# Mathematical Domain Reasoning Tasks in Natural Language Tutorial Dialog on Proofs

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Joint work with: SFB378 DIALOG Project


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http://www.ags.uni-sb.de/~chris/dialog/

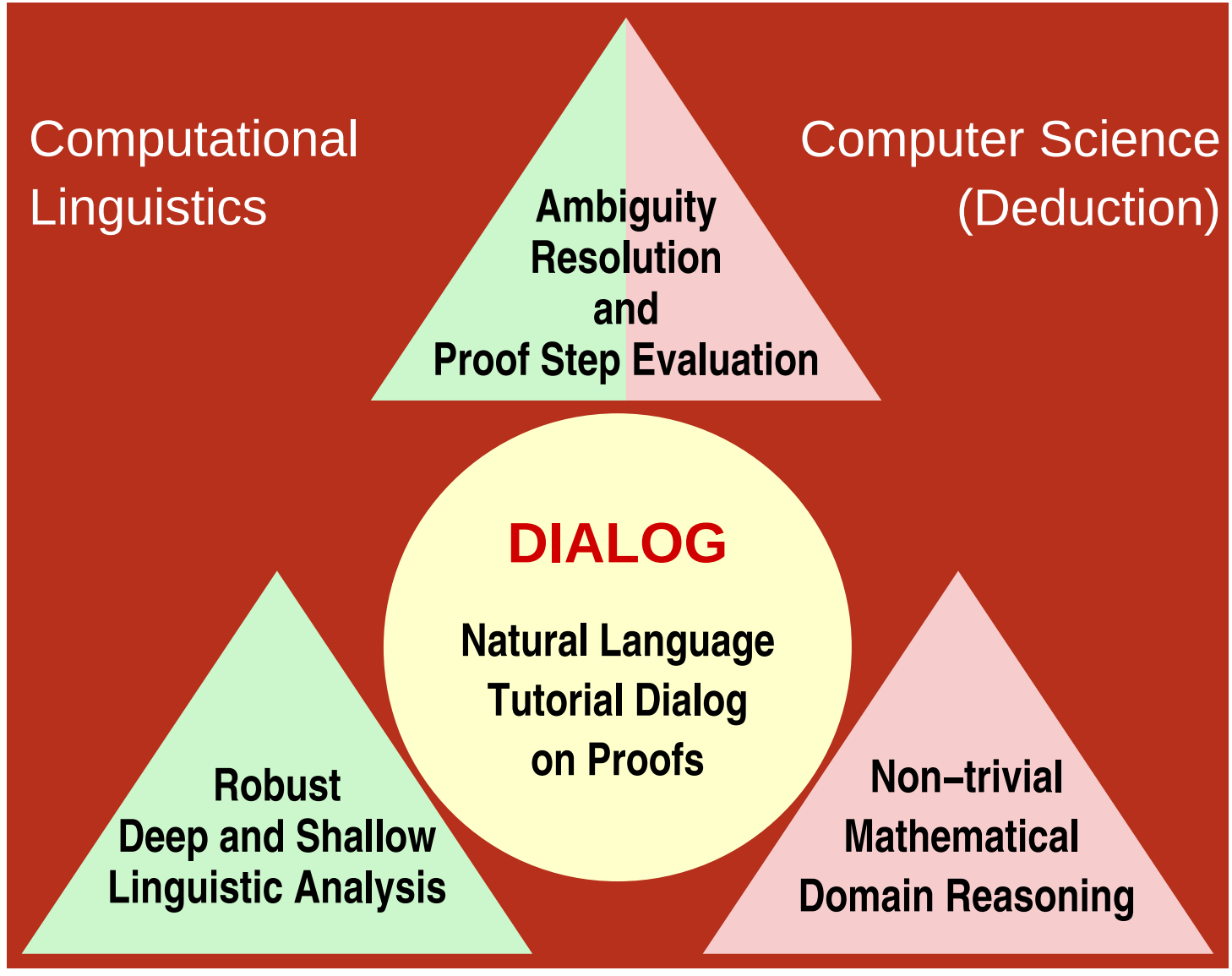
Dreamer Reunion, 1 August 2005, Edinburgh, UK

