are not represented
- Need to define position before we can give an example PST

Positions

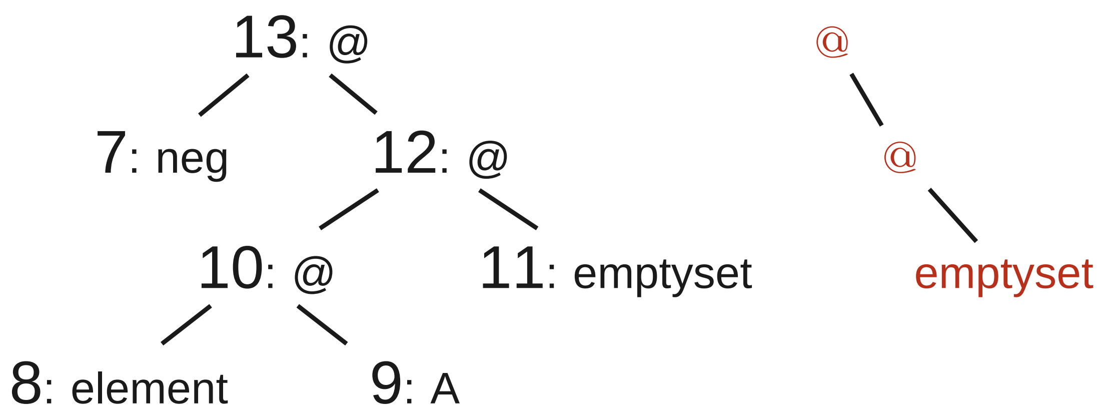
- Consider term $(\lambda.x_0)@(f@a)$:

$$
\begin{array}{lcl}
(\lambda.x_0)@(f@a) & : & [] \\
\lambda.x_0 & : & [\mathtt{func}] \\
x_0 & : & [\mathtt{func}; \mathtt{arg}] \\
f@a & : & [\mathtt{arg}] \\
f & : & [\mathtt{arg}; \mathtt{func}] \\
a & : & [\mathtt{arg}; \mathtt{arg}]
\end{array}
$$

Partial Syntax Trees and Replacement

Example

- PST for occurrences of symbol emptyset in theorem1




```
positiontable:
[arg; arg] :  emptyset
end.
```

$$p_{\text{emptyset-1}} = pst(\_, \_, p_{\text{emptyset-2}})$$
$$p_{\text{emptyset-2}} = pst(\_, \_, p_{\text{emptyset-3}})$$
$$p_{\text{emptyset-3}} = pst(\_, \_, \_) \qquad \text{– emptyset}$$

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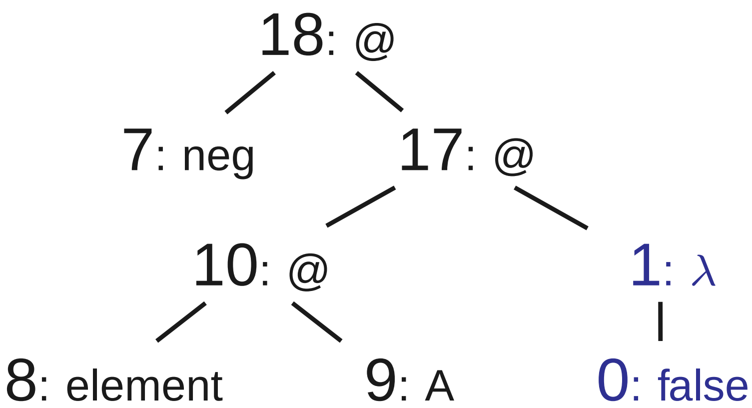
Example

