OMEGA
Resource-adaptive Proof Planning

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OMEGA Workplan

AP 1 INTEGRATION

AP 2 Wissensbasiertes Beweisplanen
AP 3 Agentenbasiertes Beweisen
AP 4 Evaluierung durch Fallstudien
AP 5 Lernen von Steuerungswissen
AP 6 Infrastruktur

Klassisches ATP

KI AGENTEN

PLANUNG KI
**APs 1 & 3: Planning and Agent-based TP**

### Deliberative Reasoning
- **Proof Planning**
  - **(PP)**

### Pro-active Reasoning
- **Agent-based Reasoning**
  - **(AR)**

#### Framework and case studies
- [PhD-Sorge-01]

#### AR as a means to combine and distribute complex reasoning procedures and external reasoners
- [Calculemus-01,KI-01]

#### Expansion of proof methods via AR
- [ARW-01]

#### Agent-based assertion retrieval
- [Festschrift-Siekmann-03,MKM-01]

#### Theory formation and PP
- [Calculemus-02]

#### AR and our new proof engine CORE
- [MSc-Huebner-03]

#### International recognition:
- [Invited-plenary-talk-Benzmüller-at-AISB-01]
AP 2: Knowledge-based Proof Planning

- Multi-strategy proof planning with MULTI [PhD-Meier-03]
- Randomization and restarts [ECP-01]
- Critical discussion and reflection
  - Proof planning and logic layer [IJCAR-WS-01]
  - Generality of proof planning [Book-35years-of-AutoMath-03]
- Mathematical representations vs. logical representations [Australien-AI-Conf-02,FLOC-02-WS]
- Semantic guidance in proof planning [TechRep-Bham-01]
- Proof planning for permutation group problems [CADE-03]
AP 4: Evaluation by Case Studies

- Exploration of residue classes
  [Journal-of-Symbolic-Computation-02,EUROCAST-01]

- Agent-based theorem proving in naive set theory
  [KI-01]

- Naturalness of proof construction, interactive island planning
  [Book-35years-of-Automath-03]

- Certifying solutions to permutation group problems
  [CADE-03]

→ Partial cooperation with University of Birmingham
AP 5: Learning

- Learning of proof methods
  [ECAI-02, CADE-WS-01]

- System LEARNOMATIC
  [CADE-02]

→ Cooperation with University of Birmingham
AP 6: Infrastructure

- New logic layer for OMEGA  [PhD-Autexier-03,UITP-03,MSc-Hübner-03]
- Proof Presentation  
  [PhD-Thesis-Fiedler,ICCS-01,NLDB-01,ICNLP-02,COLING-02,…]
- System P.rex  
  [IJCAI-01,IJCAR-01]
- MBASE: mathematical knowledge base  
  [Journal-of-Symbolic-Computation-01]
- MATHWEB-sb: mathematical software bus  
  [CADE-02,Calculemus-02,Calculemus-01]
- Completeness of OMEGAs base calculus  
  [Subm.-Journal-of-Symbolic-Logic]
Redesign of OMEGA Logic Layer

From procedural reasoning style to declarative reasoning style

- emphasis is on methods, tactics, rules
- emphasis is on abstract-level applications of assertions

- Impact on
  - Interactive theorem proving
  - Proof planning
  - Agent-based theorem proving

Autexier, Benzmüller, Siekmann, SFB Meeting, Dagstuhl, 2003
Motivating Example

Theorem Proving with OMEGA: $\sqrt{2}$ is irrational

[Book-35years-of-Automath-03]

**Theorem:** $\sqrt{2}$ is irrational.

**Proof:** (by contradiction)
Assume $\sqrt{2}$ is rational, that is, there exist natural numbers $m, n$ with no common divisor such that $\sqrt{2} = m/n$. Then $n\sqrt{2} = m$, and thus $2n^2 = m^2$. Hence $m^2$ is even and, since odd numbers square to odds, $m$ is even; say $m = 2k$. Then $2n^2 = (2k)^2 = 4k^2$, that is, $n^2 = 2k^2$. Thus, $n^2$ is even too, and so is $n$. That means that both $n$ and $m$ are even, contradicting the fact that they do not have a common divisor.
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- declarative style of argumentation: from assertions A and B follows C
- logic layer (e.g. a la ND- or Sequent-Calculus) treated implicit