# Why this talk?

DIALOG Project

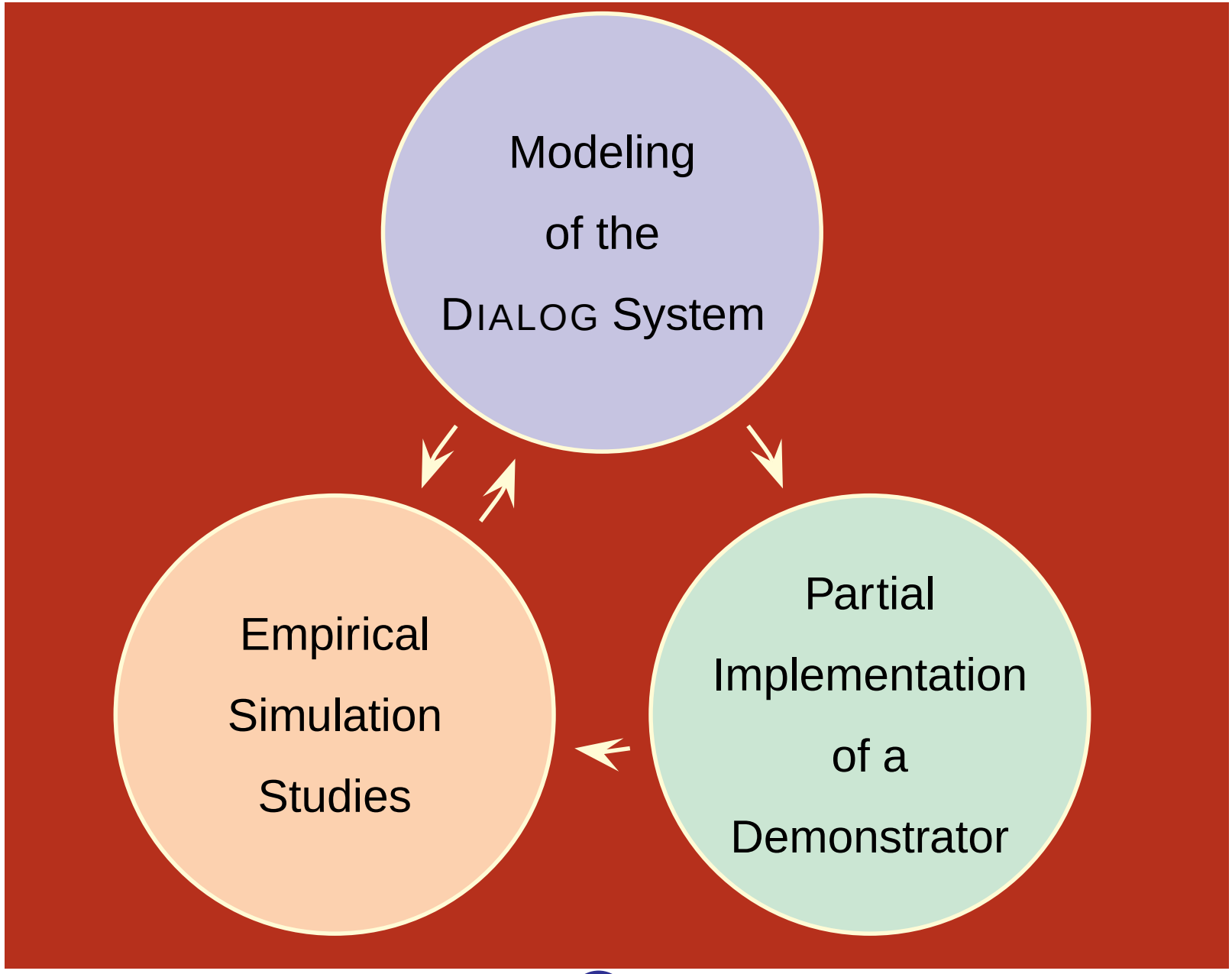
- ΩMEGA–λClam: many research links, mutual research visits, friendships
- not in this talk
  - ΩMEGA: basic research/development of an integrated mathematics assistance environment
- in this talk
  - DIALOG: NL based interaction with a mathematics assistance system
  - less well known to Dreamers
  - challenge between NL and AR
  - motivation for ‘abstract’-level reasoning: proof planning