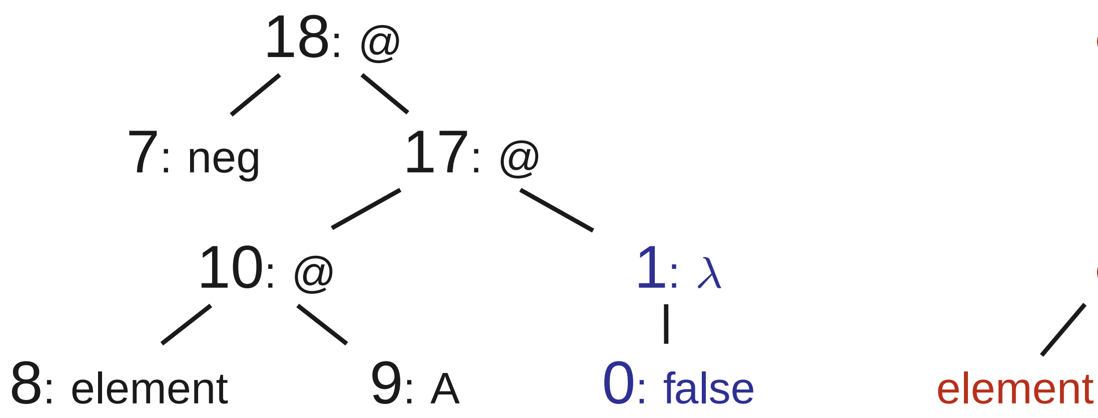
- Modified term after replacement



Partial Syntax Trees and Replacement

Example

- PST for occurrences of symbol element in term




```
positiontable:
[arg; func; func] :  element
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```

$$
\begin{aligned}
p_{\text{emptyset-1}} &= pst(\_, \_, p_{\text{emptyset-2}}) \\
p_{\text{emptyset-2}} &= pst(\_, p_{\text{emptyset-3}}, \_) \\
p_{\text{emptyset-3}} &= pst(\_, p_{\text{emptyset-4}}, \_) \\
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$$

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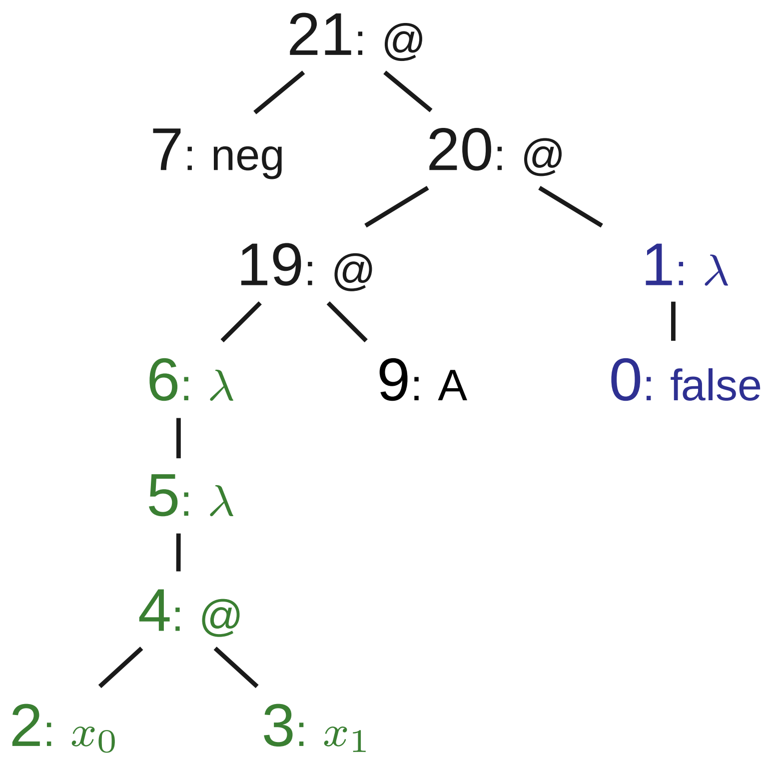