⇒ mismatch between procedural style logic-level reasoning as employed in todays theorem provers and declarative assertion level reasoning as typical for mathematical texts
**Current OMEGA**

Proof Planning: heuristically guided automated chaining of proof methods
Interactive Theorem Proving: user chains methods (tactics/rules)

⇒ Problem: full abstraction from logic layer is not achieved  
[IJCAR-WS-01]
Traditional Interactive Theorem Proving

Step 0: \texttt{PROVE (SQRT2-NOT-RAT)}
Step 1: \texttt{DECLARE ((CONSTANTS (M NUM) (N NUM) (K NUM)))}
Step 2: \texttt{NOTI default default}
Step 3: \texttt{IMPORT-ASS (RAT-CRITERION)}
Step 4: \texttt{FORALLE-SORT default default ((SQRT 2)) default}
Step 5: \texttt{EXISTSE-SORT default default (N) default}
Step 6: \texttt{ANDE default default default}
Step 7: \texttt{EXISTSE-SORT (L7) default (M) default}
Step 8: \texttt{ANDE* (L8) (NIL)}
Step 9: \texttt{LEMMA default ((= (POWER M 2) (TIMES 2 (POWER N 2)))))}
Step 10: \texttt{BY-COMPUTATION (L13) ((L11))}
Step 11: \texttt{LEMMA (L9) ((EVENP (POWER M 2))}
Step 12: \texttt{DEFN-CONTRACT default default default}
Step 13: \texttt{LEMMA (L9) ((INT (POWER N 2)))}
Step 14: \texttt{WELLSORTED default default default}
Step 15: \texttt{EXISTSI-SORT (L15) ((POWER N 2)) (L13) (L16) default}
Step 16: \texttt{IMPORT-ASS (SQUARE-EVEN)}
Step 17: \texttt{ASSERT ((EVENP M)) ((SQUARE-EVEN L10 L14)) (NIL)}
Step 18: \texttt{DEFN-EXPAND (L17) default default}
Step 19: \texttt{EXISTSE-SORT default default (K) default}
Step 20: \texttt{ANDE (L19) default default default}
Step 21: \texttt{LEMMA default ((= (POWER N 2) (TIMES 2 (POWER K 2)))))}
Step 22: \texttt{BY-COMPUTATION (L23) ((L13 L22))}
Step 23: ...
Traditional Island Planning

Network of proof ‘islands’

- Islands structure the proof in natural form
- Islands provide no argument for soundness
- Verification: expansion of island steps (automated, interactive, recursive island approach)

\[
\frac{2 \times n^2 = m^2}{\text{Island}} \quad \frac{\text{Even}(m^2)}{\text{Island}} \quad \frac{\text{Even}(m)}{\text{Island}}
\]

⇒ declarative style

Not solved by Island Approach:

Constructive assertion reasoning which still leaves logic level implicit

Autexier, Benzmüller, Siekmann, SFB Meeting, Dagstuhl, 2003
Re-design of OMEGA

Interactive Theorem Proving
Proof Planning
Agent-based Reasoning

Task Level

Logic Engine CORE

- supporting flexible assertion level reasoning
- complete hiding of logic layer

[MSc-Hübner-03]
[PhD-Autexier-03]

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Future of OMEGA

- Ongoing: Integration of CORE into OMEGA
- Resource Adaptive Agentification of
  - Inference Rules and Assertions
  - Tactics and Proof Methods
  - External Services
    - FO-ATPs and HO-ATPs
    - Computer Algebra Systems
    - Mathematical Knowledge Bases
- Agentification of the User
- Resource Adaptive Proof Planning with Agents