# The DIALOG Project in the SFB 378



# Method: Progressive Refinement

DIALOG Project



# WOZ-Experiment → Own Corpus

DIALOG Project

## Subject (Student) Room:

Audio Recording

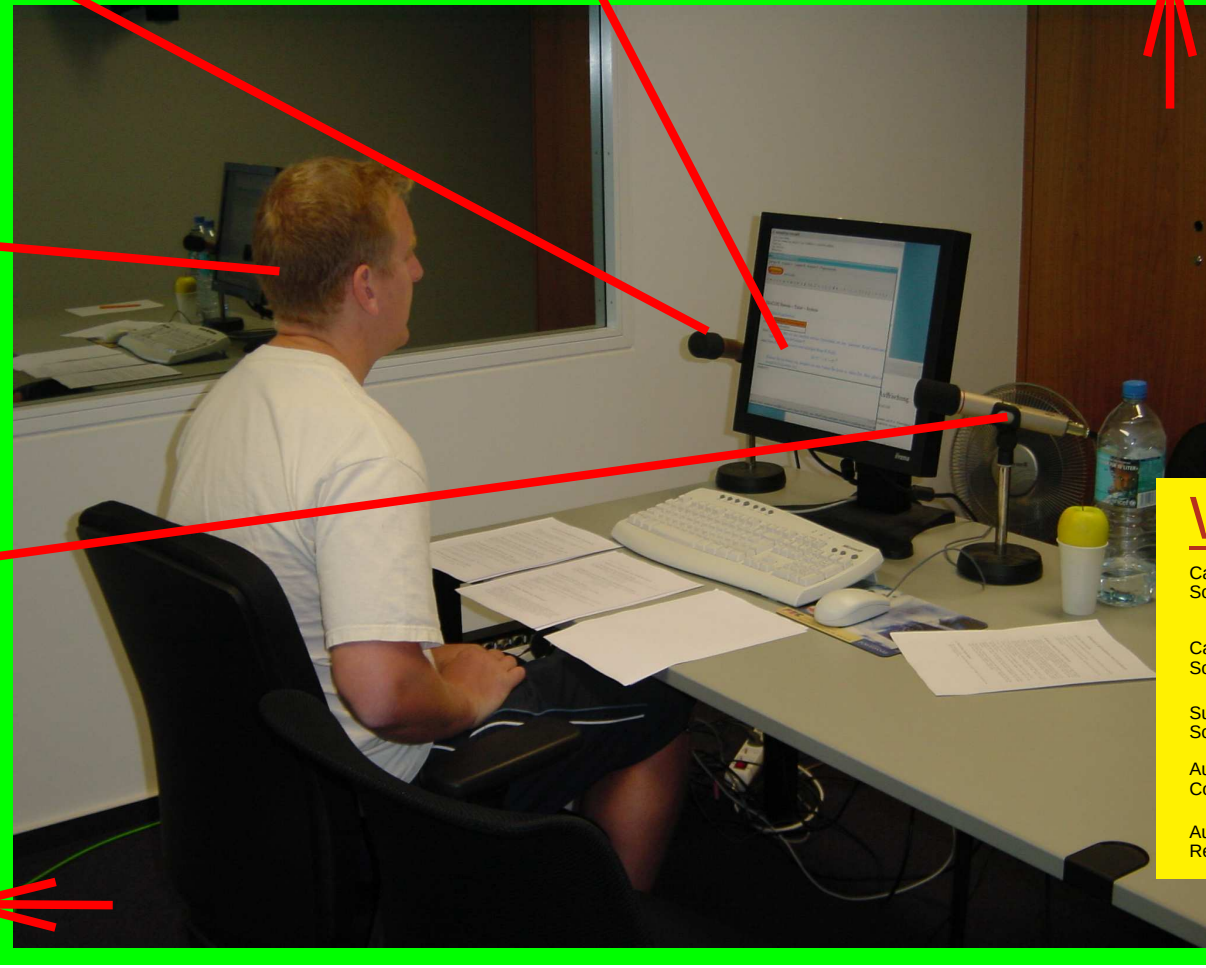
Subject GUI

Cam1

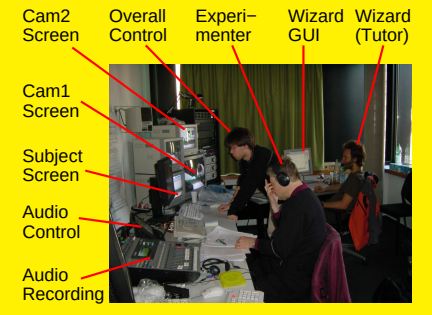
Subject

Audio Control

Cam2



## Wizard Room:



# WOZ-Experiment → Own Corpus

DIALOG Project

## Wizard (Tutor) Room:

Cam2 Screen

Overall Control

Experi−menter

Wizard GUI

Wizard (Tutor)

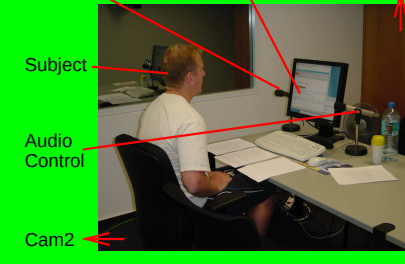
Cam1 Screen

Subject Screen

Audio Control

Audio Recording

## Subject Room:



# Corpus Example

DIALOG Project

T1: Bitte zeigen Sie: $K((A \cup B) \cap (C \cup D)) = (K(A) \cap K(B)) \cup (K(C) \cap K(D))$!
[Please show: $K((A \cup B) \cap (C \cup D)) = (K(A) \cap K(B)) \cup (K(C) \cap K(D))$!]

S1: nach deMorgan-Regel-2 ist $K((A \cup B) \cap (C \cup D)) = (K(A \cup B) \cup K(C \cup D))$.
[by deMorgan-Rule-2 $K((A \cup B) \cap (C \cup D)) = (K(A \cup B) \cup K(C \cup D))$ holds.]

T2: Das ist richtig!
[This is correct!]

S2: $K(A \cup B)$ ist laut deMorgan-1 $K(A) \cap K(B)$
[$K(A \cup B)$ is $K(A) \cap K(B)$ according to deMorgan-1]

T3: Das stimmt auch.
[That is also right.]

S3: und $K(C \cup D)$ ist ebenfalls laut deMorgan-1 $K(C) \cap K(D)$
[and $K(C \cup D)$ is also $K(C) \cap K(D)$ according to deMorgan-1]

. . .

Get corpus: `http://www.ags.uni-sb.de/~chris/dialog/`
Total figures 1. exp.: 66 dialogs / av. 12 turns / 1115 sentences

# Research Challenges

DIALOG Project

**Perspective of Mathematical Domain Reasoning (MDR):**

- Support for resolution of Ambiguities and Underspecification
- Proof Step Evaluation
  - Soundness: proof step verifiable by formal system?
  - Granularity: size/argumentative complexity of proof step?
  - Relevance: proof step needed/useful in achieving the goal?