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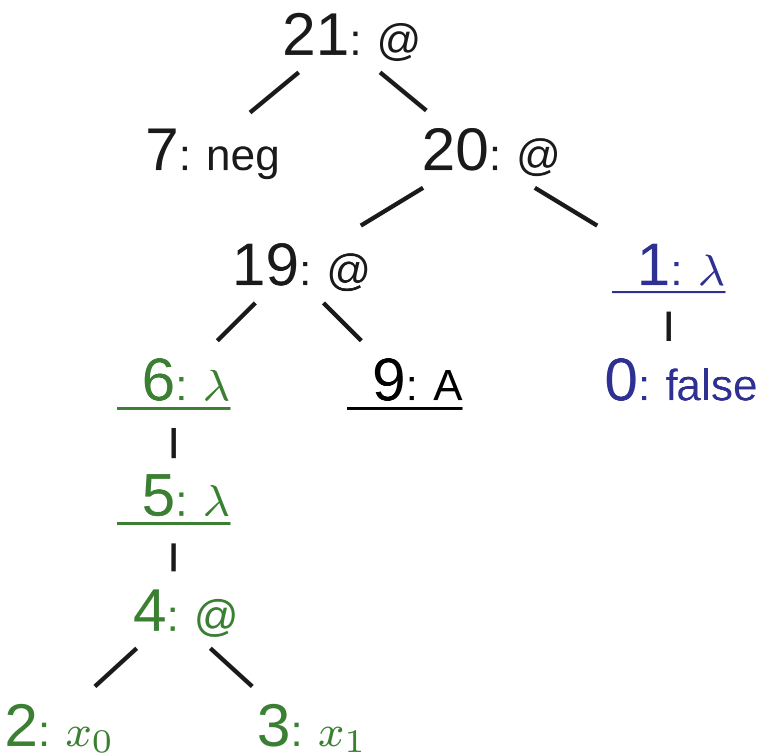
Example

- Use available information for normalisation



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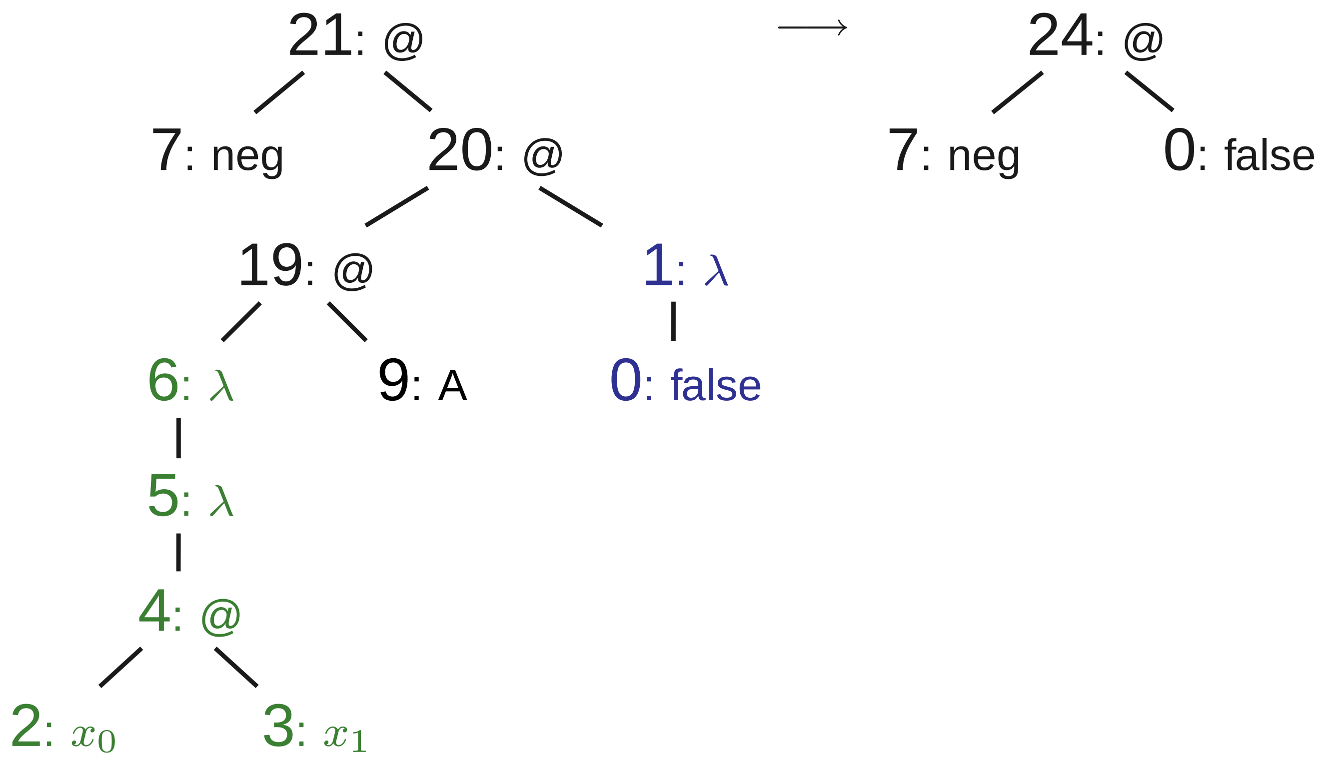
- We also index occurrences of bound variables

Partial Syntax Trees and Replacement

Example

- Normalisation (required before term goes to index again)



(LEO-II will later immediately say: Proof found!)

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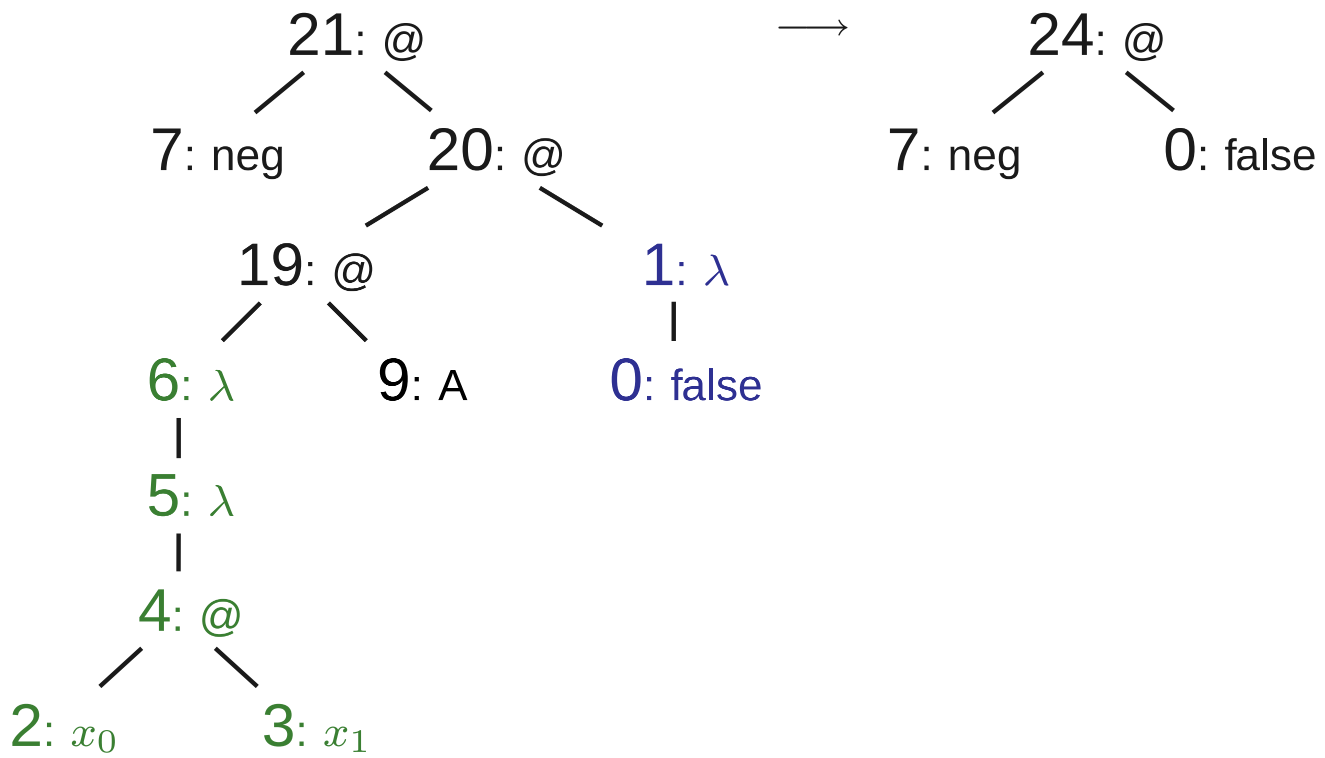

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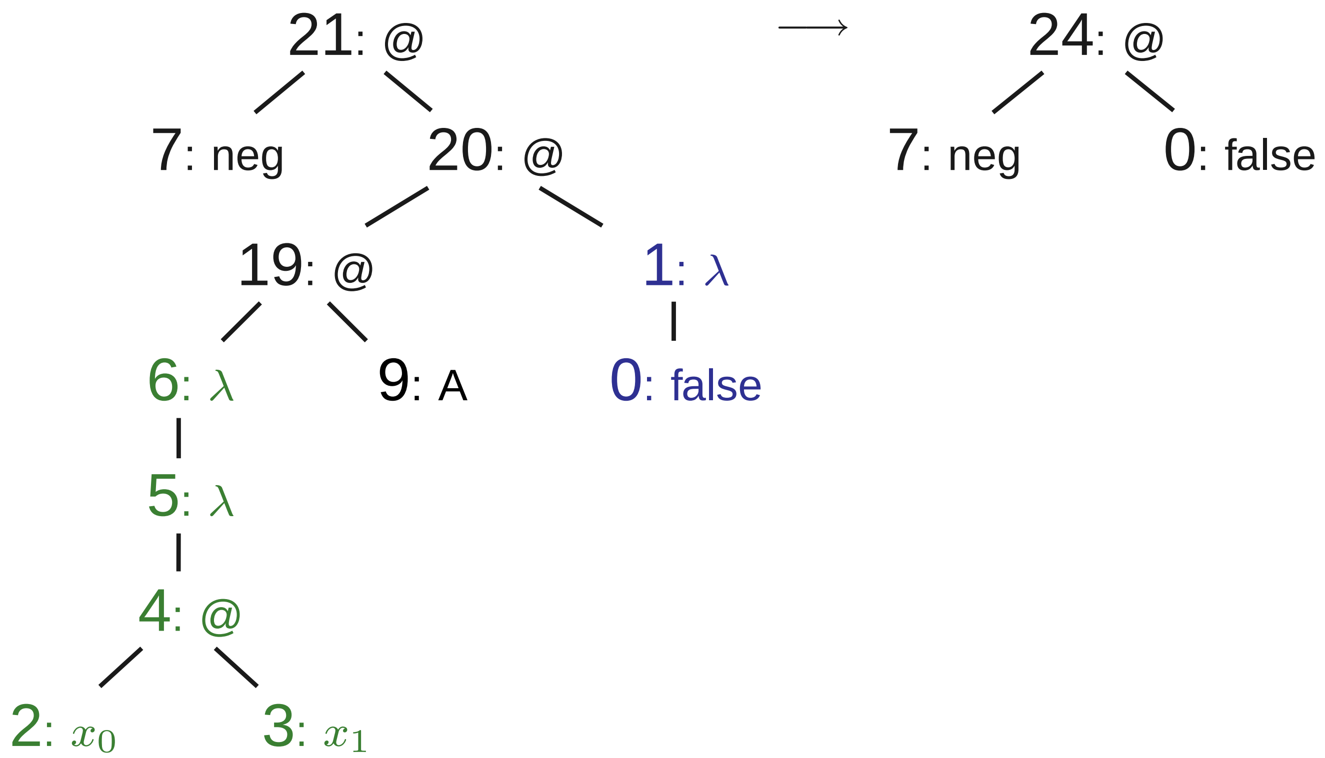

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- Provide also PSTs for bound variable occurrences

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Number of indexed terms	977
Number of created term nodes	11618
Average term size	54
Number of nodes with no parent nodes	904
Number of nodes with one parent node	9633
Number of nodes with two more more parent nodes	1083
Maximum number of parent nodes	2778 (symbol ∀)
Average number of parent nodes	1.68
Average number of terms a node occurs in	33.5
-"-(for symbols)	493.9
-"-(for nonprimitive term nodes)	24
Average PST/term size for symbol occurrences	0.21
Average PST/term size for bound variable occurrences	0.33
Average PST/term size for all term nodes	0.12

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- Occurs check for symbols, bound variables, and non primitive subterms in constant time is expected to have strong impact on crucial operations (e.g., replacement, substitution, global unfolding of definitions)
- One indicator for improvement for replacement operations is PST/term size rate: 0.21 (= speedup factor 5) for symbols and 0.33 (speedup factor 3) for bound variables

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  - Proper evaluation in LEO-II

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Conclusion

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