**Perspective of NL Analysis:** [. . . not in this talk . . . ]

**Perspective of Dialog Management:** [. . . not in this talk . . . ]

**Perspective of Tutoring Proofs:** [. . . not in this talk . . . ]

# Research Challenges

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Logical vs Tutorial Dimension

**Perspective of NL Analysis:** [. . . not in this talk . . . ]

**Perspective of Dialog Management:** [. . . not in this talk . . . ]

**Perspective of Tutoring Proofs:** [. . . not in this talk . . . ]

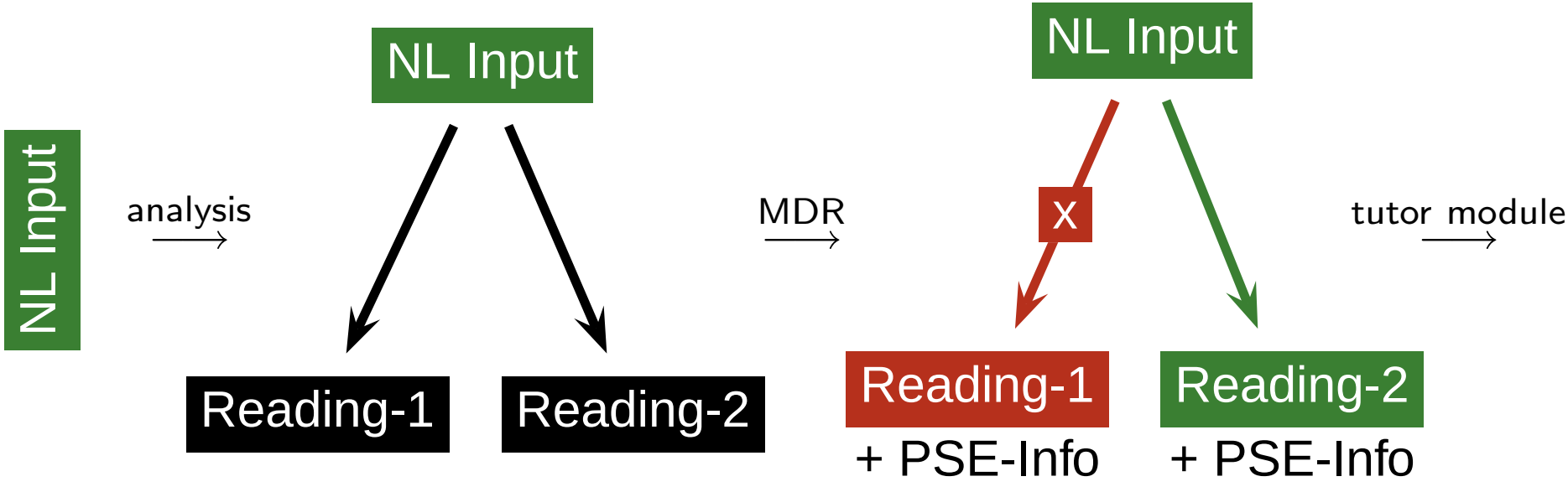
# Mathematical Domain Reasoning

DIALOG Project


— declarative abstract level sketches →

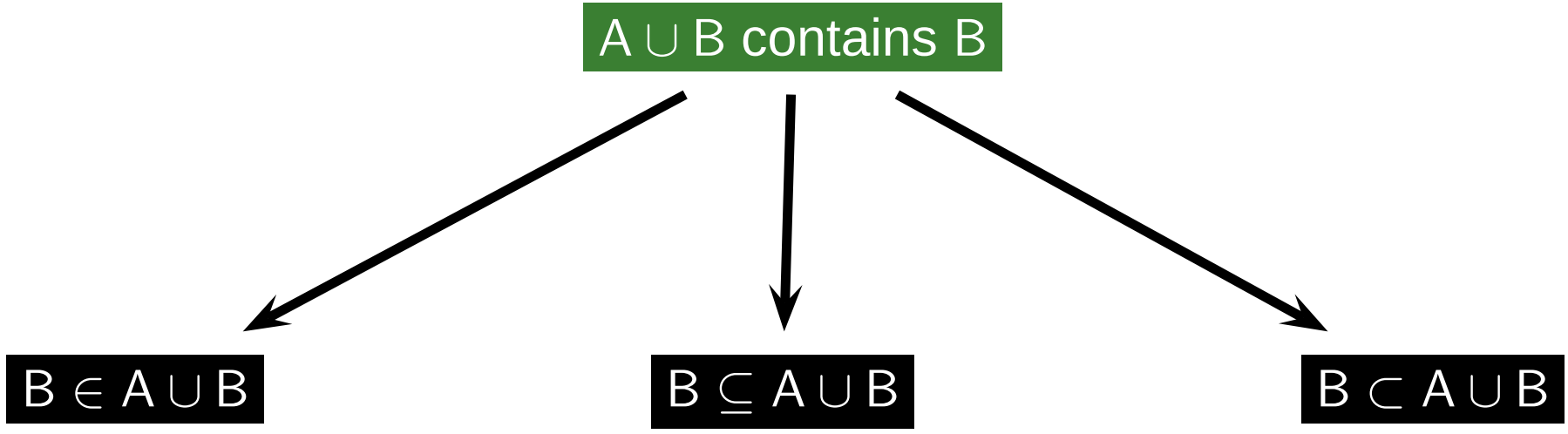
Communication Gap

← procedural calculus level proofs —



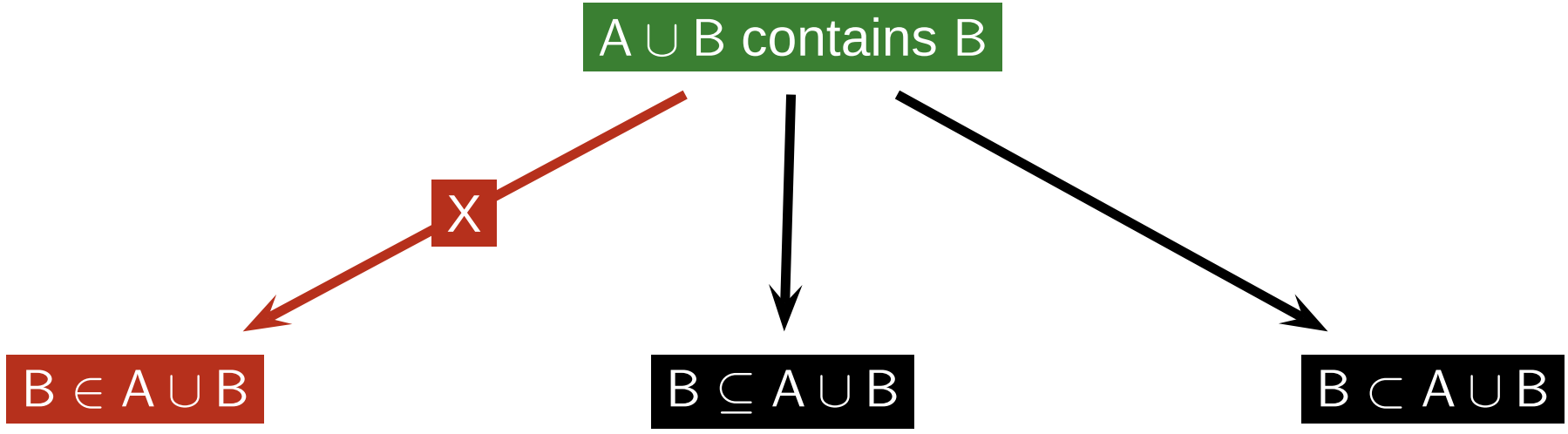
# Mathematical Domain Reasoning

DIALOG Project



# Mathematical Domain Reasoning

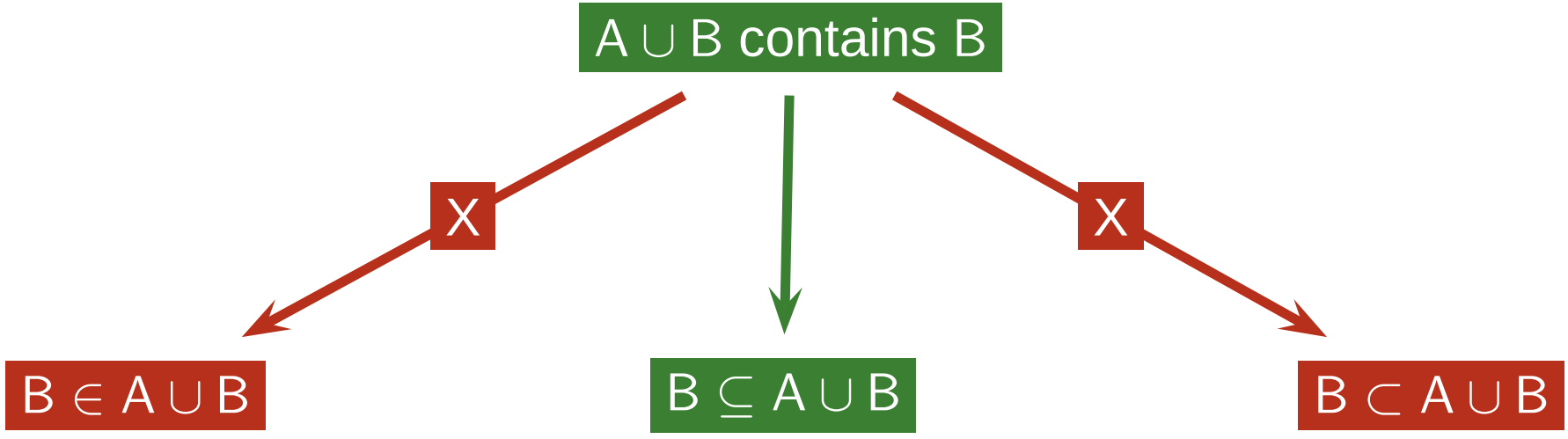
DIALOG Project




type checking

# Mathematical Domain Reasoning

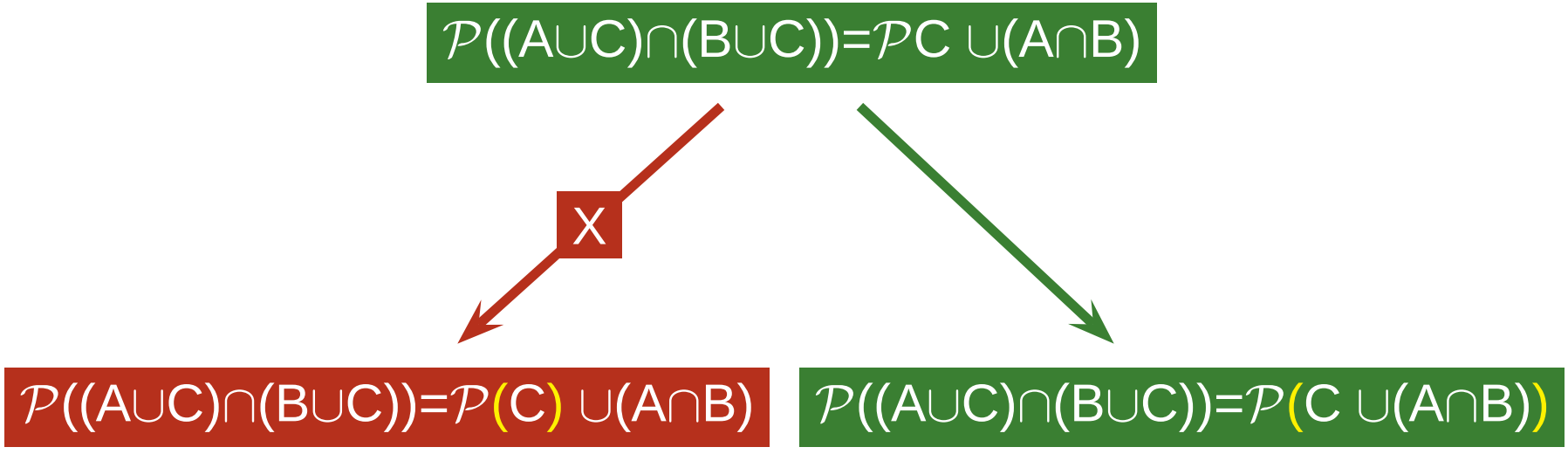
DIALOG Project




theorem proving

# Mathematical Domain Reasoning

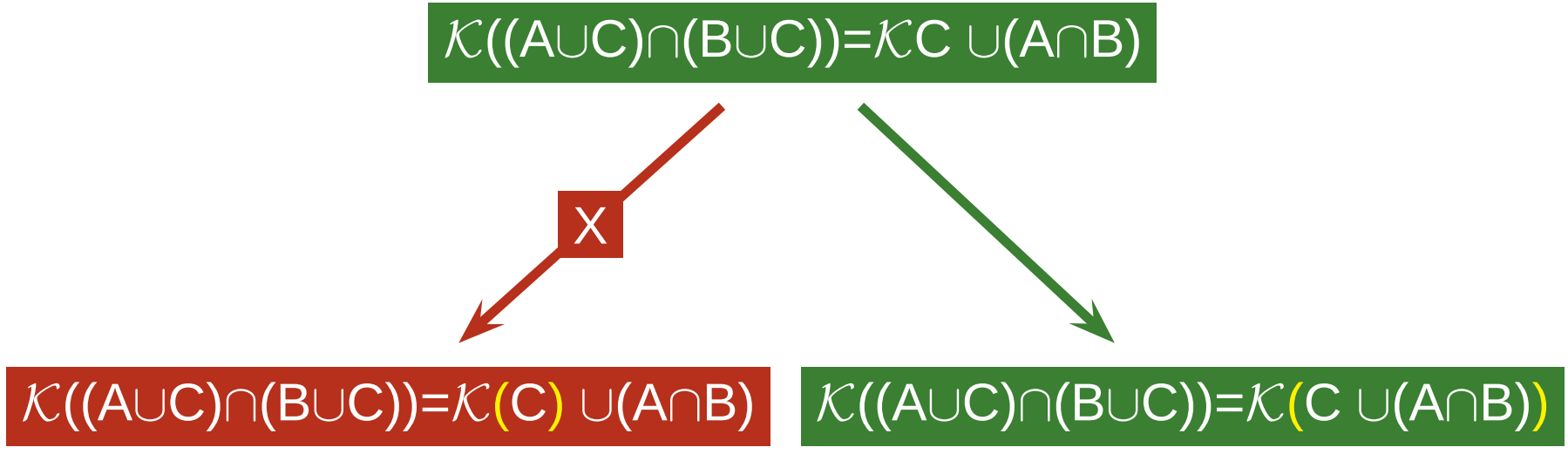
DIALOG Project




type checking

# Mathematical Domain Reasoning

DIALOG Project




theorem proving

# Proof Step Evaluation

DIALOG Project

| |
|---|
| (DM-1) . . . |
| (DM-2) . . . |
| ? |
| (G) . . . |

Given: (DM-1) $\overline{X \cup Y} = \overline{X} \cap \overline{Y}$

(DM-2) $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$

Task: Please show $\overline{(A \cup B) \cap (C \cup D)} = (\overline{A} \cap \overline{B}) \cup (\overline{C} \cap \overline{D})$

New: By deMorgan $\overline{(A \cup B) \cap (C \cup D)} = \overline{(A \cup B)} \cup \overline{(C \cup D)}$.

# Proof Step Evaluation

DIALOG Project

(DM-1) . . .
(DM-2) . . .
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(G) . . .

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New: By deMorgan $\overline{(A \cup B) \cap (C \cup D)} = \overline{(A \cup B)} \cup \overline{(C \cup D)}$.

(DM-1) . . .
(DM-2) . . .
(New) . . . ↓
?
(G) . . .

Soundness: yes
Granularity: 1x(DM-2)
Relevance: yes

(DM-1) . . .
(DM-2) . . .
?
(New) . . . ↑
(G) . . .

Soundness: yes
Granularity: 2x(DM-1)
Relevance: yes

# Proof Step Evaluation: How?

DIALOG Project

Discourse:

(1) $A \wedge B$
(2) $A \Rightarrow C$
(3) $C \Rightarrow D$
(4) $F \Rightarrow B$
?
(G) $D \vee E$

New:

We show E.

(1) . . .
(2) . . .
(3) . . .
(4) . . .
?
(G') E
(G) . . .

PSE:

Soundness

Granularity

Relevance

# Proof Step Evaluation: How?

DIALOG Project

Discourse:

(1) $A \wedge B$
(2) $A \Rightarrow C$
(3) $C \Rightarrow D$
(4) $F \Rightarrow B$
?
(G) $D \vee E$

New:

We show E.

(1) . . .
(2) . . .
(3) . . .
(4) . . .
?
(G') E
(G) . . .

PSE:

**Soundness**

- $(G') \vdash^{?} (G)$
- any proof

**Granularity**

**Relevance**

# Proof Step Evaluation: How?

DIALOG Project

Discourse:

(1) $A \wedge B$
(2) $A \Rightarrow C$
(3) $C \Rightarrow D$
(4) $F \Rightarrow B$
?
(G) $D \vee E$

New:

We show E.

(1) . . .
(2) . . .
(3) . . .
(4) . . .
?
(G') E
(G) . . .

PSE:

**Soundness**

- $(G') \vdash^{?} (G)$
- any proof

**Granularity**

- complexity($(G') \vdash^{?} (G)$)
- cognitively adequate proofs

**Relevance**

# Proof Step Evaluation: How?

DIALOG Project

Discourse:

(1) $A \wedge B$
(2) $A \Rightarrow C$
(3) $C \Rightarrow D$
(4) $F \Rightarrow B$
?
(G) $D \vee E$

New:

We show E.

(1) ...
(2) ...
(3) ...
(4) ...
?
(G') E
(G) ...

PSE:

**Soundness**

- $(G') \vdash^{?} (G)$
- any proof

**Granularity**

- complexity($(G') \vdash^{?} (G)$)
- cognitively adequate proofs

**Relevance**

- $(1), (2), (3), (4) \vdash^{?} (G')$
- detours?, shorter proofs?

# Claim

DIALOG Project

Granularity and Relevance

call for

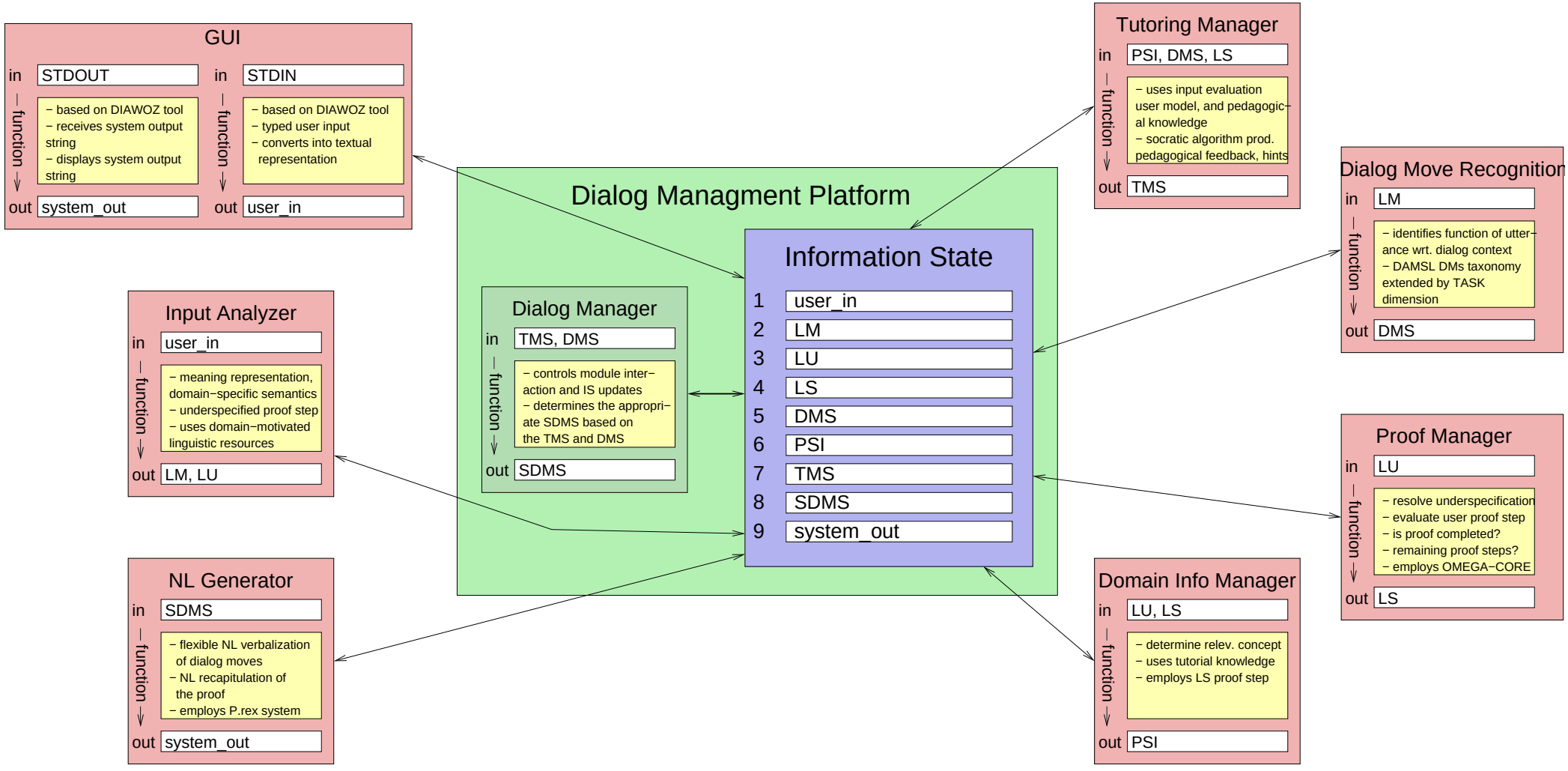
**cognitively adequate abstract level proofs**

+

**enumeration of (some) proof alternatives**

- One candidate: knowledge based proof planning [Bundy88]
- Original motivation: widen range of automatable maths
- New motivation: support for proof step evaluation

# Implementation: DIALOG Demonstrator



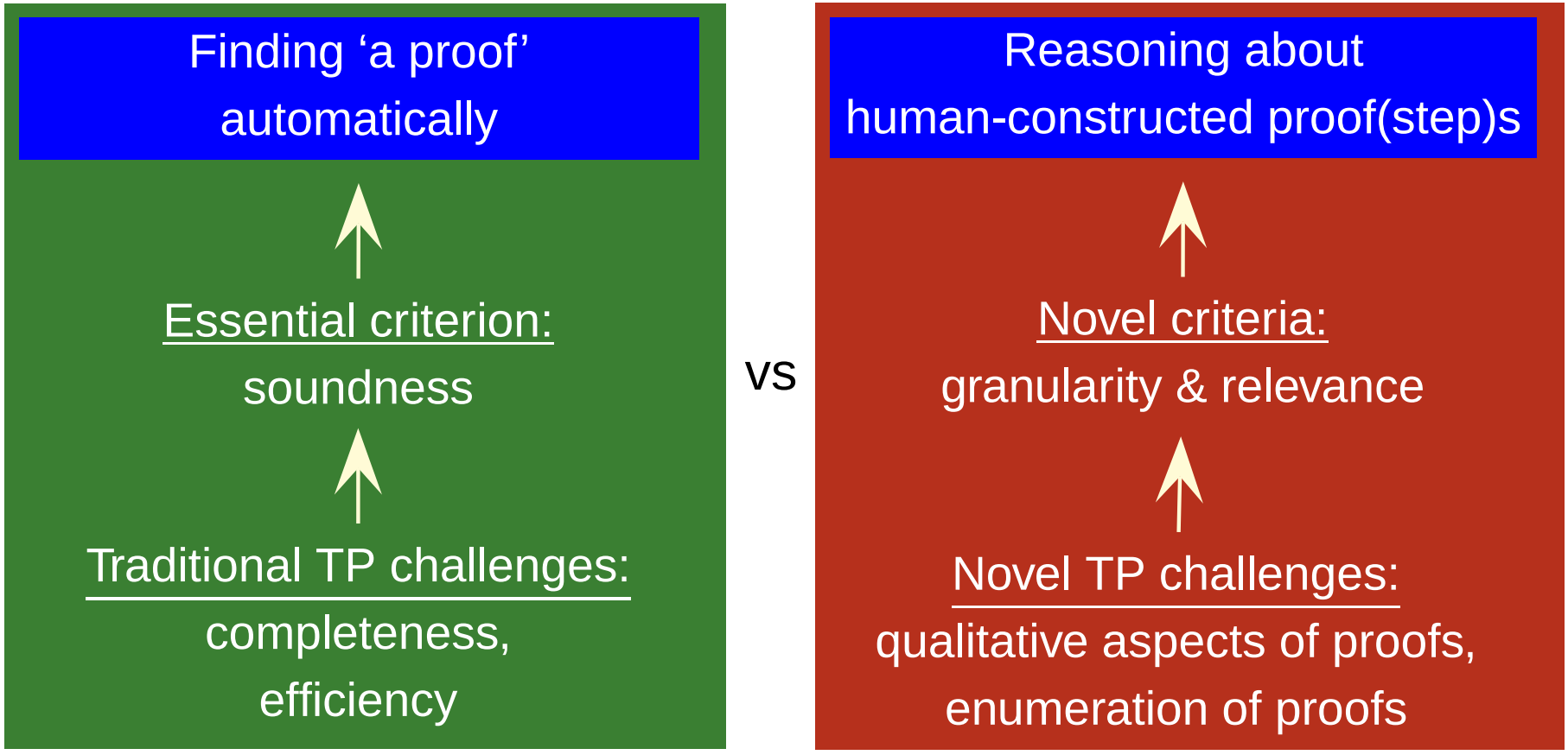
# Related Work

DIALOG Project

- Motivation: [Moore93] Flexible tutorial NL dialog supports active learning
- Closest related: [Zinn04] analyzes well structured text-book proofs; lots of interesting ongoing work
- NL analysis: shallow techniques and keyword spotting probably not suitable
- MDR: Comparison against 'golden standard solutions' [GreaserEtAl00] not suitable
- Dialog modeling: Autotutor [PersonEtAl00], Geometry Tutor [MatsudaVanLehn03], Trindi and Siridus [TraumLarsson03], Beetle [Zinn03]

# Conclusion

DIALOG Project




- Lots of ongoing work in all corners of the DIALOG Project